§34. Provisional Study on 3-D Cauchy Condition Surface Method to Identify Plasma Boundary Shape

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1. Introduction

The Cauchy condition surface (CCS) method estimates the plasma shape from magnetic sensor signals. The tokamak plasma geometry is axisymmetric so that the analysis can be made in a 2-D, r-z system [1,2]. A 3-D version of CCS method code is under development. The fundamental performance of this 3-D code is now demonstrated.

2. Three-dimensional CCS method

The 3-D CCS (Γ_C) is assumed to have a torus shape and to be located in the plasma region. The Dirichlet and the Neumann conditions along the CCS are now the vector potential and its derivative, respectively. The first step of the analysis is to obtain values of both type boundary conditions on the CCS in such a way that they will be consistent with the magnetic sensor signals.

2.1 Vector Laplacian

We adopt the Cartesian coordinate system for the analysis. The vector Laplacian in this system has the simple relationship

$$(\nabla^2 \boldsymbol{A})_k = \nabla^2 A_k \quad (k = x,\, y,\, z)\,. \tag{1}$$

2.2 Hypothetical assumption of vacuum field

One here assumes that there is no plasma current, i.e., vacuum everywhere outside the CCS. The effect of the actual plasma current is transformed into the hypothetical CCS.

2.3 Boundary integral equations

One solves two types of boundary integral equations:

(i) For points i on the CCS (Γ_C):

$$\int_{\Gamma_C}\left(\phi_i^*\frac{\partial A_k}{\partial n} - A_k\frac{\partial \phi_i^*}{\partial n}\right)d\Gamma = \frac{1}{2}A_{k,i}\,. \quad (k = x,\, y,\, z) \tag{2}$$

(ii) For the magnetic sensor locations i :

$$\int_{\Gamma_C}\left\{\left(L\,\phi_i^*\right)\frac{\partial A_k}{\partial n} - A_k\left(L\,\frac{\partial \phi_i^*}{\partial n}\right)\right\}d\Gamma = (L\,A_{k,i}) - L\,W_{k,i}\,. \tag{3}$$

The fundamental solution ϕ_i^* satisfies the 3-D scalar Laplace equation with the Dirac delta function

$$\nabla^2\phi_i^* = \delta_i\,. \tag{4}$$

The quantity $W_{k,i}$ in Eq.(3) is the contribution of external coil. The detailed form of the operator L depends on the type of magnetic sensor. Equations (2) and (3) are discretized, coupled and expressed in a matrix form. Once all the values of A_k and $\partial A_k/\partial n$ on Γ_C have been given, the vector potential can be calculated for arbitrary points.

3. Numerical examples

One assumes that 240 magnetic flux sensors and 960 magnetic field sensors are arranged outside the torus shape plasma. The CCS, which also has a torus shape, is placed within the plasma domain. The ellipse in the center of Fig.1 is the cross-section of the CCS on the r-z plane. This 3-D CCS was divided into 16 boundary elements, each of which has 9 nodal points.

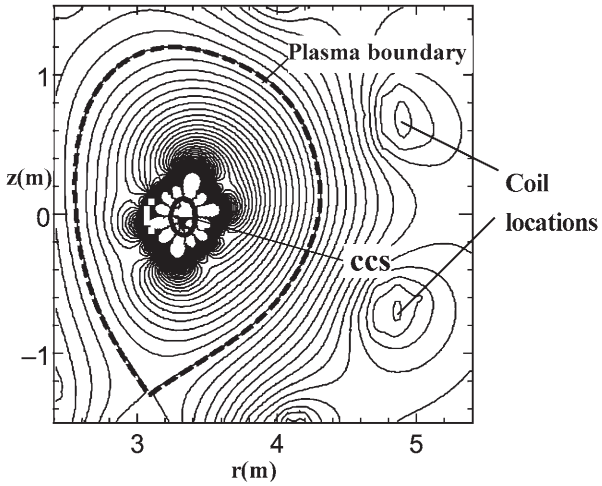

Fig.1 Contours of magnetic flux and the plasma boundary

The magnetic surface function $\psi = rA_\varphi$ can be obtained for arbitrary points. The solid lines in Fig.1 are the contours of ψ. Among the contours, the outermost closed curve represents the plasma boundary. This agrees well with the reference boundary profile (the dotted closed line) that had been given beforehand by using the reliable equilibrium code SELENE [3].

4. Conclusion and further remarks

A prototype of 3-D CCS method code has been developed. Test calculations were made for axisymmetric plasma. Results indicate that the 3-D method can also determine the plasma boundary shape accurately in the same way as the 2-D CCS method.

Our plan is to analyze the actual non-axisymmetric 3-D plasma in the LHD. To realize this, the followings should be solved:

(i) The x-, y- and z-components in Eq.(3) should be solved simultaneously for an actual magnetic field sensor. For a flux loop signal, Eq.(3) might be further integrated along the loop.

(ii) As it is difficult to derive mathematically the 3-D magnetic surface function in a helical system, one needs to seek a practical way of drawing the magnetic field line.

(iii) The rotational symmetry, which is peculiar to the LHD, should be incorporated into the boundary integral formulation in order to reduce the number of unknowns.

[1] Kurihara, K., *Fusion Eng. Des.*, **51-52,** pp. 1049-1057, 2000.
[2] Itagaki, M., et al,, T., *Nucl. Fusion*, **45**, pp. 153-162, 2005.
[3] Takeda, T., et al.., *JAERI-M 8042*, 1978.