§68. Effective Cross Volume in Collective Thomson Scattering of Strongly Focussed Beam

Kubo, S., Nishiura, M., Tanaka, K., Shimozuma, T., Yoshimura, Y., Igami, H., Takahashi, H., Ikeda, R., Tatematsu, Y., Saito, T. (Univ. Fukui, FIR FU), Tamura, N. (Nagoya Univ., Dept. Energy Sci. and Tech.)

Collective Thomson scattering (CTS) is one of the promising candidate for the direct measurements of the velocity distribution function of ions in fusion devices. Relevant frequency for the CTS probing beam source is in the millimeter wave to sub-millimeter wave range. The scattering volume of the CTS is an important parameter for determining the intensity of the scattered power, and spatial resolution of the measurement. Conventional method to estimate the scattering volume is to simply calculate the geometrical volume of probing and receiving beams. This calculation is valid when the beams are diffraction free. In the strongly focussed system which is adopted in the CTS system in LHD, careful treatment of the diffraction effect is necessary. Effective scattering volume is defined and applied to the CTS system using strongly focused beams and its validity is checked by the experimental results.

The scattering volume generally defined in the scattering measurement is purely geometrical cross-section of probing and receiving beams. Assuming the uniformity of the scattering characteristics, beam and sensitivity profile in a unit volume, scattered power density, $P_{\rm s}$ at far field, \boldsymbol{R} , in a unit angle frequency, in a unit steradian can be expressed as,

$$P_{\rm s}(\boldsymbol{R},\omega_{\rm s})\mathrm{d}\Omega\mathrm{d}\omega_{s} = \int_{V} p_{\rm i}(\boldsymbol{r}) \frac{r_{0}^{2}}{2\pi} \left| \hat{\boldsymbol{s}} \times \hat{\boldsymbol{s}} \times \hat{\boldsymbol{E}}_{\rm i} \right|^{2} n_{\rm e}(\boldsymbol{r}) S(\boldsymbol{K},\omega) \,\mathrm{d}\boldsymbol{r}$$
(1)

Here, \vec{E}_{i} and \hat{s} are the unit vectors in the oscillating electric field direction of probing beam and in the direction of scattered wave vector, $k_{\rm s}$. $k_{\rm i}$ and K are the wave vector of the probing and fluctuations that causes scattering. ω_i, ω and ω_s are angular frequency of the probing, fluctuation, and scattered radiations, respectively. Here the relations $\mathbf{k}_{s} = \mathbf{k}_{i} + \mathbf{K}$ and $\omega_{s} = \omega_{i} + \omega$ are satisfied. r_0 is the classical electron radius. $p_i(\mathbf{r})$ is the power density of the incident beam. $n_{\rm e}(\mathbf{r})$ and $S(\mathbf{K},\omega)$ are the local electron density and form factor that depends on local plasma parameters. Integration of this equation should be performed all over the space where the scattering can occur. In the case when the plasma parameters n_e , $S(\mathbf{K}, \omega)$ and $|\hat{\mathbf{s}} \times \hat{\mathbf{s}} \times \hat{\mathbf{E}}|^2$ are constant over the volume, eq.(1) can be simplified to get a conventional formula for the total receiving power $P_{\rm r}$ with the antenna

efficiency function $\eta_r(\mathbf{r})$ as,

$$P_{\mathbf{r}} = \int p_{\mathbf{i}}(\boldsymbol{r})\eta_{\mathbf{r}}(\boldsymbol{r})\frac{r_{0}^{2}}{2\pi} \left| \hat{\boldsymbol{s}} \times \hat{\boldsymbol{s}} \times \hat{\boldsymbol{k}} \right|^{2} n_{e}(\boldsymbol{r})S(\boldsymbol{K},\omega) \, \mathrm{d}\boldsymbol{r}$$
$$= \frac{r_{0}^{2}}{2\pi} \frac{P_{\mathbf{i}}}{A} V \left| \hat{\boldsymbol{s}} \times \hat{\boldsymbol{s}} \times \hat{\boldsymbol{k}} \right|^{2} \overline{n_{e}}\overline{S(\boldsymbol{K},\omega)}$$
(2)

here, $\overline{n_e}$ and $S(\mathbf{K}, \omega)$ are the averaged electron density and form factor in the scattering volume. V and A are the scattering volume, and the cross-section of the probing beam, respectively. These values can be independently calculated from the geometrical overlap between probing and receiving beams and cross-section of probing beam, in the case when the spot size changes of both beams near the scattering center are negligible. The precise combined expression of A and V can be derived from eq.(2) as the integral form,

$$\frac{P_{\rm i}}{A}V = \iint p_{\rm i}(\boldsymbol{r})\eta_{\rm r}(\boldsymbol{r'}) \times \delta(\boldsymbol{r'} - \overleftarrow{\boldsymbol{T}} \cdot \boldsymbol{r})\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r'}$$
(3)

here, $p_i(\mathbf{r})$ is power density distribution function and $\eta_r(\mathbf{r'})$ is the antenna efficiency function defined by the Gaussian parameters of the probing and receiving beams.

The scattering volume scan experiment was performed using 32 channel receiver system. Subtracting the background ECE, the spectral intensity ratio for the bottom, center,top cases were 2.9:1.0:1.8. Recalling eq.(2), the scattered spectral intensity is proportional to the volume, V and inversely proportional to probing beam cross section, A. The relative spectral intensity except for the plasma parameter scales to the effective length, V/A. In Fig.1 are shown the calculated V/A with the measured intensity normalized at the center. Good agreement of the relative intensity of the scattered spectra to the effective length V/A for the using effective scattering volume $V_{\rm eff}$ indicates that the observed spectra reflects the local ion velocity distribution function defined by the designed Gaussian beam optics.



Fig. 1: Dependence of the geometrical and effective scattering length, V_{geom}/A and V_{eff}/A , during the vertical scan of the volume center. Open circles indicate the relative intensity of the scattered spectra normalized by that at the center.