§5. Analytic Study of Magnetic Islands Induced by Resistive Interchange Mode in Heliotron Plasma

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Previously, we numerically studied the nonlinear evolution of a resistive interchange mode with single helicity in a straight heliotron configuration. By using the NORM code[1)] based on the reduced MHD equations, we found generation of magnetic islands in the nonlinear saturation phase[2)]. A remarkable property of the islands is that the number of the island on a poloidal cross section is twice of the poloidal mode number of the dominant mode, as shown in Fig.1. In this study, we analytically obtain such an island in the cylindrical geometry.

We assume that the perturbations of the stream function Φ, the poloidal flux Ψ and the pressure P can be expanded as follows:

$$\Phi = \epsilon\Phi_1 \sin(m\theta - n\zeta) + \epsilon^2\Phi_2 \sin(2m\theta - 2n\zeta) \quad (1)$$

$$\Psi = \epsilon\Psi_1 \cos(m\theta - n\zeta) + \epsilon^2[\Psi_0 + \Psi_2 \cos(2m\theta - 2n\zeta)] \quad (2)$$

$$P = \epsilon P_1 \cos(m\theta - n\zeta) + \epsilon^2[P_0 + P_2 \cos(2m\theta - 2n\zeta)]. \quad (3)$$

Here, ϵ is a perturbation parameter. We substitute these equations into the reduced MHD equations and solve the equation for each order of ϵ. As for Φ_1, we utilize an expression of the linear eigenfunction for large resistivity given by

$$\Phi = r^{-1/2}\Phi_0 \exp\left[-\frac{(r - r_s - \delta)^2}{2W^2}\right], \quad (4)$$

where W, r_s and δ denote the half width of the function, the position of the resonant surface and the shift of the peak position from the surface, respectively. By utilizing the numerical results for these parameters, we obtain the functions of Eqs.(1)-(3). Figure 2 shows the comparison of the stream function between the numerical and the analytic results. Good agreement is obtained. Figure 3 shows the comparison of the poloidal flux. The tendency of the profiles are quite similar between the analytic and the numerical results, although the absolute values are a little different. Particularly, the substantial value of the $(m, n) = (4, 2)$ component at the resonant surface is obtained analytically as well as numerically, which corresponds to the $m = 4$ islands in Fig.1. These results indicate that the analytic treatment based on Eqs.(1)-(4) is appropriate for the estimation of the induced islands and second harmonic property of the islands results from the nonlinear coupling of the dominant mode.

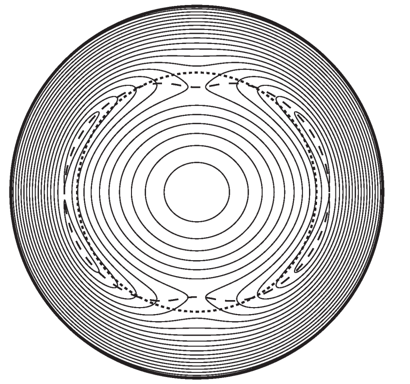

Fig.1 Contour of helical magnetic flux in the nonlinear phase of a resistive interchange mode. Dominant component is $(m, n) = (2, 1)$.

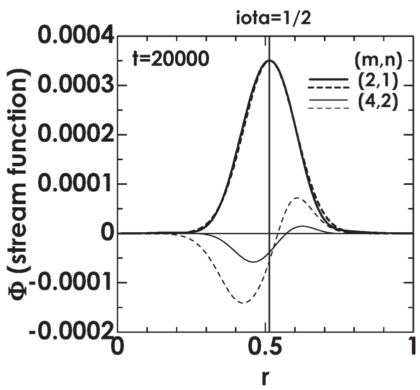

Fig.2 Profile of stream function. Solid and dashed lines show numerical and analytic results, respectively.

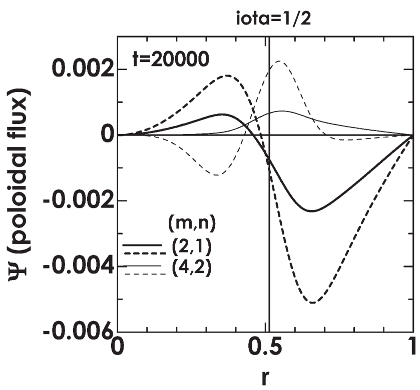

Fig.3 Profile of poloidal flux. Solid and dashed lines show numerical and analytic results, respectively.

1) Ichiguchi, K. et al. : Nucl. Fusion **43** (2003) 1101.
2) Ichiguchi, K., Carreras, B. A. : J. Plasma Fusion Res. SERIES, **6** (2004) 589-592.