# §22. Study of Statistical Laws and Spatial Structure of Passive Scalar in NS and MHD Turbulence by Using Massive Parallel Direct Numerical Simulation

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It is quite common that there exists a mean temperature gradient in the fusion reactor where the gradient goes from the reactor center to the edge, in the atmosphere where the temperature decreases from the ground to the sky in the day time atmospheric boundary layer, and in the ocean where the mean gradients of the temperature and salinity exist but in the opposite directions. One of the fundamental issues of the scalar transfer in this situation is to obtain reliable estimate of the scalar transfer flux under the existence of the mean uniform scalar gradient $\Gamma = \langle \nabla \Theta \rangle$ in the homogeneous isotropic steady turbulence. The scalar flux in the direction parallel to the mean gradient (say $x_3$) is defined by $q = -u_3\theta$, where $\theta$ is the scalar fluctuation, and the total scalar field is $\Theta = \Gamma x_3 + \theta$. We have studied the dependence of the Nusselt number $Nu = -\langle u_3\theta\rangle/(\Gamma\kappa)$ on the Péclet number $Pe = UL/\kappa$, the statistical distribution of the fluctuations of the scalar flux around the mean, and the spatial structure of the scalar field by using very high resolution direct numerical simulation (DNS).[1)]

In the steady state, we can put $\Gamma = 1$ without lose of generality, and put the Schmidt number $Sc = \nu/\kappa$ to be unity. Figure 1 shows the variation of $Nu$ against $Pe$. The DNS data shows that $Nu$ increases as $Nu \propto Pe$, while the experimental data tends to be $Nu \propto Pe^{0.55}$. [2,3)] The reason for the discrepancy is not known. An analysis using the statistical theory of turbulence has shown that the correlation $\langle u_3\theta\rangle$ is given by the time integral of the Lagrangian velocity auto-correlation as

$$\langle u_3(\boldsymbol{x},t)\theta(\boldsymbol{x},t)\rangle = \int_{-\infty}^{t} \langle v_3(\boldsymbol{x},t|t)v_3(\boldsymbol{x},t|s)\rangle ds,$$

where $\boldsymbol{v}(\boldsymbol{x},t|s)$ is the velocity of the fluid particle measured at time $s$ whose trajectory passes $(\boldsymbol{x},t)$.[1)] From this expression we have $Nu \propto Pe$.

Figure 2 shows the velocity $u_3$, the scalar fluctuation $\theta$, and the scalar flux $u_3\theta$ which are plotted along the direction of the mean gradient. Three lines indicate the integral scales of the velocity and scalar, and the half the velocity integral scale from the top, respectively. It is clearly seen that the total scalar field is of the form of stairs whose size is about the velocity integral scale and the amplitude of the scalar jump is about a few times $u_{\rm rms}$, and the sign of $u_3$ is opposite to that of $\theta$. The probability density function (PDF) of the scalar flux is negatively skewed and has a longer negative tail irrespective of $Pe$. The theory has predicted the PDF as $P(q) \propto \exp(-c_\pm |q|/\sigma_q)$ where $c_\pm = \frac{1}{1\pm r}\left(\frac{\sigma_q}{\sigma_w \sigma_\theta}\right)$.[4)] Since $r = \langle u_3\theta\rangle < 0$, we have $c_- < c_+$, meaning that $P(q)$ has the longer negative tail, which is consistent with the DNS data.

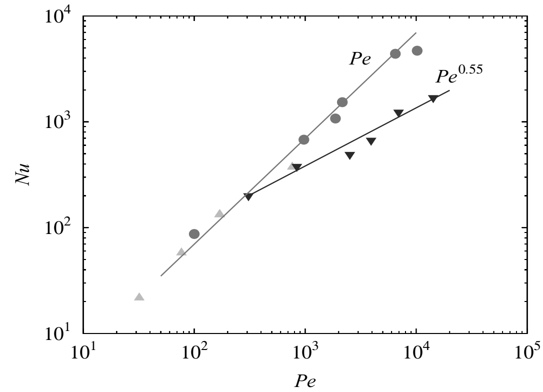


FIG. 1: Variation of the Nusselt number against Péclet number. • (red) : present DNS, ▲ (green): DNS by Overholt and Pope,[2)] ▼ (blue): experiment by Mydlarski,[3)] Straight lines show $Nu \propto Pe$ for the theoretical prediction, and $Nu \propto Pe^{0.55}$ for the experiment, respectively.

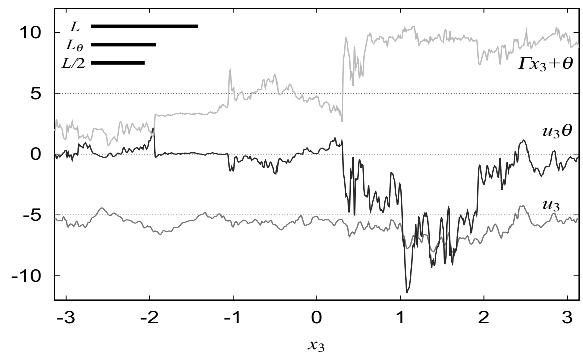


FIG. 2: Variation of $\Gamma x_3 + \theta$ (green), $q = u_3\theta$ (blue) and $u_3$ (red) with respect to $x_3$ at $(x_1 = 0, x_2 = -\pi)$. Stepwise increase in the scalar fluctuation is observed.

1) Gotoh T. and Watanabe T., Physica D, to appear (2011).
2) Overholt M. R. and Pope S. B. Phys. Fluids **8** (1996) 3128–3148.
3) Mydlarski, L. J. Fluid Mech. **475** (2003) 173–203.
4) Thoroddsen S. T. and Van Atta C. W. J. Fluid Mech. **244** (1992) 547–566.