§33. Development of Extended Finite/ Boundary Node Method

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1. Introduction

The finite element method (FEM) and the boundary element method (BEM) have been applied to various fields in fusion engineering and plasma physics and, consequently, they have produced a number of fruitful results. Before using a numerical code based on the FEM/BEM, a region must be divided into a set of elements.

For the purpose of resolving these difficulties, many types of meshless methods have been so far proposed[1, 2]. However, all of them have been plagued by an inherent demerit that originates from the implementation method of essential boundary conditions. For example, the Lagrange multiplier method and the penalty method are adopted as the implementation method in the Element-Free Galerkin (EFG) method[1] and the Meshless Local Petrov-Galerkin (MLPG) method[2], respectively.

The purpose of the present study is to propose a new implementation method of elementary boundary conditions and to numerically investigate the performance of the proposed method.

2. Novel Meshless Method

For simplicity, we consider the following 2D Poisson problem on the domain Ω bounded by a simple closed curve $\partial\Omega$:

$$-\nabla u = p \text{ in } \Omega,\ u = \bar{u} \text{ on } \Gamma_{\mathrm{D}},\ \frac{\partial u}{\partial n} = \bar{q} \text{ on } \Gamma_{\mathrm{N}}, \qquad (1)$$

where Γ_{D} and Γ_{N} are parts of $\partial\Omega$ such that $\Gamma_{\mathrm{D}} \cup \Gamma_{\mathrm{N}} = \partial\Omega$ and $\Gamma_{\mathrm{D}} \cap \Gamma_{\mathrm{N}} = \phi$. In addition, $\boldsymbol{n}$ denotes an outward unit normal on $\partial\Omega$. Furthermore, the superposed bar indicates prescribed boundary values and $p(\boldsymbol{x})$ is a given function on Ω. As is well known, both the 2D Poisson equation and the natural boundary condition are satisfied if and only if the following weak form is fulfilled:

$$\forall w \text{ s.t. } w|_{\Gamma_{\mathrm{D}}} = 0:$$
$$\iint_{\Omega} \nabla w \cdot \nabla u \, d^2\boldsymbol{x} = \iint_{\Omega} wp \, d^2\boldsymbol{x} + \int_{\Gamma_{\mathrm{N}}} wq \, ds.$$

Although the essential boundary condition is included in the weak form of the EFG and the MPLG, it is not included in the above weak form.

In the present study, the above weak form and the essential boundary condition are discretized separately. First, the test function $w(\boldsymbol{x})$ and the trial function $u(\boldsymbol{x})$ are assumed as

$$w \in \mathrm{span}(\psi_1, \psi_2, \cdots, \psi_N),\ u \in \mathrm{span}(\phi_1, \phi_2, \cdots, \phi_N).$$

In addition, the function space on Γ_{D} is also assumed to be $\mathrm{span}(N_1, N_2, \cdots, N_K)$. Under the above assumptions, the discretized form of the Poisson problem can be written in the form,

$$\begin{bmatrix} A & C \\ D^T & O \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{g} \end{bmatrix}, \qquad (2)$$

$$A \equiv \sum_{i=1}^{N}\sum_{j=1}^{N} \iint_{\Omega} \nabla\psi_i \cdot \nabla\psi_j d^2\boldsymbol{x}\ \boldsymbol{e}_i^* \boldsymbol{e}_j^{*T},$$

$$C \equiv \sum_{i=1}^{N}\sum_{p=1}^{K} \int_{\Gamma_{\mathrm{D}}} \psi_i N_p ds\ \boldsymbol{e}_i^* \boldsymbol{e}_p^T, D \equiv \sum_{i=1}^{N}\sum_{p=1}^{K} \int_{\Gamma_{\mathrm{D}}} \phi_i N_p ds\ \boldsymbol{e}_i^* \boldsymbol{e}_p^T.$$

Especially when the test and trial functions are chosen such that $\psi_i = \phi_i \quad (i = 1, 2, \cdots, N)$, (2) coincides with the matrix equation for the EFG method[1]. Also, for the case with $N_p(s) = \delta(s - s_p) \quad (p = 1, 2, \cdots, K)$, C and D can be calculated analytically. Moreover, the essential boundary condition is exactly fulfilled for this case. In the following, the meshless method with $N_p(s) = \delta(s - s_p)$ $(p = 1, 2, \cdots, K)$ are called the collocation meshless method.

3. Performance Evaluation of Proposed Meshless Method

On the basis of the collocation meshless method, a numerical code has been developed for solving the 2D Poisson problem. By means of the code, we have investigated the performance of the collocation meshless method. The results of computations show that, from the standpoint of the accuracy, the collocation meshless method is superior to the standard one (see Fig.1).

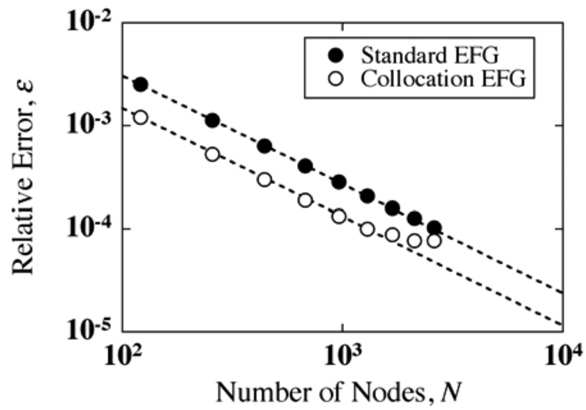

Fig. 1. The relative error ε as functions of the number N of nodes. Here, the Poisson problem is solved by means of either the standard EFG or the collocation EFG.

1) Belytschko, T. et al.: Int. J. Numer. Mech. Engng. **37** (1994) 229.
2) Atluri, S. N. et al.: Comput. Mech. **22** (1998) 117.