§15. Linearized Model Collision Operators for Multiple Ion Species Plasmas and Gyrokinetic Entropy Balance Equations

Sugama, H., Watanabe, T.-H., Nunami, M.

Linearized model collision operators for multiple ion species plasmas are derived, which conserve particles, momentum, and energy, and satisfy adjointness relations and Boltzmann's H-theorem even for collisions between different particle species with unequal temperatures [1]. The linearized collision operator for collisions between species a and b is written as the sum of the test-particle and field-particle parts, $C^L_{ab}(\delta f_a, \delta f_b) = C^T_{ab}(\delta f_a) + C^F_{ab}(\delta f_b)$. Detailed expressions of $C^T_{ab}(\delta f_a)$ and $C^F_{ab}(\delta f_b)$ in our model are written in [1]. The model collision operators are also given in the gyrophase-averaged form $C^{(GK)}_{ab} \equiv (2\pi)^{-1} \oint d\varphi\, e^{i\mathbf{k}_\perp \cdot \boldsymbol{\rho}_a} C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp})$ that can be applied to the gyrokinetic equation for the perturbed distribution function $\delta f_{a\mathbf{k}_\perp}$ with the perpendicular wave number vector $\mathbf{k}_\perp$.

Balance equations for zonal-mode and nonzonal-mode parts of the turbulent entropy density, the energy of electromagnetic fluctuations, the turbulent transport fluxes of particle and heat, and the collisional dissipation are derived from the gyrokinetic equation including the collision term and the Maxwell equations. Figure 1 shows the entropy balance schematically. Zonal modes are defined as fluctuations which have the wave number vectors in the direction perpendicular to flux surfaces, $\mathbf{k}_\perp = k_s \nabla s$. The summation over wave number vectors are divided into regions of zonal and nonzonal modes, $\sum_{\mathbf{k}_\perp} = \sum_{\mathbf{k}_\perp(\mathrm{Z})} + \sum_{\mathbf{k}_\perp(\mathrm{NZ})}$, where (Z) and (NZ) represent zonal and nonzonal modes, respectively. The entropy balance equations for nonzonal and zonal turbulent fluctuations of distribution functions and electromagnetic fields are written as

$$
\begin{aligned}
&\frac{\partial}{\partial t} \sum_{\mathbf{k}_\perp(\mathrm{NZ})} \left[\sum_a T_a \left\langle \left\langle \int d^3v \frac{|\delta f_{a\mathbf{k}_\perp}|^2}{2 f_{aM}} \right\rangle \right\rangle + \frac{1}{8\pi} \left\langle \left\langle |\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2 \right\rangle \right\rangle \right] \\
&= \sum_a T_a (J^A_{a1} X^A_{a1} + J^A_{a2} X^A_{a2}) - \mathcal{T}(\mathrm{NZ} \to \mathrm{Z}) \\
&\quad + \sum_{\mathbf{k}_\perp(\mathrm{NZ})} \sum_{a,b} T_a \left\langle \left\langle \int d^3v \frac{\delta f^*_{a\mathbf{k}_\perp}}{f_{aM}} C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp}) \right\rangle \right\rangle, \quad (1)
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial}{\partial t} \sum_{\mathbf{k}_\perp(\mathrm{Z})} \left[\sum_a T_a \left\langle \left\langle \int d^3v \frac{|\delta f_{a\mathbf{k}_\perp}|^2}{2 f_{aM}} \right\rangle \right\rangle + \frac{1}{8\pi} \left\langle \left\langle |\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2 \right\rangle \right\rangle \right] \\
&= \mathcal{T}(\mathrm{NZ} \to \mathrm{Z}) + \sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_{a,b} T_a \left\langle \left\langle \int d^3v \frac{\delta f^*_{a\mathbf{k}_\perp}}{f_{aM}} C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp}) \right\rangle \right\rangle, \quad (2)
\end{aligned}
$$

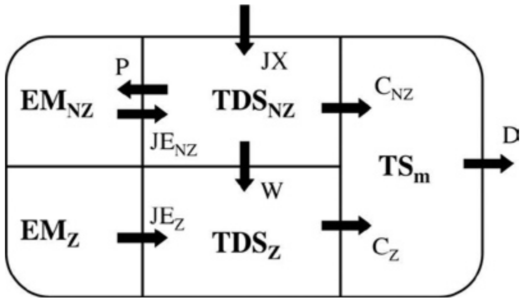

Fig.1. The schematic diagram representing the entropy balance equations. The entropy and electromagnetic energy quantities are represented by bounded regions while the transfer terms in the entropy balance equations are delineated by arrows. See Table I which shows in detail what quantities the bounded regions and the arrows represent.

Table 1: Quantities represented by bounded regions and arrows in Fig. 1.

Region	Quantity
$\mathrm{TS_m}$	$-\sum_a T_a \left\langle\left\langle \int d^3v (f_{aM} + \delta f_a) \log(f_{aM} + \delta f_a) \right\rangle\right\rangle$
$\mathrm{TDS_{NZ}}$	$\sum_{\mathbf{k}_\perp(\mathrm{NZ})} \sum_a T_a \left\langle\left\langle \int d^3v |\delta f_{a\mathbf{k}_\perp}|^2 / 2 f_{aM} \right\rangle\right\rangle$
$\mathrm{TDS_Z}$	$\sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_a T_a \left\langle\left\langle \int d^3v |\delta f_{a\mathbf{k}_\perp}|^2 / 2 f_{aM} \right\rangle\right\rangle$
$\mathrm{EM_{NZ}}$	$\sum_{\mathbf{k}_\perp(\mathrm{NZ})} \left\langle\left\langle |\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2 \right\rangle\right\rangle / 8\pi$
$\mathrm{EM_Z}$	$\sum_{\mathbf{k}_\perp(\mathrm{Z})} \left\langle\left\langle |\mathbf{E}_{\mathbf{k}_\perp}|^2 + |\mathbf{B}_{\mathbf{k}_\perp}|^2 \right\rangle\right\rangle / 8\pi$
Arrow	**Quantity**
JX	$\sum_a T_a (J^A_{a1} X^A_{a1} + J^A_{a2} X^A_{a2})$
W	$\mathcal{T}(\mathrm{NZ} \to \mathrm{Z})$
$\mathrm{C_{NZ}}$	$-\sum_{\mathbf{k}_\perp(\mathrm{NZ})} \sum_{a,b} T_a \langle\langle \int d^3v (\delta f^*_{a\mathbf{k}_\perp} / f_{aM}) \times C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp}) \rangle\rangle$
$\mathrm{C_Z}$	$-\sum_{\mathbf{k}_\perp(\mathrm{Z})} \sum_{a,b} T_a \langle\langle \int d^3v (\delta f^*_{a\mathbf{k}_\perp} / f_{aM}) \times C^L_{ab}(\delta f_{a\mathbf{k}_\perp}, \delta f_{b\mathbf{k}_\perp}) \rangle\rangle$
D	$\sum_a (T_a/V')(\partial/\partial s)[V' \{ (S_{aM}/n_a) J^A_{a1} + J^A_{a2} \}]$
P	$-(c/4\pi V')(\partial/\partial s)[V' \langle\langle (\mathbf{E} \times \mathbf{B}) \cdot \nabla s \rangle\rangle]$
$\mathrm{JE_{NZ}}$	$\sum_a e_a n_a \sum_{\mathbf{k}_\perp(\mathrm{NZ})} \mathrm{Re} \langle\langle (\mathbf{u}^*_{a\mathbf{k}_\perp} \cdot \mathrm{E}_{\mathbf{k}_\perp})^{(3)} \rangle\rangle$
$\mathrm{JE_Z}$	$\sum_a e_a n_a \sum_{\mathbf{k}_\perp(\mathrm{Z})} \mathrm{Re} \langle\langle (\mathbf{u}^*_{a\mathbf{k}_\perp} \cdot \mathrm{E}_{\mathbf{k}_\perp})^{(3)} \rangle\rangle$

respectively. It is important to note that the zonal modes never contribute to the turbulent (or anomalous) particle and heat fluxes (J^A_{a1} and J^A_{a2}) driven by pressure and temperature gradient forces (X^A_{a1} and X^A_{a2}). The source terms given by the product of the fluxes J^A_{aj} and X^A_{aj} ($j = 1, 2$) appear in Eq. (1) while they don't in Eq. (2). Here, $\mathcal{T}(\mathrm{NZ} \to \mathrm{Z})$ represents the nonlinear entropy transfer from the nonzonal modes to the zonal modes, the detailed expression of which is given in [1]. In the steady turbulence, we find from Eq. (2) that $\mathcal{T}(\mathrm{NZ} \to \mathrm{Z}) > 0$ because of the H-theorem. Thus, we see from Eqs. (1) and (2) that the nonlinear entropy transfer from nonzonal to zonal modes occurs and contributes to reduction of the amplitudes of the nonzonal modes and the turbulent transport.

1) H. Sugama, T.-H. Watanabe, and M. Nunami, Phys. Plasmas **16**, 112503 (2009).