§20. Fundamental Relation between Diagonal and Off-diagonal Components of Conductivity Tensor in the Quantum Hall System

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We investigate the relation between the diagonal (σ_{xx}) and off-diagonal (σ_{xy}) components of the conductivity tensor in the quantum Hall system. First, we calculate the conductivity components for a short-range impurity potential using the linear response theory, employing an approximation that simply replaces the self-energy by a constant value $-i\hbar/(2\tau)$ with τ the scattering time. The approximation is equivalent to assuming that the broadening of a Landau level due to disorder is represented by a Lorentzian with the width $\Gamma = \hbar/(2\tau)$.1) Analytic formulas are obtained for both σ_{xx} and σ_{xy} within the framework of this simple approximation at low temperatures. By examining the leading terms in σ_{xx} and σ_{xy}, we find a proportional relation between $\mathrm{d}\sigma_{xy}/\mathrm{d}B$ and $B\sigma_{xx}^2$.

$$\frac{\mathrm{d}\sigma_{xy}}{\mathrm{d}B} \simeq \lambda B \sigma_{xx}^2, \tag{1}$$

with the coefficient λ determined by scattering parameters and Fermi energy, ε_F.

Next, we investigated the relation considering the long-range impurity potential. To account for the long-range nature of the impurity potential, we modify the analytic expressions of $\sigma_{xx} = e^2/h\tilde{\sigma_{xx}}$ and $\sigma_{xy} = e^2/h\tilde{\sigma_{xy}}$, introducing two types of scattering times, namely, the quantum scattering time $\tau_q = \hbar/(2\Gamma)$ that describes the impurity broadening of the Landau levels and the momentum relaxation time τ related to the conductivity at $B = 0$. The relation, after slight modification of the expression, is given by

$$\frac{\mathrm{d}\tilde{\sigma}_{xy}^{\mathrm{LR}}(X_\mathrm{F},\gamma_q,\gamma_m)}{\mathrm{d}B} \simeq \pi \frac{\mu_m^2}{\mu_q} \frac{\hbar\omega_c}{\varepsilon_\mathrm{F}} \left[\tilde{\sigma}_{xx}^{\mathrm{LR}}(X_\mathrm{F},\gamma_q,\gamma_m)\right]^2 + \mu_m \left[1 - \frac{\pi}{\mu_q B} \coth\left(\frac{\pi}{\mu_q B}\right)\right] \tilde{\sigma}_{xx}^{\mathrm{LR}}(X_\mathrm{F},\gamma_q,\gamma_m). \tag{2}$$

where ω_c is the cyclotron frequency. We made use of the mobility $\mu_i = e\tau_i/m^*$ for $i = q$, m with effective electron mass m^*. $X_\mathrm{F} = \varepsilon_\mathrm{F}/\hbar\omega_c - 1/2$ is the dimensionless parameter.

In Fig.1, we show the longitudinal and the Hall conductivities calculated by the expressions of $\tilde{\sigma}_{xx}^{\mathrm{LR}}(X_\mathrm{F},\gamma_q,\gamma_m)$ and $\tilde{\sigma}_{xy}^{\mathrm{LR}}(X_\mathrm{F},\gamma_q,\gamma_m)$ with parameters $\varepsilon_\mathrm{F} = 7.5$ meV, $\mu_q = 7.1\ \mathrm{m}^2/(\mathrm{Vs})$, and $\mu_m = 78\ \mathrm{m}^2/(\mathrm{Vs})$. In Fig.2, we show $\mathrm{d}\tilde{\sigma}_{xy}/\mathrm{d}B$ attained by the numerical differentiation of experimentally obtained σ_{xy} obtained in the GaAs/AlGaAs. The relation, equation(2), is shown to be in quantitative agreement with experimental results obtained in the GaAs/AlGaAs two-dimensional electron system at the low magnetic-field regime where spin splitting is negligibly small.

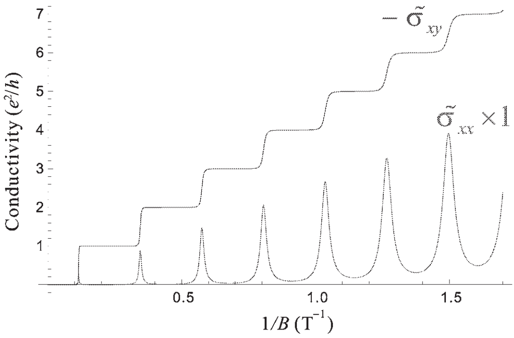

Fig. 1: The diagonal and the off-diagonal components of the conductivity tensor. The horizontal axis is the inverse magnetic field.

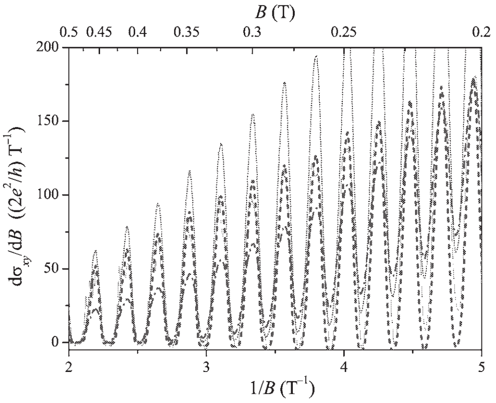

Fig. 2: Experimental traces of $\mathrm{d}\tilde{\sigma}_{xy}/\mathrm{d}B$. Thin solid red line corresponds to numerical differentiation of experimentally obtained σ_{xy}. $\mathrm{d}\tilde{\sigma}_{xy}/\mathrm{d}B$ approximated by the r.h.s. of equation(2) (thick dashed green line) or by the first term of the r.h.s. of equation(2) (thin dotted green line) calculated using experimentally obtained σ_{xx}. $\beta\tilde{\sigma}_{xx}$ with the experimentally obtained σ_{xx} and $\beta = 2\tau_m/\tau_q$ is also plotted by dot-dashed blue line.2)

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2) Chang A. M., and Tsui D. C., 1985, *Solid State Commun.* **56** 153.