§18. Relative Reduction of Nonlinear Energy Transfer by Nonlinear Mode Interaction in Hall MHD Turbulence

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Depression of nonlinearity in fully developed turbulence attracted interests of turbulent researchers and is investigated for the neutral fluid case (e.g.[1, 2]) and the magnetohydrodynamic (MHD) fluid case [3]. Their analysis were mainly based on the comparison of turbulent fields with the phase randomized fields. In the present study, we analyzed the freely decaying, isotropic homegeneous turbulent Hall MHD solution whose energy spectrum shows self-similar feature [4], and attempt to compare the amplitudes of triad interactions during the time development.

We analyzed the time development of the shell averaged triad mode interactions. In order to compare the amplitudes, triad interactions are normalized in terms of the resistivity η and the disspation rate of the magnetic energy of each snapshot data $\epsilon_B(t)$. Since characteristic scales of turbulence such as Kolmogorov scale changes gradually in time, the boundary of shells in the Fourier space are determined according to the wave number corresponding to Kolmogorov scale $k_\eta(t) = (\epsilon_B(t)/\eta^3)^{1/4}$.

Furthermore, the band width of the analyzing shells are given by a geometric sequence in order to the characteristic oscillation wavelength of the band pass filtered field λ_j and the window width of the band pass filter Δ_j obey the scaling relations given by

$$\lambda_j \propto \sqrt{2}^j/k_\eta(t),\ \Delta_j \propto \sqrt{2}^j/k_\eta(t).$$

In the present study, the common ratio of the neighbouring shells is set to $\sqrt{2}$, i.e., the band pass filtered field is given by

$$\boldsymbol{f}_j(\vec{x},t) = \sum_{|\vec{k}|=k_\eta(t)/(\sqrt{2})^{j+1}}^{k_\eta(t)/(\sqrt{2})^j} \hat{\boldsymbol{f}}(\vec{k},t)\exp\left(i\vec{k}\cdot\vec{x}\right). \quad (1)$$

According to this normalization, their amplitudes become comparable quantitatively for the different time data.

In the Figure given below we show the time development of the energy transfer due to mode interactions given by

$$T_{UU}(j,k,l) = -\int ((\boldsymbol{u}_k\cdot\nabla)\boldsymbol{u}_j)\cdot\boldsymbol{u}_l \mathrm{d}^3\vec{x},$$

$$T_{UB}(j,k,l) = \int (\boldsymbol{j}_j\times\boldsymbol{b}_k)\cdot\boldsymbol{u}_l \mathrm{d}^3\vec{x},$$

$$T_{BU}(j,k,l) = \int (\nabla\times(\boldsymbol{u}_j\times\boldsymbol{b}_k))\cdot\boldsymbol{b}_l \mathrm{d}^3\vec{x},$$

$$T_{BB}(j,k,l) = -\epsilon\int (\nabla\times(\boldsymbol{j}_j\times\boldsymbol{b}_k))\cdot\boldsymbol{b}_l \mathrm{d}^3\vec{x}.$$

It is remarkable that the energy transfer by the induction term and the Lorentz force term shows somewhat stationary characteristics, while the inertial term and the Hall term effects gradually decreases. Investigation of the physical reasons of this novel selective reduction phenomenon is now underway.

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3) Servidio S., Matthaeus W.H., and Dmitruk P.: *Phys. Rev. Lett* **100**:095005, 2008.

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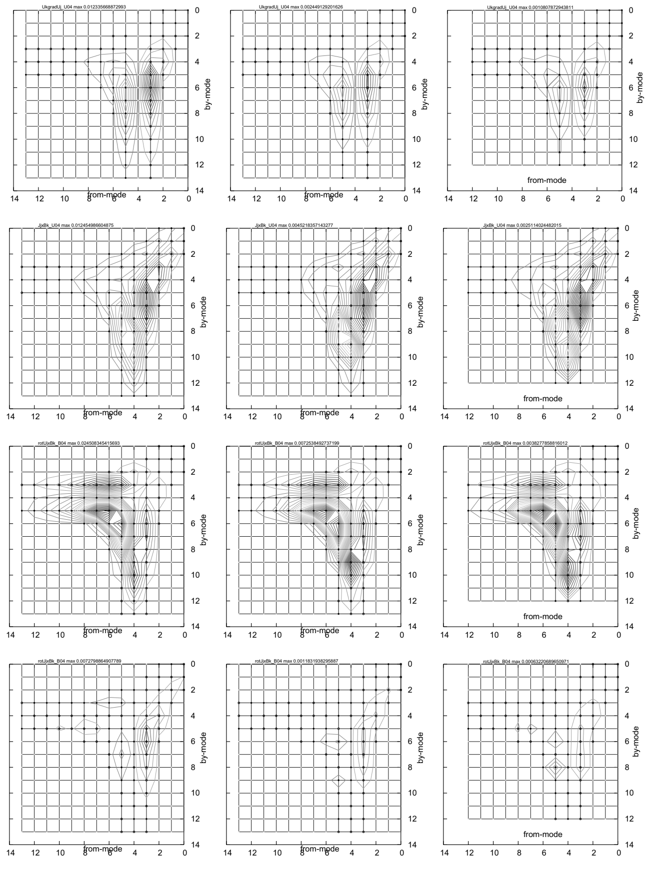

Figure: time evolution of the typical profiles of shell averaged triad mode interactions $T_{XX}(j,k,4;t)$. Each set of three panels from top to bottom: T_{UU}, T_{UB}, T_{BU} and T_{BB}. Three panels in each set correspond to the dataset at $t = 1.5$, 2.5 and 3.5 (left to right) in dimensionless time. Abscissa: number of j, ordinate: number of k; contours in green to red: energy receiving interactions, blue to purple: energy losing ones.