Due to rapid progress in ultrahigh intensity lasers over last two decades, based on the invention of the chirped pulse amplification (CPA) and advanced femtosecond techniques, it is possible to drive electrons with relativistic energy opening new avenues of relativistic nonlinear optics and plasma physics. Currently, this wide field is diverging in two main broad areas; the first, related to laser fusion, high energy densities and laboratory astrophysics, and the second, related to ultra high field science, high energy particle acceleration and photon beam and ultra-fast attosecond phenomena. Laser plasma interactions are a useful test bed for exploring rich variety of strongly nonlinear complex plasma phenomena. They include a number of important three-wave (3WI) resonant instability models, involving the relativistic laser pump parametrically coupled to plasma eigen-modes; mostly of a decay type, with the laser energy transfer to lower frequency daughter waves, typically, to electrostatic plasma modes. Stimulated Raman backscattering is a 3WI resonant instability of the laser pump against excitation of an electron plasma wave and another electromagnetic (laser) wave which propagates backwards. In laser fusion, great concern is related to stimulated Raman (SRS) and Brillouin scattering on electron plasma waves (EPW) and ion sound waves, respectively, which can result in undesirable loss of laser energy at the target and production of energetic particles.

The nonlinear backward SRS model for the spatiotemporal evolution of complex amplitudes of the pump $A_0$, scattered $A_1$ and EPW $A_2$ in a weak coupling approximation is obtained from Maxwell’s and fluid equations in WKB approximation, assuming that resonant matching between frequencies and wave numbers of three waves ($\omega_0 = \omega_1 + \omega_2$, $k_0 = k_1 + k_2$) are satisfying their linear dispersion relations

$$\frac{\partial a_0}{\partial \tau} + V_0 \frac{\partial a_0}{\partial \xi} = -a_1 a_2, \quad (1)$$
$$\frac{\partial a_1}{\partial \tau} - V_1 \frac{\partial a_1}{\partial \xi} = a_0 a_2^*, \quad (2)$$
$$\frac{\partial a_2}{\partial \tau} + V_2 \frac{\partial a_2}{\partial \xi} = \beta_0^2 a_0 a_1^* - \Gamma a_2 + i\delta |a_2|^2 a_2, \quad (3)$$

with time and space variables $\tau = \omega_0 t$, $\xi = x/L$, and dimensionless amplitudes of laser $E_0$ and scattered wave $E_1$ electric fields and electron density fluctuations $n_e$. Normalized group velocities and the damping rate $\Gamma$ are expressed by

$$V_0 = \frac{c^2 k_0}{\omega_0^2 L}, \quad V_1 = \frac{c^2 k_1}{\omega_0 \omega_1 L}, \quad V_2 = \frac{3k_2\nu_e^2}{\omega_0\omega_{pe} L}, \quad \Gamma = \frac{\nu_e}{2\omega_0}, \quad (4)$$

and the laser intensity is ratio of the electron quiver velocity to the speed of light.

$$\beta_0 \equiv \frac{\gamma_{0ac} e}{m_e\omega_0 c}. \quad (5)$$

We note a self-modal nonlinearity in the equation for EPW, given as a relativistic phase detuning (shift) $\sim \delta |a_2|^2$, due to large EPWs excited through the SRS.

Break up of Manly-Rowe Invariants and Transition to Nonstationarity and Chaos

Assuming steady state ($\partial/\partial t \to 0$) in the system (1-3), conserved quantities (Manly-Rowe invariants) are

$$m_0 = V_0 \rho_0 (x) - V_1 \rho_1 (x) = \text{const.}, \quad (6)$$
$$m_1 = V_0 \rho_0 (x) + V_2 \rho_2 (x) = \text{const.}, \quad (7)$$
$$K(x) = A_0 A_1 A_2 \sin \phi - \frac{\sigma}{4} A_2^4 - \frac{\delta}{2} A_2^2 = \text{const.}(8)$$

with $\rho_i (x) = A_i (x)^2$, $\sigma = 0, 1, 2$, and $a_i (x, t) = A_i (x, t) e^{i\phi_i (x, t)}$ where $A_i$ and $\phi_i$ is the amplitude and phase of the wave, with the total phase shift given as $\phi = \phi_0 - \phi_1 - \phi_2$. For boundary conditions

$$n_0 (0) = 1, n_1 (L) = 0, n_2 (0) = 0 \quad (9)$$

the third invariant is $K(0) = 0$. However, at $x = L$, as generally, $A_2 (L) \neq 0$, gives $K(L) \neq 0$; which breaks up the invariance, i.e., $K(x) \neq \text{const.}$; hence, contradicts the steady state assumption. Such simple argument, predicts a generic cause of nonstationarity in SRS processes, e.g., as was observed in nonlinear saturation of backward SRS experiments in laser-plasmas. Indeed, by solving eqs. (1-3) for increased laser intensity, reveals a bifurcation sequence with a quasi-periodic transition to intermittency and spatiotemporal chaos. This generic complex property of nonlinear SRS saturation was exposed in early original papers. We note the above relevance to recently proposed schemes for ultra-fast compression and amplification of laser pulses based on SRS; which contrary to expectations find poor intensity scaling and coherence loss. Furthermore, the nonstationary saturation of SRS may impose a difficulty in current attempts to develop experimentally validated integrated simulation code for laser underdense-plasma instabilities (e.g. pF3D code) based on fluid and weak coupled-mode approximations.