§33. Statistical Analysis of Time-Resolved X-Ray Images – Generalized N-Dimensional Principal Component Analysis –

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In laser fusion researches, time-resolved x-ray images are important for observation of laser-imploded targets. The time-resolved x-ray images have been obtained by using a multi-imaging x - ray pinhole camera coupled to an x - ray streak camera and the time-resolution is up to 10-ps [1]. The time-resolved x-ray image can be considered as a three-dimensional image. It is difficult to extract important information (core information) by using a simple way or conventional methods because of its large dimension. To date, Principal component analysis (PCA) is generally used for core information or feature extraction and has been widely used in a lot of research fields, such as computer vision and pattern recognition. Essentially, it is to transform data to a subspace where the data can be represented compactly and efficiently. The projections on the subspace bases are called principal components. PCA is the method for 1-dimensional (1D) vector data. If the time-resolved x-ray image, which is a three-dimensional data is used for analysis, the three-dimensional data should be unfolded to be a 1D vector in order to make it to be processed by PCA. Usually the unfolded vector is very long, so the dimension of the covariance matrix is so huge that it is very difficult to calculate the bases in the unfolding vector subspace. In this research, we propose a tensor-based generalized N-dimensional principal component analysis (GND-PCA) for statistical analysis of multi-dimensional data [2, 3]. In the proposed method, the multi-dimensional data is treated as a tensor. For example, the time-resolved x-ray image is treated as a 3^{rd}-order tensor.

The basic idea of GND-PCA is that we want to reconstruct the original N-th order tensor $\mathcal{A} \in \mathbf{R}^{I_1 \times I_2 \times \cdots \times I_N}$ with a lower rank core tensor $\mathcal{B} \in \mathbf{R}^{J_1 \times J_2 \times \cdots \times J_N}$, where $J_n < I_n$, and try to find a set of optimal matrices $\mathbf{U}^{(n)} \in \mathbf{R}^{I_n \times J_N}$, $n = 1,2,\cdots,N$ with orthogonal column for each mode. The reconstruction of N-th order tensor $\mathcal{A}$ can be expressed as $\hat{\mathcal{A}} = \mathcal{B} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times \cdots \times_N \mathbf{U}^{(N)}$. Illustration for 3^{rd} order tensor reconstruction is shown in Fig.1.

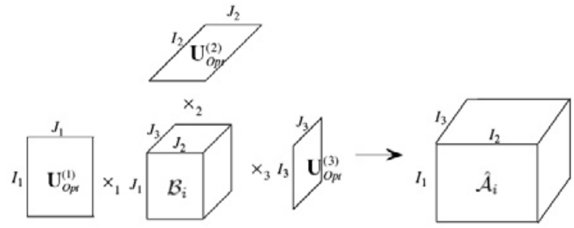

Fig.1 Illustration of reconstructing a 3^{rd} order tensor

The optimal orthogonal matrices $\mathbf{U}^{(n)}$ can be determined by minimizing a cost function as Eq.(1).

$$\begin{aligned} S &= \sum_{i=1}^{M} \left\| \mathrm{A}_i - \hat{\mathrm{A}}_i \right\|^2 \\ &= \sum_{i=1}^{M} \left\| \mathrm{A}_i - \mathrm{B}_i \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times \cdots \times_N \mathbf{U}^{(N)} \right\|^2 \end{aligned} \quad (1)$$

There is no close-form solution to simultaneously resolve the matrices $\mathbf{U}^{(n)}$. So we use an iteration algorithm to simultaneously calculate the optimal matrices $\mathbf{U}^{(1)}_{opt}, \mathbf{U}^{(2)}_{opt}, \cdots, \mathbf{U}^{(N)}_{opt}$.

The proposed method has been successfully applied to statistical analysis of medical three-dimensional images such as CT volumes [2, 3] and MR volumes [2]. We are going to applied GND-PCA to real time-resolved x-ray images of laser-produced plasma.

1) H. Shiraga et al., *Review of Scientific Instrumen*. 66 (1995) 722-724.
2) R. Xu and Y.-W.Chen, *Neurocomputing*, 72 (2009) 2276-2287.
3) X. Qiao and Y.-W. Chen, Proc. of International Conference on Pattern Recognition, (2010) 2488-2491.