§21. Localness of Mode Interactions in Fully Developed, Freely Decaying, Isotropic, Homegeneous MHD and Hall-MHD Turbulences

Araki, K. (Okayama Univ. Sci.)

In this study we solved the incompressible Hall-MHD equations given by

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla P + \boldsymbol{j}\times\boldsymbol{b} + \nu\nabla^2\boldsymbol{u}, \quad (1)$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla\times((\boldsymbol{u}-\epsilon\boldsymbol{j})\times\boldsymbol{b}) + \eta\nabla^2\boldsymbol{b}, \quad (2)$$

where $\boldsymbol{u}$ is the bulk velocity field and satisfies $\nabla\cdot\boldsymbol{u}=0$, $\boldsymbol{b}$ is the magnetic field, $\boldsymbol{j} := \nabla\times\boldsymbol{b}$ is the current density field, P is the total pressure, ν is the kinematic viscosity, η is the resistivity, and ϵ is the parameter for relative strength of the Hall term. The parameters are set to $\nu=\eta=2\times10^{-3}$ and $\epsilon=0.05$. The case of $\epsilon=0$ is studied for comparison.

The velocity and magnetic fields are expanded in the wavelet modes as

$$\boldsymbol{f}(\vec{x},t) = \sum f_{j\epsilon\vec{l}\sigma}(t)\ \boldsymbol{\psi}_{j\epsilon\vec{l}\sigma}(\vec{x}), \quad (3)$$

where $\boldsymbol{f}$ stands for $\boldsymbol{u}$ or $\boldsymbol{b}$, $\boldsymbol{\psi}$'s are the wavelet basis functions, and the expansion coefficients are given by $f_{j\epsilon\vec{l}\sigma}(t) := \int \boldsymbol{f}(\vec{x},t)\cdot\boldsymbol{\psi}_{j\epsilon\vec{l}\sigma}(\vec{x})\mathrm{d}^3\vec{x}$. The physical implications of the wavelet indices j, ϵ, $\vec{l}$, and σ are summarized in Ref.[1]. In the present study we only use the information on the spatial scale of the wavelets. So the fields are decomposed into *wavelet scale spectrum* which is given by

$$\boldsymbol{f} = \sum_j \boldsymbol{f}_j \quad \text{where} \quad \boldsymbol{f}_j(\vec{x},t) = \sum_{\epsilon,\vec{l},\sigma} f_{j\epsilon\vec{l}\sigma}(t)\,\boldsymbol{\psi}_{j\epsilon\vec{l}\sigma}(\vec{x}). \quad (4)$$

It should be noted that as the scale index j increases the corresponding spatial scale become small by the factor 1/2.

In Fig.1, the spectra of the magnetic induction energy transfer which is given by

$$\langle \boldsymbol{b}_k|\boldsymbol{b}|\boldsymbol{u}_j\rangle_{Ind} := \int \boldsymbol{b}_k\cdot\nabla\times(\boldsymbol{u}_j\times\boldsymbol{b})\,\mathrm{d}^3\vec{x}$$

are shown. Although their average modulii is different, it is very remarkable that the following features are found in common for the MHD and HMHD cases:

1. As a whole, the energy is transferred from larger scale components to smaller ones due to the magnetic induction (or the Lorentz force) irrespective of the kind of field.
2. Intense transfer is localized around $j\sim k$. This implies that energy transfer is *local*, i.e., the transfer brackets that dominantly contribute to the energy budget is constituted by such modes that have very close spatial scales with each other.

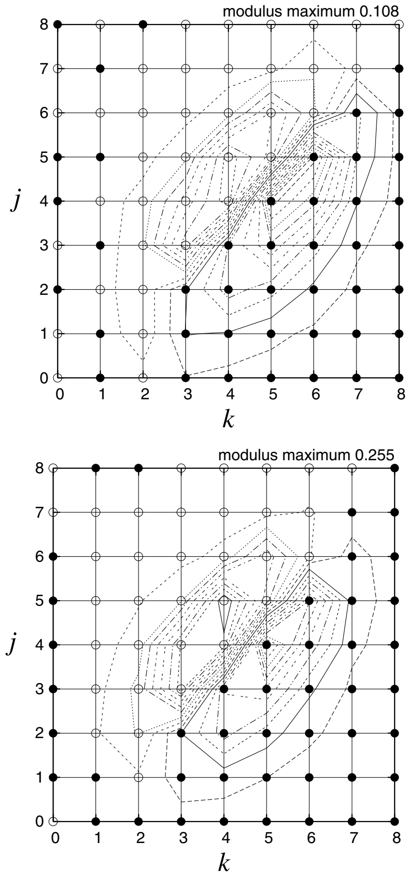

Fig. 1: Wavelet scale-to-scale energy transfer spectrum for the magnetic induction: $\langle \boldsymbol{b}_k|\boldsymbol{b}|\boldsymbol{u}_j\rangle_{ind}$. Top: MHD case, bottom: Hall MHD case. Solid circles: $\langle \boldsymbol{b}_k|\boldsymbol{b}|\boldsymbol{u}_j\rangle_{ind} > 0$, i.e., the transfer enhances the magnetic energy $E_k^{(B)}$, open ones: $\langle \boldsymbol{b}_k|\boldsymbol{b}|\boldsymbol{u}_j\rangle_{ind} < 0$, i.e., the transfer enhances the kinetic energy $E_j^{(u)}$. Contours are added to grasp the transfer amplitudes visually. Levels of contours are given by $(0.1n+0.05)\times\max\{\langle \boldsymbol{b}_k|\boldsymbol{b}|\boldsymbol{u}_j\rangle_{ind}\}$ where n is an integer.

Dominance of local transfer makes sharp contrast to the results of Alexakis et al.[2], Mininni et al.[3], and our previous result[4], all of which report the importance of the nonlocal energy transfer.

These results are discussed in Ref.[5].

1) K. Araki and H. Miura, "Orthonormal Divergence-free Wavelet Analysis of Nonlinear Energy Transfer in Rolling-Up Vortices", Y. Kaneda (ed.), IUTAM symposium on Computational Physics and New Perspectives in Turbulence, 149 (2008).

2) A. Alexakis, P. D. Mininni, and A. Pouquet, Phys. Rev. E, **72**, 046301 (2005).

3) P. D. Mininni, A. Alexakis, and A. Pouquet, J. Plasma Phys., **73**, 377 (2007).

4) K. Araki and H. Miura, JPFR Ser. **8**, 96 (2009).

5) K. Araki and H. Miura, JPFR Ser., to appear.