§38. Simulation Study of Scale Hierarchy in Plasma Turbulence

Yoshida, Z. (Grad. Sch. Frontier Sci., Univ. Tokyo)

Homogeneous and isotropic turbulence in plasmas was studied numerically on the basis of the magnetohydrodynamics (MHD) model. Especially the effect of the Hall term on the energy spectra was paid attention.

The Hall term is included as a singular perturbation (a small parameter multiplied by a higher-order derivative term) in the Ohm's law in the MHD equations. If a small-scale structure in space is created, the higher-order derivative term becomes large, and then the Hall term cannot be neglected compared with the other terms in the Ohm's law even if the Hall parameter (or ion skin depth) itself is small. Therefore, the Hall term is expected to affect the energy spectra in the high wave number or short wave length regime.

As a theoretical study, we have developed the scaling law of the energy spectra for the MHD equations including the Hall term, which is an extension of the Kolmogorov's scaling for neutral fluids. Similar to the Kolmogorov's argument, we assumed that the energy spectra E(k) has an inertial range in which the properties of the turbulence is determined only by the wave number and the energy transfer rate, and that the energy transfer rate does not depend on the wave number (the energy does not accumulate at a particular wave number, and is transferred from small to high wave number regime). Here, the important fact is that the plasma has more degrees of freedom for the energy transfer compared to the neutral fluids where the energy transfer is only by the vortices. This leads to the scaling of the energy spectra different from the Kolmogorov's $E(k)\propto k^{-5/3}$ scaling. If we take account of the energy transfer due to the Hall term, we obtain $E(k)\propto k^{-7/3}$ in the high wave number regime. In the small wave number region, on the other hand, the Hall term does not play a dominant role, and we obtain $E(k)\propto k^{-5/3}$. Those two regions may be separated around the wave number corresponding to the ion skin depth.

According to the above theoretical prediction, we have developed a shell model for the Hall-MHD equations, and simulated homogeneous and isotropic turbulence. Figure 1 shows the energy spectra of the magnetic field. We see that the scaling changes from the $k^{-5/3}$ law to the $k^{-7/3}$ law around $k=10^2$, which corresponds to the ion skin depth assumed in the simulation. The energy spectrum does not show drastic decrease even for $k=10^6$, which may be because the dissipation term is comparable to the Hall term. On the contrary, the energy spectrum for the velocity field does not show such features; the $k^{-5/3}$ law was obtained in the entire inertial range, and the energy spectrum showed the drastic decrease around $k=10^6$.

Reference: D. Hori, "Formation of the Scale Hierarchy in the Plasma Turbulence", PhD Thesis, Dept. Adv. Energy, Grad. Sch. Frontier Sci., Univ. Tokyo, Japan (2007).

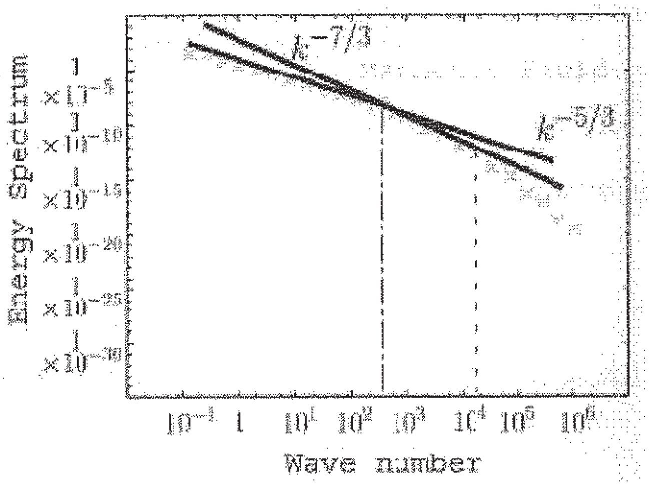

Fig. 1: Energy spectrum of the magnetic field in a homogeneous and isotropic turbulence obtained by a numerical simulation of the shell model for the Hall MHD equations.