§11. Application of Photo-absorption to X-ray Spectroscopy in LHD

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In the present research an application of photo-absorption by a material is evaluated as an x-ray spectroscopy in Large Helical Device (LHD). The x-ray transmission through a filter is the basis of the present application. Generally, it is recognized that the transmission intensity is obtained from an incident spectrum, absorption coefficient, and the thickness of the filter as follows,

$$I(t) = \int_0^\infty dE I_0(E) f(E) e^{-\alpha(E)t} \quad , \tag{1}$$

where E, t, $\alpha(E)$, $I_0(E)$, $I(t)$, and $f(E)$ are photon energy, the thickness of the filter, the absorption coefficient, the intensity of incident spectrum, the transmission intensity, and detection efficiency, respectively. On the contrary, it is the concept of the application that the incident spectrum is obtained from the transmission intensity, the absorption coefficient and the thickness of the filter as follows,

$$I_0(E) = -\frac{1}{2\pi i f(E)} \int_{c-i\infty}^{c+i\infty} ds \frac{g(s)}{G(s)} \quad , \tag{2}$$

$$\because g(s) \equiv \int_0^\infty dt I(t) t^{s-1} \quad , \tag{3}$$

$$\because G^{-1}(s) \equiv \alpha^s(E) \Gamma^{-1}(s) \frac{d}{dE} \ln \alpha(E) \quad , \tag{4}$$

where c, i and Γ are a positive constant, an imaginary unit ,and a gamma function, respectively. Eq.(2) is obtained from Eq.(1) by using Mellin transform given in Eq.(3). The transform Eq.(3) indicates that the thickness of the filter must be continuously changed for the application.

In order to calculate Eq.(2) numerically by a computer, an approximation must be taken into account as follows,

$$I_\sigma(E) = -\frac{1}{2\pi i f(E)} \int_{c-i\infty}^{c+i\infty} ds \frac{g(s)}{G(s)} e^{-\left[\frac{\sigma}{2}(s-c)\right]^2} \quad , \tag{5}$$

where σ is a parameter meaning the finite integration range. Eq.(5) is exactly equal to Eq.(2), if the parameter σ is equal to zero. In an assumption with an infinitesimally narrow line at an energy of E_0, the analyzed result by using Eq.(5) is as follows,

$$I_\sigma(E) = \frac{1}{\sqrt{\pi}\sigma} \frac{f(E_0)}{f(E)} e^{cX} e^{-\left(\frac{X}{\sigma}\right)^2} \frac{dX}{dE_0} \quad , \tag{6}$$

$$\because X \equiv \ln \alpha(E) - \ln \alpha(E_0) \quad . \tag{7}$$

The energy resolution is corresponding to the width of a gaussian in Eq.(6). The width is approximately obtained as follows,

$$\Delta E^{-1} \approx \left| \sigma^{-1} \frac{d}{dE} \ln \alpha(E) \right| \quad , \tag{8}$$

where ΔE is the width of the gaussian. The parameter σ is approximately equal to 0.01 ,which is corresponding to the ability of a personal computer. In the x-ray region, the absorption coefficients are approximately expressed as follows,

$$\alpha(E) \approx aE^{-b} \quad , \tag{9}$$

where a and b are constants. Then, the energy resolution is obtained as follows,

$$\frac{E}{\Delta E} \approx \frac{b}{\sigma} \quad . \tag{10}$$

In the case of beryllium, the constant b is equal to 3.12. Consequently, the energy resolution $E/\Delta E$ is equal to 312. The energy resolution is approximately one order better than that of conventional semi-conductor detectors[1)].

The energy resolution given in Eq.(10) is the upper limit, since no static error is assumed. The static error is proportional to the square of counting rate. The counting rate is obtained from the emissivity of the plasma, the time resolution of x-ray detection, the configuration of a detection system, the pass length of the sightline across the plasma, and the detection efficiency. The pixel size and viewing sight of the system are assumed to be 0.6×0.6 mm^2 and $4\pi\times10^{-6}$ strad, respectively. The viewing sight is corresponding to a spatial resolution of 24 mm at the mid plane of LHD. The pass length and the detection efficiency are also assumed to be 0.2 m and 100 %, respectively. From the experimental results by using a pulse height analyzer, the integrated intensity between 1.0 keV and 10 keV is estimated to be 6.0×10^{20} photons/m^3/s/ in the case of typical NBI plasma of LHD[2)]. Figure 1 shows simulated transmission intensity. Then, the relative static error is estimated to be 4.0 % in a time resolution of 10 μs. As is shown in the figure, the energy deviation due to the statistical error is estimated to be ±0.05 keV. Then, the energy resolution $E/\Delta E$ is estimated to be100.

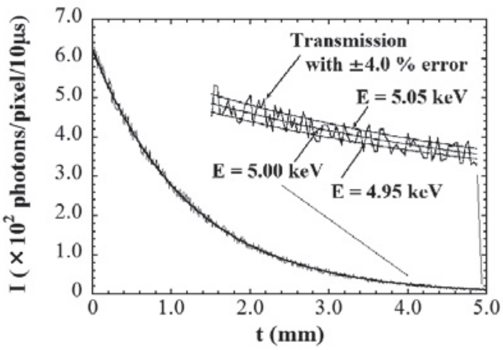

Fig. 1. The transmission intensity through a beryllium filter. The horizontal and vertical axes represent the thickness of the filter and the transmission intensity, respectively. The incident spectrum is assumed to be infinitesimally narrow at an energy of 5.0 keV.

1) S.Muto *et al.*: Rev.Sci.Instrum.**72**,(2001),p.1206.
2) S.Muto *et al.*: J.Plasma Fus.Res.SERIES **7**, (2006) p.27.