§18. Momentum Balance and Radial Electric Fields in Axisymmetric and Nonaxisymmetric Toroidal Plasmas

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We have investigated how symmetry properties of toroidal magnetic configurations influence mechanisms of determining the radial electric field such as the momentum balance and the ambipolar particle transport [1]. Both neoclassical and anomalous transport of particles, heat, and momentum in axisymmetric and nonaxisymmetric toroidal systems are taken into account. Results obtained in this work are summarized in Table 1, where Γ_a, q_a, and $\langle (P_a)^s_\theta \rangle$ ($\langle (P_a)^s_\zeta \rangle$) represent the radial fluxes of particles, heat, and poloidal (toroidal) momentum for species a, respectively, and $J_\parallel$ is the parallel current.

Generally, in nonaxisymmetric systems, the radial electric field is determined by the neoclassical ambipolarity condition. Figure 1 shows an example of nonaxisymmetric toroidal systems with stellarator symmetry. We see from Fig. 1 that there exists an axis lying on the equatorial plane such that the system looks unchanged by a rotation about it by 180 degrees. When defining the origin $(\theta, \zeta) = (0, 0)$ at the position where this symmetry axis intersects the flux surface, the magnetic field strength $B(s, \theta, \zeta)$ has the same value at the points (s, θ, ζ) and $(s, -\theta, -\zeta)$: $B(s, \theta, \zeta) = B(s, -\theta, -\zeta)$. In axisymmetric systems with up-down symmetry, $B(s, \theta) = B(s, -\theta)$ is satisfied. It is shown by using a novel parity transformation that, for axisymmetric systems with up-down symmetry and quasisymmetric systems with stellarator symmetry, the particle fluxes are automatically ambipolar up to $\mathcal{O}(\delta^2)$ and the determination of the radial electric field E_s requires solving the $\mathcal{O}(\delta^3)$ momentum balance equations, where δ denotes the ratio of the thermal gyroradius to the characteristic equilibrium scale length.

In axisymmetric systems with large $\mathbf{E} \times \mathbf{B}$ flows on the order of the ion thermal velocity v_{Ti}, the radial fluxes of particles, heat, and toroidal momentum are dependent on E_s and its radial derivative while the time evolution of the E_s profile is governed by the $\mathcal{O}(\delta^2)$ toroidal momentum balance equation. In nonaxisymmetric systems, $\mathbf{E} \times \mathbf{B}$ flows of $\mathcal{O}(v_{Ti})$ are not generally allowed even in the presence of quasisymmetry because the nonzero local radial current is produced by the large flow term in the equilibrium force balance for which the Boozer coordinates cannot be constructed.

Table 1. Remarks on the transport fluxes Γ_a, q_a, $\langle J_\parallel B \rangle$, $\langle (P_a)^s_\zeta \rangle$, $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle$, the viscosity terms $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle$, $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle$, the ambipolarity condition $\sum_a e_a \Gamma_a = 0$, and the radial electric field E_s in toroidal systems with the $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\delta v_T)$. Here, ϵ_A is used to represent a measure of up-down asymmetry or stellarator-symmetry breaking.

Axisymmetric system with up-down symmetry

$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = 0$ and $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $\sum_a e_a \Gamma_a = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $E_s = \mathcal{O}(\delta)$ is determined from the $\mathcal{O}(\delta^3)$ toroidal momentum balance equation.

Axisymmetric system without up-down symmetry

$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = \mathcal{O}(\epsilon_A \delta^2)$ and $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\epsilon_A \delta^2)$. $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta^2)$ drives $E_s = \mathcal{O}(\epsilon_A) \Longrightarrow$ requires treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ when $\epsilon_A \gg \delta$.

Nonaxisymmetric system without quasisymmetry

$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are dependent on $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = \mathcal{O}(\delta^2)$ and $\langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\delta)$. $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta)$ determines E_s.

Quasisymmetric system with stellarator symmetry

$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle = 0$ and $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $\sum_a e_a \Gamma_a = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. E_s is determined from the $\mathcal{O}(\delta^3)$ momentum balance equation.

Quasisymmetric system without stellarator symmetry

$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle = \mathcal{O}(\epsilon_A \delta^2)$ and $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\epsilon_A \delta^2)$. $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta^2)$ drives $E_s = \mathcal{O}(\epsilon_A) \Longrightarrow$ requires treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ when $\epsilon_A \gg \delta$.

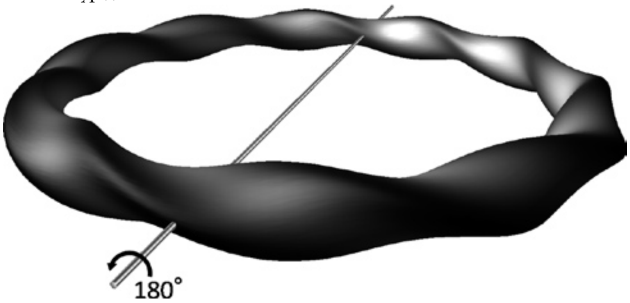

Fig. 1. A nonaxisymmetric toroidal system with stellarator symmetry. The system looks unchanged by a rotation about the shown axis by 180 degrees.

1) H. Sugama, T.-H. Watanabe, and M. Nunami, Plasma Phys. Control. Fusion **53**, 024004 (2011).