§12. Development of a Next-Generation High-Resolution Scheme for Magnetohydrodynamics

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We expect that a connection between macro- and micro-scale magnetohydrodynamic (MHD) dynamics and structure would be clarified in the next decades because an MHD turbulence in a global simulation can be realized by super-high-resolution simulations on next-generation computers. However, existing high-resolution schemes for MHD cannot be applied in those next-generation simulations because such schemes often suffer from various numerical instabilities which become crucial in high-resolution grids. Therefore, in this study, we construct a very robust high-resolution numerical scheme for MHD on next-generation massively parallel petaflops-scale computers. In particular, in this paper, robust and accurate divergence-cleaning and divergence-free techniques for the magnetic field are newly proposed and comparatively investigated with existing numerical techniques.

For simplicity, we restrict our attention to a two-dimensional finite volume method with a uniform Cartesian grid system, where a cell center is denoted by (i, j) and cell boundaries are denoted by $(i \pm 1/2, j)$ for x and $(i, j \pm 1/2)$ for y, respectively. Grid sizes are assumed that $\Delta x = \Delta y = \Delta$. The extension to other grid systems is straightforward.

In the projection method, the magnetic field at the cell center computed by a multi-dimensional MHD scheme $\boldsymbol{B}^*$ is projected into the subspace of divergence-free field after every time step as $\boldsymbol{B}^{n+1} = \boldsymbol{B}^* + \nabla\phi$ where $\nabla^2\phi = -\nabla\cdot\boldsymbol{B}^*$. However, this method sometimes leads to a checkerboard instability or an odd-even decoupling. Moreover, in this method, the magnetic field itself at the cell center which is computed by a basic high-resolution MHD scheme is corrected. In another respect, the thermal energy is modified after every time step. Therefore, we propose herein another algorithm of the projection method[1). First, we assume that

$$\begin{aligned} b_{x\,i+1/2,j} &= (B_{x\,i+1,j} + B_{x\,i,j})/2 + (\phi_{i+1,j} - \phi_{i,j})/\Delta, \\ b_{y\,i,j+1/2} &= (B_{y\,i,j+1} + B_{y\,i,j})/2 + (\phi_{i,j+1} - \phi_{i,j})/\Delta, \end{aligned} \quad (1)$$

respectively. Using the discrete divergence-free condition, the correction terms in Eqs. (1) and (2) are calculated by

$$\begin{aligned} &(\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1} - 4\phi_{i,j})/\Delta^2 \\ &= -(B_{x\,i+1,j} - B_{x\,i-1,j} + B_{y\,i,j+1} - B_{y\,i,j-1})/2\Delta. \end{aligned} \quad (2)$$

In this method, the normal component of the magnetic field at the cell boundaries is corrected before every time step. Then, numerical fluxes are computed using the magnetic fields (1) and (2). Because the time evolution of the magnetic field at the cell center is simply computed by the basic MHD scheme without correction, the thermal energy is not modified in the divergence-cleaning step. It was confirmed from some numerical tests that the proposed method can remove unwanted odd-even decoupling while the original method cannot. We also found that the magnitude of the divergence errors in the proposed method becomes smaller than that in the original method.

We also construct a new variant of the Constrained Transport (CT) method[1). In the CT method, Stokes' theorem is applied for the cell boundaries as

$$\begin{aligned} \Delta b_{x\,i+1/2,j}/\Delta t &= -(E_{z\,i+1/2,j+1/2} - E_{z\,i+1/2,j-1/2})/\Delta, \\ \Delta b_{y\,i,j+1/2}/\Delta t &= (E_{z\,i+1/2,j+1/2} - E_{z\,i-1/2,j+1/2})/\Delta. \end{aligned} \quad (3)$$

After the time integration of (3), the magnetic field at the cell center is reconstructed by some average. In particular, we adopt the flux-CT method. The flux-CT method is well suitable for the high-resolution MHD scheme because the electromotive forces (EMF) E_z of the induction equation are evaluated by the numerical fluxes for the induction equation. However, the original flux-CT method, where the EMF at a cell edge is evaluated by the arithmetic mean of the numerical fluxes at neighbor cell boundaries, is not reduced to the one-dimensional basic scheme for plane-parallel grid-aligned flows. On the other hand, consistent flux-CT methods can be reconstructed using the derivatives of the EMF on the cell boundaries. In order to achieve high-resolution and robustness, the derivatives of the EMF are given by the HLL fluxes of the evolution equations for the derivatives of the magnetic field on the cell boundaries. It was indicated from numerical tests that negative pressures are generated both in the present flux-CT method and the original method in extremely severe situations. It is guessed that because the thermal energy is incorrectly modified at the reconstruction step of the magnetic field at the cell center. Therefore, in our scheme, we also apply an energy correction to the total energy density. As a result, it was confirmed that the present method can provide correct solutions while the original may amplify some MHD waves.

As a practical problem, a high-resolution simulation of magnetic reconnection was performed using the new flux-CT method proposed in this paper and its result was compared with that for a recent excellent work[2). It was clearly indicated that complicated and fine structures of various discontinuities and shocks are well reproduced.

1) Miyoshi, T., Kusano, K.: Plasma Fusion Research, in press.
2) Zenitani, S., Miyoshi, T.: Physics of Plasmas **18** (2011) 022105.