§17. Electron Absorption Cross-sections to Spherical Probe in Weak Magnetic Field


For the weak magnetic field, i.e. small $\mu_\varepsilon$, the closest radius $r_{\text{min}}$, which is corresponding to the probe radius, is approximated exponential dependence to the strength of magnetic field or $\mu_\varepsilon$.

$$
\frac{2}{\sqrt{\pi}} \alpha_\varepsilon (\mu_\varepsilon, \zeta_{\text{sc}}) = 1 + \left[ \frac{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)}{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)} - 1 \right] \exp \left[ - \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \mu_\varepsilon \right].
$$

where the distances, velocity and time are normalized by the impact parameter $b_{\text{in}}$, the initial speed $V_{j, \text{in}}$ and $b_{\text{in}}/V_{j, \text{in}}$, respectively. In the case of the strong magnetic field, the closest radius $r_{\text{min}}$ approaches the impact parameter $b_{\text{in}}$ due to the straight motion along the magnetic line of force. Here $\zeta_{\text{sc}} \equiv \zeta_{\text{sc}} / \Sigma_{D}$ is the parameter of the effect of the plasma shielding and $\eta_\varepsilon$ is the parameter, which depends on $\alpha_\varepsilon$ and $\zeta_{\text{sc}}$. The quantity $r_{\text{min}0}$ is the closest radius in the absence of magnetic field, which is obtained from the OML theory:

$$
\frac{2}{\sqrt{\pi}} \alpha_\varepsilon (\mu_\varepsilon, \zeta_{\text{sc}}) = 1 + \left[ \frac{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)}{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)} - 1 \right] \exp \left[ - \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \mu_\varepsilon \right].
$$

In this study the parameter $\eta_\varepsilon$ is determined by the relation of $\mu_\varepsilon = 1.0$:

$$
\eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) = \ln \left[ \frac{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)}{\ln \left( \eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) \right)} - 1 \right]
$$

In the case of $\alpha_\varepsilon = 1.0$, which corresponds to the negative applied voltages, and $\zeta_{\text{sc}} = 0$, the $\eta_\varepsilon$ becomes 0.602. The $\eta_\varepsilon$s for the cases of the weak shielding, $\zeta_{\text{sc}} = 0.3$, and strong one, $\zeta_{\text{sc}} = 1.0$, the parameter $\eta_\varepsilon$s decrease to 0.421 and 0.222, respectively. On the other hand, in the case of positive $V_\varepsilon$ ($\alpha_\varepsilon = -1.0 < 0$), the $\eta_\varepsilon$s for the case of $\zeta_{\text{sc}} = 0$, 0.3 and 1.0 become 0.751, 0.553 and 0.339, respectively. The parameter $\eta_\varepsilon$ is approximated by the polynomial of degree three as a function of $\alpha_\varepsilon$:

$$
\eta_\varepsilon (\alpha_\varepsilon, \zeta_{\text{sc}}) = c_0 (\zeta_{\text{sc}}) + c_1 (\zeta_{\text{sc}}) \alpha_\varepsilon + c_2 (\zeta_{\text{sc}}) \alpha_\varepsilon^2 + c_3 (\zeta_{\text{sc}}) \alpha_\varepsilon^3.
$$

These formulae determine the realistic relation:

$$
R_p = b_{\text{in}} + (R_{\text{min}0} - b_{\text{in}}) \exp (-\eta_\varepsilon \mu_\varepsilon),
$$

where $\eta_\varepsilon$ is expressed by Eq. (4) and

$$
\alpha_\varepsilon = -eR_p V_\varepsilon / b_{\text{in}} e V_{j, \text{in}}, \quad \mu_\varepsilon = b_{\text{in}} [e B_0] / \sqrt{2 m_\varepsilon e V_{j, \text{in}}},
$$

and $R_{p0}$ is the closest radius in the absence of the magnetic field, which satisfies the following relation:

$$
R_{p0}^2 - \alpha_\varepsilon b_{\text{in}} R_{p0} \exp (-\zeta_{\text{sc}} R_{p0} / b_{\text{in}}) - b_{\text{in}}^2 = 0.
$$

As an example, the absorption cross-sections are shown in Fig. 1 as a function of the strength of the uniform magnetic field $B_0$ for the case $R_p = 1$ cm, $\varepsilon_{\text{in}} = 10$ eV, (a) $V_\varepsilon = -10$ eV and (b) $V_\varepsilon = 10$ V. In the case of negative applied voltage, (a) $V_\varepsilon < 0$, the cross-sections at $B_0 = 100$ G increase from 1.57 cm$^2$ ($\zeta_{\text{sc}} = 0$), 2.06 cm$^2$ (0.3), and 2.62 cm$^2$ (1.0) to 2.01 cm$^2$ (+28.1 %), 2.29 cm$^2$ (+11.8 %), and 2.69 cm$^2$ (+6.9 %), respectively. On the other hand for the positive applied voltage, (b) $V_\varepsilon > 0$ the cross-sections decrease from 4.71 cm$^2$ ($\zeta_{\text{sc}} = 0$), 4.36 cm$^2$ (0.3), and 3.77 cm$^2$ (1.0) to 3.98 cm$^2$ (-15.5 %), 3.93 cm$^2$ (-9.9 %), and 3.19 cm$^2$ (-15.5 %), respectively. The relatively strong magnetic field enables an electron approach to the probe, which indicates the absorption cross-section increases or decreases for the case of negative and positive applied voltages, respectively. The plasma shielding has the same tendency. These effects make the absorption cross-section approach the geometrical cross-section of the probe ($= \pi R_p^2$).