§70. Algorithm for the Rotational Transform of the LHD Magnetic Field

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Chaotic field line region, produced by high magnetic shear and nonaxisymmetry of the magnetic field, is present outside the last closed flux surface (LCFS) of the LHD. There exits many magnetic islands of various sizes both inside and outside the LCFS. It is necessary to examine the size, the rotational transform and the distribution of the magnetic islands because it seems that these magnetic islands get involved very mach in the stability and the transport process of the LHD plasma.

Under the rotating helical coordinate system, the lines force $\boldsymbol{r}(\ell)$, the trace of magnetic axis $\boldsymbol{r}_{\rm ax}(\ell)$ and the rotational transform $\iota/2\pi$ are given by

$$\boldsymbol{r}(\ell) = \Big(X(\ell)\,, Y(\ell)\,, \phi(\ell) \Big)\,, \tag{1}$$

$$\boldsymbol{r}_{\rm ax}(\ell) = \Big(X_{\rm ax}(\ell)\,, Y_{\rm ax}(\ell)\,, \phi_{\rm ax}(\ell) \Big)\,, \tag{2}$$

$$\frac{\iota}{2\pi} = \lim_{\ell\to\infty} \frac{\theta(\ell)-\theta(0)}{\phi(\ell)-\phi(0)} + p\,, \tag{3}$$

where $p\,(=5)$ is the helical pith parameter of the magnetic field and ℓ is the length along the magnetic field lines. The poloidal angle around the magnetic axis is given by

$$\tan\{\theta(\ell)\} = \frac{Y(\ell)-Y_{\rm ax}(\ell)}{X(\ell)-X_{\rm ax}(\ell)}\,.$$

In the LHD magnetic field, very accurate approximate formula for the magnetic axis is possible to be used.[1)]

The expression (3) of the rotational transform $\iota/2\pi$ gives same values on lines of force on a magnetic island. Then a plot of the rotational transform gives the information of the size and the splitting of a island.

When we trace numerically a lines of force in the range

$$0 \le \ell \le L_{\max}\,,$$

the toroidal and the poloidal angles, $\phi(\ell)$ and $\theta(\ell)$ behave as follows,

$$\theta(\ell) = a\cdot\ell + g_1(\ell)\,, \tag{4}$$

$$\phi(\ell) = b\cdot\ell + h_1(\ell)\,, \tag{5}$$

where a and b are some constants and $g_1(\ell)$ and $h_1(\ell)$ are fluctuating parts. The unknown fluctuating parts $g_1(\ell)$ and $h_1(\ell)$ can be dropped by an average filter in starting position ℓ_0 with a fixed length L_0.

$$\begin{aligned}\langle\theta(L+\ell_0)-\theta(\ell_0)\rangle_{\ell_0} &= a\cdot L + \langle g_1(L+\ell_0)-g_1(\ell_0)\rangle_{\ell_0}\\ &\simeq a\cdot L\,,\end{aligned}$$

$$\begin{aligned}\langle\phi(L+\ell_0)-\phi(\ell_0)\rangle_{\ell_0} &= b\cdot L + \langle h_1(L+\ell_0)-h_1(\ell_0)\rangle_{\ell_0}\\ &\simeq b\cdot L\,.\end{aligned}$$

For the average filter, a window function $W(\ell_0)$ are introduced. The rotational transform $\iota/2\pi$ can be calculated with a high degree of accuracy by

$$\frac{\iota}{2\pi} = \frac{a}{b} = \frac{\int_0^{L_0}\{\theta(L+\ell_0)-\theta(\ell_0)\}\,W(\ell_0)\mathrm{d}\ell_0}{\int_0^{L_0}\{\phi(L+\ell_0)-\phi(\ell_0)\}\,W(\ell_0)\mathrm{d}\ell_0}\,. \tag{6}$$

We use a following window function.

$$\begin{aligned}W(\ell_0) &= \frac{\tanh\xi_+ - \tanh\xi_-}{2}\,, \qquad (7)\\ \xi_+ &\equiv \frac{\ell_0-2d}{d} + \left(\frac{\ell_0-2d}{d}\right)^3,\\ \xi_- &\equiv \frac{\ell_0-L_0+2d}{d} + \left(\frac{\ell_0-L_0+2d}{d}\right)^3,\end{aligned}$$

$$L_0 = L = \frac{L_{\max}}{2}\,,\quad d = \frac{5}{440}L_{\max}\,.$$

This algorithm has been built into the visualization system[1)] for lines of force in the LHD. A numerical example of a plot of rotational transforms is shown in Fig.1.

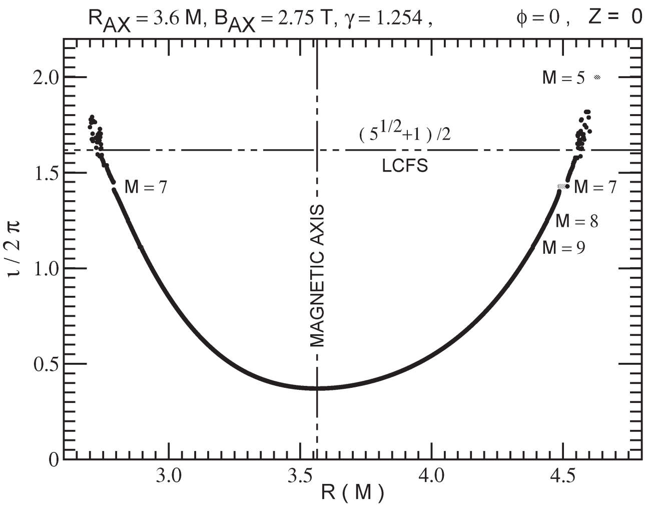

Fig.1 Plots of rotational transforms of the LHD magnetic field with $R_{\rm ax} = 3.6\,$m. The rotational transform of the LCFS and the line of the golden mean $(=(\sqrt{5}+1)/2)$, is shown. A lot of islands exist in and outside of the LCFS. The poloidal mode number M of corresponding island are also shown.

Reference

1) Watanabe, T., Yoshida, M., Masuzaki, S., Emoto, M., and Nagayama, Y., Reasearch Report NIFS-TECH Series-14, Sept, 2006).