§32. Statistical Laws of Fluctuations and Effects of Spatial Dimensions in NS, Superfluid, and MHD Turbulence

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Singular structure of flow field of Navier Stokes turbulence is vortex point in 2D, vortex filament in 3D and their geometrical dimensions are important in understanding of the energy cascade and energy dissipation. In 2D, two vortex points collide each other with almost zero probability and thus do not contribute to the energy dissipation, which is consistent with no forward cascade of the energy. In 4D classical turbulence, it is known that the energy cascade to high wavenumbers is stronger than in 3D [1, 2], but geometry of the singular structure at small scales are not well understood. In quantum turbulence, vortex filaments with quantized circulation entangle and reconnect each other, which result in the energy cascade as in the classical turbulence [3–5]. In both classical and quantum turbulence, geometry, amplitude, characteristic time and distribution of the singular structures in space in the D dimensional space are key ingredients.

Here we studied the topological dimensions of the singular structure in classical and quantum turbulence in 4 dimensions in terms of Direct Numerical Simulation (DNS). Navier Stokes and Gross-Pitaevskii equations were numerically integrated and the spectral method was used for the nonlinear terms.

In the NS turbulence (hereafter classical turbulence), the 2-form $\Omega_{ij} = \partial u_j/\partial x_i - \partial u_i/\partial x_j$ in 4D corresponds to the vorticity vector in 3D. We considered (1) to examine 3D visualized field of the 2-form by changing the fourth coordinate, (2) to compute the generalized dimension of $\sum_{ij} \Omega_{ij}^2$ and Ω_{12}^2, and (3) to study the distribution of the eigenvalues of strain tensor. We have found that the singular structure in the classical turbulence in 4D is of the sheetlike. Figure 1 shows the generalized dimension $\sigma = \sum_{ij} \Omega_{ij}^2$ computed by the box counting method. For large q, it is about 2.7 which is greater than 2 for the sheet. σ is the sum of $4 \times 3/2 = 6$ components while Ω_{ij} is just one component so that Ω_{12} tends to show stronger singularity of the field than would for the sum. $\tilde{D}_q^{(4)}$ in Fig.1 is about 2.3 for large q which is closer to the sheet than in the case of σ.

Also $D_q^{(3)}+1$ and $\tilde{D}_q^{(3)}+1$ for the classical turbulence in 3D at the similar Reynolds numbers are plotted in the figure. It can be seen that $\tilde{D}_q^{(4)} \approx \tilde{D}_q^{(3)}+1$ holds for $q > 0$, especially for large q, which is consistent with observation that the topological dimensions of the singular structure in 4D is 2.

In order to examine the singular structure of the quantum turbulence in 4D, as in the same way as in the above we have numerically integrated the GP equation and visualized isosurface of the density which corresponds to the vortex tubes in the classical turbulence. Torus structure in 3D picture is observed for a fixed fourth dimension, and remains almost unchanged during which the fourth coordinate is changed. Although definite conclusion can not be drawn from this single observation, it is quite likely for the singular object of the quantum turbulence in 4D to have the topological dimension 2. Further studies are under way.

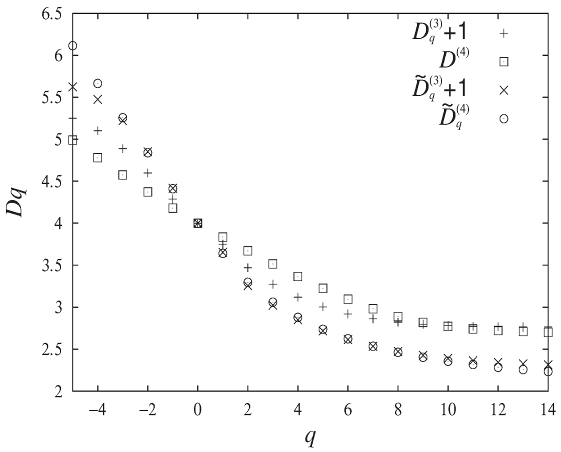

FIG. 1: Generalized dimension $D_q^{(4)}$ of singular structure in 4-D turbulence. Also curves $D_q^{(3)}+1$ and $\tilde{D}_q^{(3)}+1$ are shown for comparison.

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