§13. Charging of Dust Particles in Weak Magnetic Field - Orbits of Charged Particles -

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The charged particle orbits of j-th species ($j = e, i$) in the magnetic field are investigated in the cylindrical coordinates (r, θ, z), where the uniform magnetic field B_0 is applied to the axial z-direction. The unmovable point dust particle is located at the origin with the charge q_d. The charged particle with the charge q_j starts to move from the initial position ($r = b_{in}$, $\theta = 0$, $z = z_{in}$) with the velocity $(v_\rho = v_\theta = 0, v_z = v_{j,in})$. The azimuthal motion is determined from the conservation of canonical angular momentum P_q:

$$\frac{d\theta}{dt} = \frac{1}{m_j \rho^2}(P_\theta - \frac{q_j B_0}{2}\rho^2). \qquad (1)$$

The initial conditions ($r = b_{in}$, $d\theta / dt = 0$) give the constant value of P_q:

$$P_\theta = \frac{q_j B_0}{2} b_{in}^2. \qquad (2)$$

The normalized equations of motion become

$$\frac{d^2\bar{\rho}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{\rho}}{\bar{r}^3} + \frac{\mu_j^2}{4}\frac{1-\bar{\rho}^4}{\bar{\rho}^3}, \qquad (3)$$

$$\frac{d^2\bar{z}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{z}}{\bar{r}^3}, \qquad (4)$$

where the distances, velocity and time are normalized by the impact parameter b_{in}, the initial speed $v_{j,in}$ and $b_{in}/ v_{j,in}$, respectively. The system in the absence of magnetic field is determined by the parameter α_j,

$$\alpha_j \equiv \frac{q_j q_d}{4\pi\varepsilon_0 b_{in}} / \frac{m_j v_{j,in}^2}{2}, \qquad (5)$$

which is the ratio of the electrostatic potential energy at the distance of the impact parameter to the initial kinetic energy and the parameter μ_j indicates the effect of the static magnetic field,

$$\mu_j \equiv b_{in} / \frac{m_j v_{j,in}}{|q_j B_0|}, \qquad (6)$$

which is the ratio of the impact parameter b_{in} to the Larmor radius with respect to the initial speed $v_{j,in}$. The parameter μ_e of the electron is much larger than that of the ion for the case of ions with the sound speed c_s and the thermal speed of the electron v_{the}:

$$\frac{\mu_e}{\mu_i} = \frac{m_i v_{i,in}}{Z_i m_e v_{e,in}} \simeq \frac{m_i c_s}{Z_i m_e v_{the}} \simeq \sqrt{\frac{m_i}{m_e}}, \qquad (7)$$

where Z_i is the charge state of the ion. This relation indicates the effect of the magnetic field on the ion is much smaller than that of the electron (see Eq. 3).

The typical orbit of an electron near the negatively charged dust, which is located at the origin ($\rho = z = 0$), is shown in Fig.1, where $\alpha_e = 1.0$ and $\mu_e = 0.01$. The closest radius in the presence of in the axial magnetic field (solid line in Fig.1) becomes smaller than that in the absence of magnetic field. The orbit of a charged particle in magnetic field is characterized by three-dimensional nature rather than the two dimensional orbit. The particle has a radial velocity due to the radial electric field. This radial velocity pushes an electron in the azimuthal direction by the Lorentz force. This azimuthal velocity makes the radial force by the axial magnetic field. Thus the magnetic field has the second order effect on the orbit without magnetic field, see Eq. 3. As a result the radial force balance of the particle between the Lorentz force and the centrifugal force determines the radial motion of a charged particle. The radial equation of motion is expressed by

$$m_j \frac{d^2\rho}{dt^2} = q_j E_r \frac{\rho}{r} + m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt}, \qquad (8)$$

where the first term of the RHS is the electrostatic force by the dust particle, the second one is the centrifugal force and the third one indicates the Lorentz force. From the relation of the conservation of the canonical angular momentum (Eqs. 1 and 2), the summation of the centrifugal force and the Lorentz force is expressed as:

$$m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt} = \frac{1}{m_j \rho^3}(P_\theta^2 - \frac{q_j^2 B_0^2}{4}\rho^4)$$

$$= \frac{q_j^2 B_0^2}{4 m_j \rho^3}(b_{in}^4 - \rho^4). \qquad (9)$$

For the case of the orbit of the charged particle with the same sign as the dust charge, its radius r is larger than the initial one (b_{in}) due to the radial electrostatic force, which means the Lorentz force is stronger than the centrifugal force all the time. This deference makes the closest radius smaller than that without magnetic field, Fig. 1. On the other hand the charged particle with the opposite sign of the dust charge leaves further the dust.

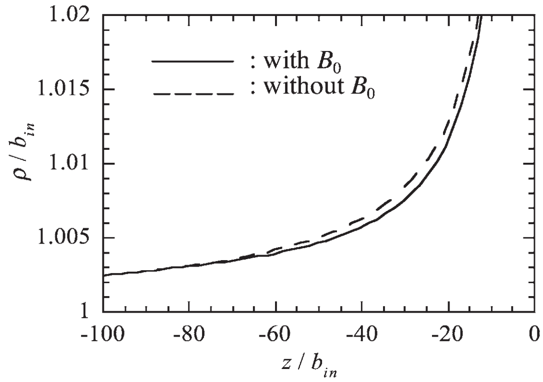

Fig.1 Typical orbit of an electron near the negatively charged dust for the case $\alpha_e = 1.0$ and $\mu_e = 0.01$. The solid and dashed lines are the orbits with and without magnetic field, respectively.