§27. Possible Global Magneto-Fluid Structure of the Stellar Convection Zone

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Structures of the magnetic field and velocity in stars are discussed based on the mean field MHD equations [1]. A special case is presented, where the solution is constructed by the Beltrami solution in the stellar convection zone with the symmetry in the azimuthal direction. Magnetic field lines form concentric toroidal magnetic surfaces. The cross-helicity dynamo mechanism induces a mean flow of plasmas. The structure of this driven flow is also shown to constitute toroidal surfaces. Considering the symmetry and the relation of this toroidal magnetic structure with the polarity, it is shown that the latitudinal component of this flow is pole-ward in the northern as well as southern hemispheres. This gives an insight into the role of magnetic field for the meridional flow.

The basic equations we start with are the mean-field MHD equations [2]. In order to obtain the possible solution with a global structure, we consider the case where the turbulent viscosity is large enough in comparison with the molecular viscosity, and the resistive diffusion of the magnetic field is neglected. For the transparency of the argument, the case of constant dynamo coefficients of (α, β, γ) is considered. The basic equation takes the form:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \mathrm{rot}\left(u \times \boldsymbol{B} + \alpha \boldsymbol{B} - \beta \boldsymbol{J} + \gamma(\omega + 2\omega_F)\right).$$

We are interested in a special case of $u // B$, and search for the solution employing the approximation of dropping $\boldsymbol{u} \times \boldsymbol{B}$. From the relation $\boldsymbol{J} = \nabla \times \boldsymbol{B}$, the solutions that satisfy

$$\nabla \times \boldsymbol{B} - \frac{\alpha}{\lambda_u \beta} \boldsymbol{B} = \frac{2\gamma}{\lambda_u \beta} \omega_F$$

where $\lambda_u = \left(1 - \gamma^2 \beta^{-2}\right)$ and $\boldsymbol{u} = \gamma \boldsymbol{B}/\beta$ are searched for.

General solutions for B_ζ, and B_θ are formally written in terms of the n-th order spherical Bessel functions (j_n and n_n) and the associated Legendre function of $P_n^{(m)}$ as

$$B_\zeta(r, \theta) = \sum_n \left(a_n j_n(y) + b_n n_n(y)\right) P_n^{(1)}(\cos\theta)$$

$$B_\theta(r, \theta) = -\sum_n \left(a_n\left(\frac{n+1}{y} j_n(y) - j_{n+1}(y)\right) + b_n\left(\frac{n+1}{y} n_n(y) - n_{n+1}(y)\right)\right) P_n^{(1)}(\cos\theta)$$

$$B_r(r, \theta) = \sum_n \left(a_n j_n(y) + b_n n_n(y)\right) y^{-1} \left(\frac{2\cos\theta}{\sin\theta} P_n^{(1)}(\cos\theta) - P_n^{(2)}(\cos\theta)\right)$$

where $y = \alpha r / \lambda_u \beta$.

Noting the fact that the turbulent resistivity β is a scalar quantity but the turbulent helicity α and turbulent cross-helicity γ are pseudo-scalar, the ratios α/β and γ/β change the sign under the mirror transformation. Thus, B_ζ changes sign between the upper and lower hemisphere, but the poloidal magnetic field has the same sign. Figure 1 illustrates the polarity of the magnetic field on the magnetic surface. The flow velocity, which is induced by the cross-helicity dynamo effect, is parallel to the magnetic field line. The flow velocity is indicated in Fig.1. The toroidal flow is symmetric across the equatorial plane, but the latitudinal flow is anti-symmetric. That is, the poloidal flow (latitudinal flow) directs to the north pole in the northern hemisphere, and to the south pole in the southern hemisphere. The meridional flows are pole-ward in both hemispheres. This solution naturally reveals the meridional flow. The direction of the magnetic field changes associated with the magnetic cycle.

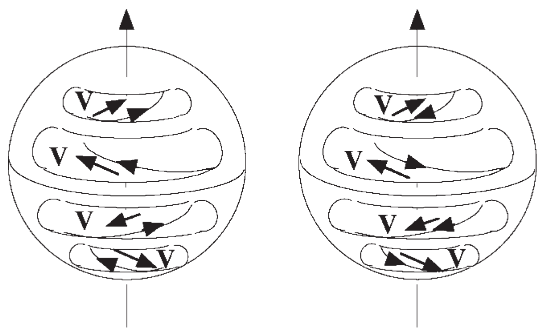

Fig.1 Schematic drawing of the toroidal magnetic and flow structures. Thin line with arrows indicate the magnetic field and the thick arrows indicate the flow velocity. Cases (a) and (b) show the two time slices when the sign of the magnetic field is reversed, owing to the solar magnetic cycle. In both cases, the poloidal flow which is driven by the cross-helicity dynamo is pole-ward.

References

[1] S.-I. Itoh, K. Itoh, P. H. Diamond, A. Yoshizawa: PASJ **58** (2006) 605
[2] A. Yoshizawa, S.-I. Itoh, & K. Itoh: Plasma and Fluid Turbulence: Theory and Modelling (Bristol: Institute of Physics, 2003)