§5. Theoretical Studies of Equilibrium Beta Limit in Heliotron Plasmas

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Generating and keeping good magnetohydrodynamic (MHD) equilibrium are aims of magnetic confinement researches. Usually, MHD equilibrium degrades due to increasing $\beta$ and then reaches the equilibrium beta limit. In the conventional theory, the equilibrium beta limit is defined by the Shafranov shift $\Delta/a$, where $\Delta=(R_{\text{in}}(\beta)-R_{\text{in}}(0))$ and $a$ is the effective minor radius. If the Shafranov shift achieves about 0.5, the equilibrium is limited. For tokamaks, at the equilibrium beta limit the separatrix appears, because the poloidal field is canceled by the external vertical field to keep the MHD equilibrium. The separatrix in the core leads to the degradation of the confinement. In stellarator/heliotron, the equilibrium beta limit can be defined by the Shafranov shift as well as tokamaks. However, Hayashi et al. pointed out the equilibrium beta limit is also defined by the stochasticity of magnetic field lines and it is more severe than the limit defined by the Shafranov shift. Since the pressure-induced perturbation breaks the symmetry of the magnetic field, the stochasticity of magnetic field lines by the equilibrium response is an intrinsic property in stellarator/heliotron. The problem how to define and represent the equilibrium beta limit is still an open question. Thus, understanding the equilibrium beta limit is a critical and urgent issue from the viewpoint to aim stellarator/heliotron reactor.

In order to understand the stochasticity of magnetic field lines due to increasing $\beta$, positions of the magnetic axis ($R_{\text{ax}}$), inward and outward LCFS ($R_{\text{in}}$ and $R_{\text{out}}$) as the function of $<\beta>$ are plotted in fig.1. The magnetic axis $R_{\text{ax}}$ monotonically changes due to increasing $\beta$. On the other hand, though $R_{\text{in}}$ and $R_{\text{out}}$ shift outward due to increasing $\beta$, $R_{\text{out}}$ is fixed near the vacuum LCFS until $<\beta>\sim5.2\%$. However, for higher $\beta$, $R_{\text{out}}$ shrinks inside the vacuum LCFS and magnetic field lines become strongly stochastic. In more than $<\beta>\sim6.3\%$, the force balance starts breaking inside the vacuum LCFS. To keep the force balance, the pressure gradient in the stochastic region decreases and the fixed profile is reduced with decreasing $R$ to fix the pressure profile. For $<\beta>\sim6.3\%$, $s=1$ flux surface is a surface pass through $R=4.6m$. However, for $<\beta>\sim6.3\%$, the stochastic field cannot keep the pressure gradient until $R=4.6m$. Thus, fixed point $R$ to prescribe the pressure profile is decreased to $R=4.4m$. As the result, total stored energy $W_s$ decreases. We summarize the MHD equilibrium in heliotron plasmas involves following three phases in fig.1. (I) the plasma shifts due to the finite beta but Rout is fixed near the vacuum LCFS. (II) Increasing beta, the stochastic region increases and the LCFS shrinks compared with the vacuum. (III) Increasing the stochasticity, the residual force balance is broken. The magnetic configuration cannot keep the plasma.

In order to confirm our speculation, the volume averaged beta obtained from the calculation is plotted as the function of the beta on the axis. Figure 2 shows the relation between $\beta_0$ and $<\beta>$. Two auxiliary lines are plotted as the reference. For $<\beta>\sim6.3\%$, the volume averaged beta increases almost linearly. Since the pressure profile is prescribed as the function of the toroidal flux, which means the function of the cross section, it is not completely linear. However, for higher $\beta$ ($<\beta>\sim6.3\%$), the stochasticity becomes strong and the fixed pressure profile is reduced to keep the force balance. Thus, an inflection points appeared for $<\beta>\sim6.3\%$. In the considered condition, the equilibrium beta limit is more than $6\%$.

Fig. 1. The change of the axis ($R_{\text{ax}}$), inward and outward positions ($R_{\text{in}}$ and $R_{\text{out}}$) of the LCFS on the horizontal elongated cross section (along Z=0 plane) are plotted as the function of $<\beta>$.

Fig. 2. The volume averaged beta $<\beta>$ is plotted as the function of the beta on the axis $\beta_0$. 