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We develop a new Monte Carlo simulation code for overcoming difficulty in conventional Monte Carlo methods, where the difficulty is caused by nonlinear terms in fluid equations. To confirm the computational principle of the new method, we solve Dirichlet problems in one-dimensional (1D, i.e., radial) coordinate space in the first trial. The code is called DIPS-1D (Dirichlet Problem Solver in 1D coordinate space).

In general, the fluid equation expressed in the form of the Fokker-Planck type equation can be rewritten as the following initial-boundary value problem (t is replaced by $t_1 - t$):

$$\left( L + \eta_s \right) u + \frac{\partial u}{\partial t} = h_s(t, x) \quad \text{in } \Omega,$$

$$u(t_1, x) = \Phi(x) \quad \text{on } \partial \Omega,$$

$$u(t, x) = G(t, x) \quad \text{on } \partial S,$$

where $\partial \Omega$ is a bounded domain with the boundary $\partial \Omega$, $\Omega = \Omega \times [0, t_1)$, $S = \partial \Omega \times [0, t_1)$, and

$$Lu = \left\{ \frac{1}{2} D_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + U_{ij} \frac{\partial}{\partial x^j} \right\} u,$$

$$U_{ij} = -U_{ji} + \frac{1}{2} \frac{\partial D_{ij}}{\partial x^l},$$

$$\eta_s = -\eta - \frac{\partial U_{ij}}{\partial x^j},$$

$$h_s = -h.$$

If $\Phi, G, h, \eta, D$ and $U$ are assumed to be given-smooth functions, the solutions of Eqs. (1)-(3) are known to be described as

$$u(t, x; \Phi, G, h_s, \eta_s, D, U_s) = E_{t, x} \left[ \Phi(X(t_1)) \exp \left\{ \int_t^{t_1} \eta_s(s, X(s)) ds \right\} \chi_{t=t_1} \right] + E_{t, x} \left[ G(\tau, X(\tau)) \exp \left\{ \int_t^{\tau} \eta_s(s, X(s)) ds \right\} \chi_{t<\tau} \right] - E_{t, x} \left[ \int_t^{\tau} h_s(s, X(s)) \exp \left\{ \int_t^s \eta_s(\nu, X(\nu)) d\nu \right\} ds \right].$$

where $E_{t, x}$ is the expectation operator given by the diffusion process $X(s)$ in coordinate space:

$$dX^i(s) = \sigma^i_j(t, X(s)) dW^j(s) + U_{ij}^s(t, X(s)) ds,$$

satisfying $X(t) = x$. Here $D_{ij} = \sigma^i_k g^{kl} \sigma^l_j$, $g^{kl}$ is the metric, $W(s)$ is a Brownian process, $\chi_A$ is the indicator function of a set $A$ (e.g., $\chi_{t<t_1} = 1$ if $t < t_1$, and $\chi_{t=t_1}$ = 0 otherwise), and $\tau$ is defined as

$$\tau = \begin{cases} \text{the first time } \nu \in [t, t_1) \text{ that } X(\nu) \text{ leaves } \Omega \text{ if such a time exists,} \\
1 \quad \text{otherwise.} \end{cases}$$

Using the Monte Carlo simulation code DIPS-1D based on Eq. (8), we solve iteratively the following radial energy transport equation for electrons:

$$\frac{\partial}{\partial t} \left( \frac{3}{2} n T_e \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \sigma v_{\text{rec}} n T_e \right) - s \frac{\partial}{\partial r} T_e + S = 0,$$

where the initial condition is given as $\Phi(r) = T_{\text{ax}}(0) - \left( T_{\text{ax}}(0) - T_{\text{edge}}(0) \right) r/a$ with $T_{\text{ax}}(0) = 2$ keV and $T_{\text{edge}}(0) = 200$ eV. The boundary conditions are $G(t, r_0) = T_e(t, r_0) = T_e(t, r_0 + \delta r) - \delta r \partial T_e/\partial r(t, r_0)$ and $G(t, a) = T_e(t, a) = 200$ eV with sufficiently small $\delta r$ and $r_0$ (i.e., $0 < \delta r/a, r_0/a < 1$), and $\partial T_e/\partial r(t, r_0) = -1.8$ keV/m. $T_e$ is the electron temperature, and the constant density is assumed to be $n = 1 \times 10^{19}$ m$^{-3}$. $\kappa_e^B$ is the gyro-Bohm thermal conductivity, $a = 1$ m is the minor radius, $\sigma v_{\text{rec}}$ is the radiative recombination rate coefficient, and $S$ is the heat source. The steady-state solution of Eq. (10) is given in Fig. 1. Details are shown in Ref. [1].