Strong Shear Formation by Poloidal Chain of Magnetic Islands

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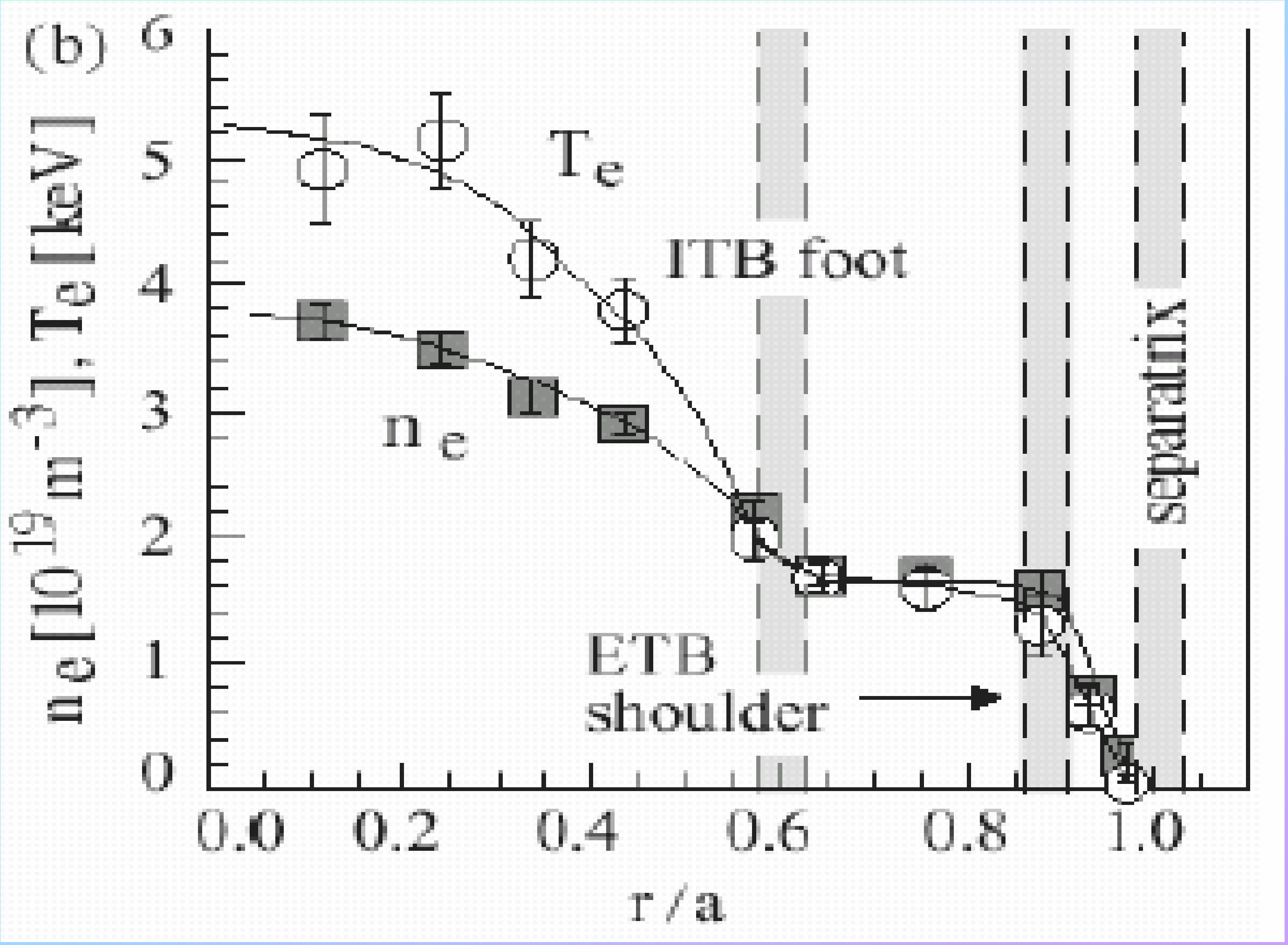
Objectives

We will shown that:

• vortical convective cells are narrower for larger shear and for abrupt plasma density profile. Also amplitude of vortex saturation is inversely proportional to shear. It promotes abrupt plasma density profile and ITB formation.

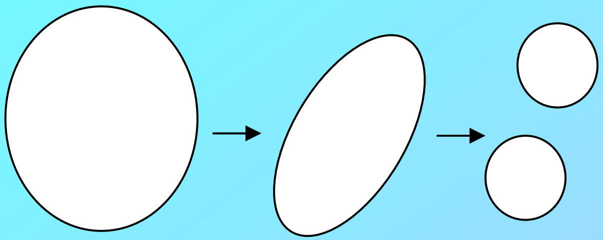
• plasma heating near low order rational surface with poloidal chain of narrow magnetic islands can lead to shear formation.

We consider ITB.



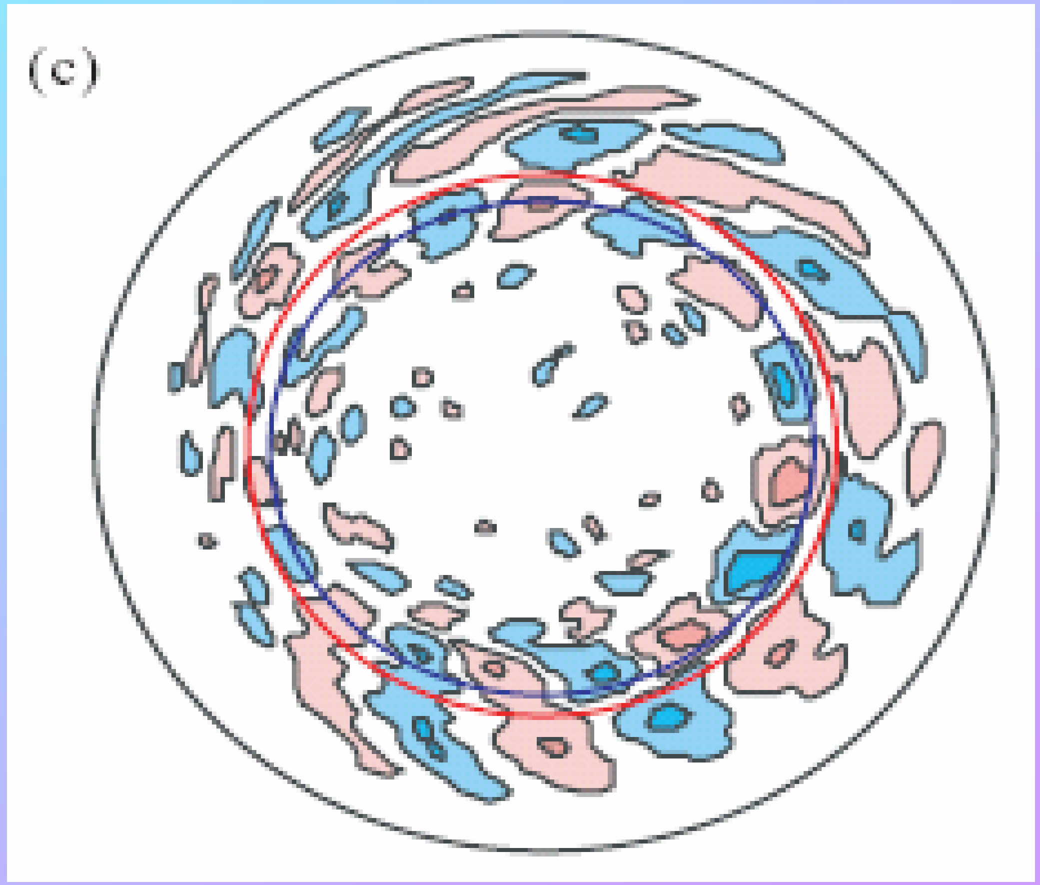
J.W. Connor et al. 2004

There is mechanism of turbulence and anomalous transport damping, providing shear of angular velocity $\partial_r \omega_\theta \neq 0$ of plasma particles. In other words, providing propagation of plasma layers relative to each other. By this way the plasma particle bunches or plasma particle holes as parts of plasma perturbations are damped.



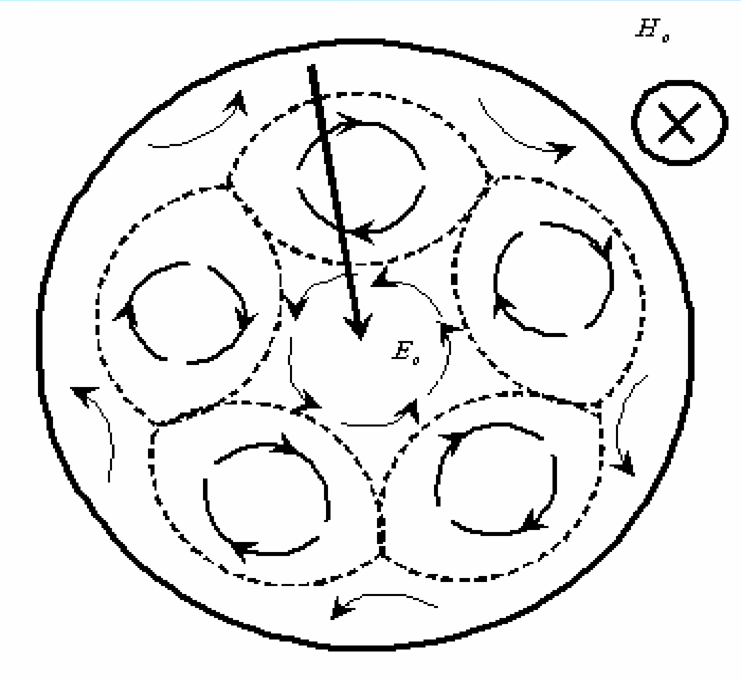
Mince vortices and their amplitude saturation by shear

J.W. Connor et al. 2004

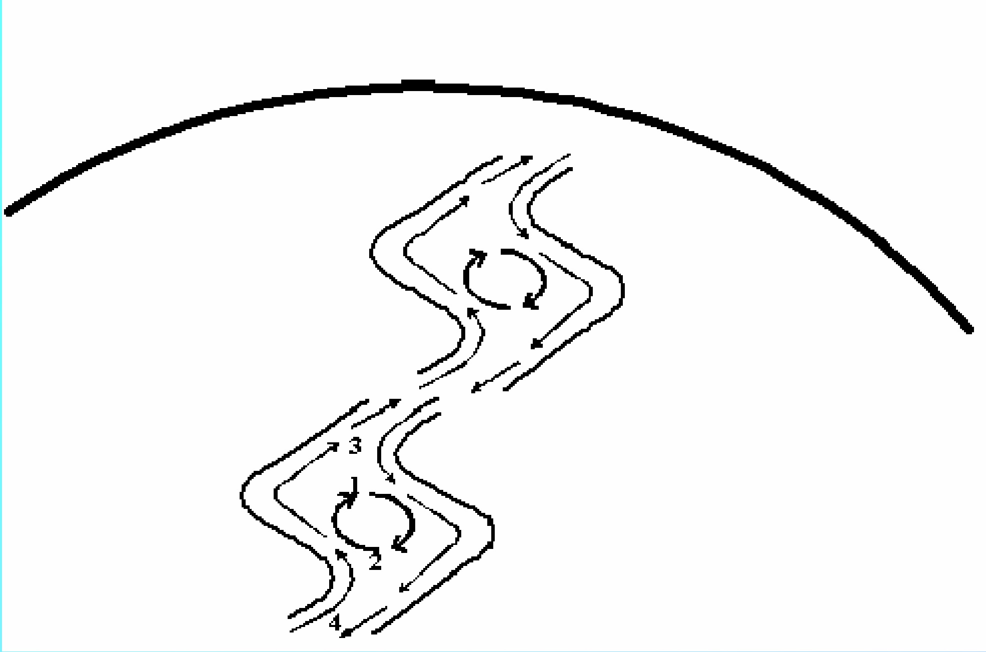


Spatial structure of vortices

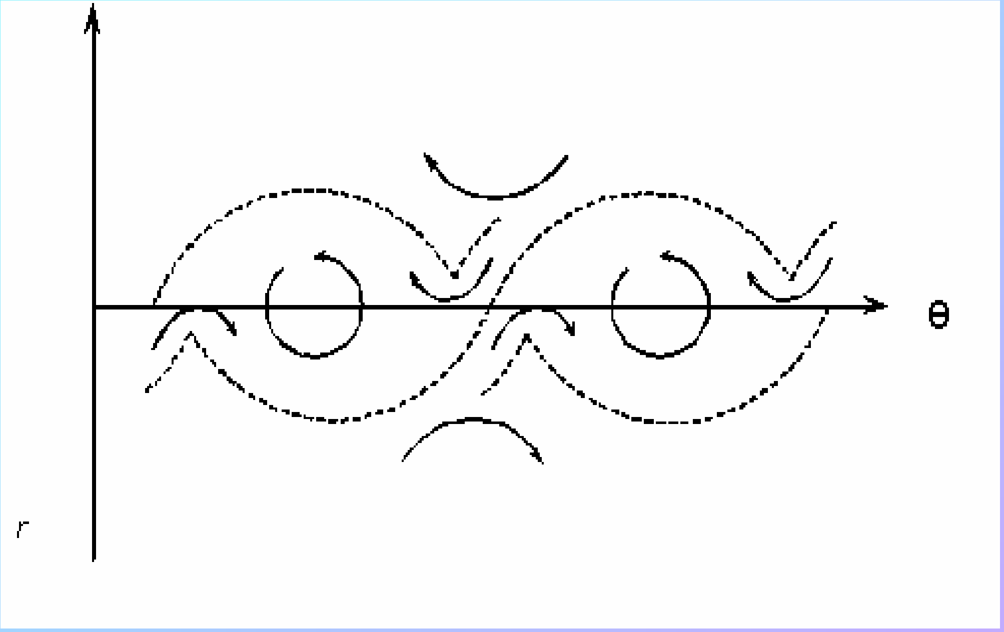
But let us at first consider other effect of turbulence and anomalous transport damping. For that we consider spatial structure of vortices in crossed magnetic $\vec{H}_0$ and radial electrical $\vec{E}_{r0}$ fields in rest-frame rotated with $\omega_{ph} \equiv V_{ph} / r_v$. r_v is radius of vortex localization, $V_{ph} = V_{\theta 0}\big|_{r=r_v}$. For simplicity let us consider single chain of vortices in cylindrical approximation.



Anomalous transport is determined by set of chains.



The convective – diffusive transport, performed by vortices



Trap particles by vortices in plane approximation.

Neglecting nonstationary and nonlinear members, from electron motion eq we have

$$\vec{V}_{\perp} = \vec{V}_{\theta 0} + \left(\frac{e}{m_e \omega_{ce}}\right)\left[\vec{e}_z, \vec{\nabla}_{\perp}\phi\right] - \left(\frac{1}{n_e m_e \omega_{ce}}\right)\left[\vec{e}_z, \vec{\nabla}\delta p_e\right]$$

$$\vec{V}_{\theta 0} = -\left(\frac{e}{m_e \omega_{ce}}\right)\left[\vec{e}_z, \vec{E}_{ro}\right] - \left(\frac{1}{n_e m_e \omega_{ce}}\right)\left[\vec{e}_z, \vec{\nabla} p_{0e}\right]$$

Both members in $\vec{V}_{\theta 0}$ have the same sign and effect in one direction as against ions. Using $d_t\theta = d_t\theta_1 + \omega_{ph}$ and decomposition $V_{\theta o}(r)$ on $\delta r \equiv r - r_v$ near r_v we obtain eq., describing electron dynamics in vortex

$$(\delta r)^2 + \frac{4}{r_v m_e \omega_{ce}(r_v)\partial_r \omega_{\theta o}\big|_{r=r_v}}\left(e\phi - \frac{\delta p_e}{n_e(r_v)}\right) = \text{const}$$

$$\omega_{\theta o} \equiv V_{\theta o}(r)/r$$

Let us connect $e\phi - \delta p_e / n_e(r_v)$ with $\alpha \equiv \vec{e}_z \mathrm{rot} \vec{V}_e$. From electron eq. of motion we derive

$$\frac{\delta r_v^2}{r_0 L} \approx \left(\frac{2T_e}{e\Delta\varphi}\right) \frac{\alpha}{\omega_{ci}(k\rho_{ci})^2} \qquad \frac{\alpha}{V_{thi}\rho_{ci}} \approx \left(\frac{e}{T_e}\right)\Delta\varphi - \left(\frac{1}{n_e}\right)\Delta n_e$$

If $\alpha \to \omega_{\theta o}$ for $k\rho_{ci} \approx 1$ $\qquad \delta r_v^2 \approx 2\rho_{ci}^2$

Frequency of electron oscillation in vortex

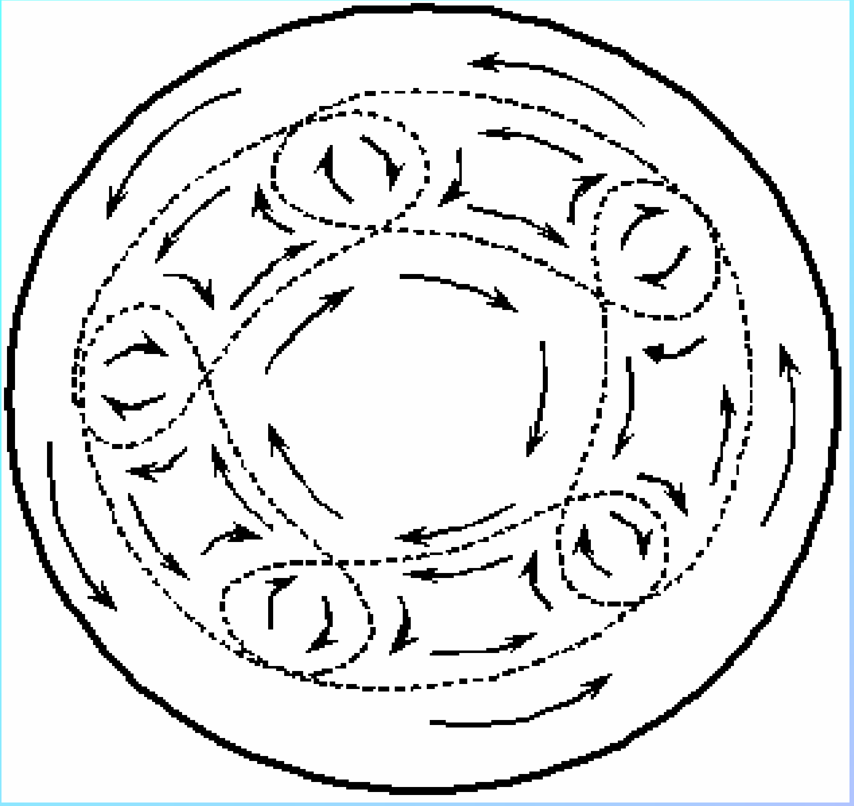
$$\Omega_r = \ell_\theta \left[\frac{\left| \partial_r \omega_{\theta o} \right|_{r=r_v} \Big|}{r_v m_e \omega_{ce}(r_v)} \left| e\phi_o - \frac{\delta p_{e0}}{n_e(r_v)} \right| \right]^{1/2}$$

$$\boxed{\delta r_v \approx 2\left[\frac{2}{r_v m_e \omega_{ce}(r_v)\left|\partial_r \omega_{\theta o}\right|_{r=r_v}\Big|}\left|e\phi_o - \frac{\delta p_{e0}}{n_e(r_v)}\right|\right]^{1/2}}$$

$$\delta r_v \propto \frac{1}{\left|\partial_r \omega_{\theta o}\right|_{r=r_v}\Big|}$$

It helps ITB formation.

Vortexes of large amplitudes



Counter-flows in electron bunches. Opposite rotation of electron holes and bunches.

$$\delta r_h = \left[(\delta r_b)^2 - \frac{8}{r_v m_e \omega_{ce}(r_v) \partial_r \omega_{\theta o}\big|_{r=r_v}} \left| e\phi_o - \frac{\delta p_{e0}}{n_e(r_v)} \right| \right]^{1/2}$$

Radial dimension of slow vortices

$V_{ph} \approx V_{\theta 0} \rightarrow \quad V_{ph} << V_{\theta 0}$ for example of Rossby kind

V.D. Larichev, G.M. Reznik. 1976

At first we derive general nonlinear eq for electrons.

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\left(n_e \vec{V}\right) = 0 \qquad \vec{\nabla} \times \quad \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}\vec{\nabla}\right)\vec{V} = \left(\frac{e\vec{\nabla}\varphi}{m_e}\right) + \left[\vec{\omega}_{ce}, \vec{V}\right] - \left(\frac{\vec{\nabla} p_e}{n_e m_e}\right)$$

Similar M.V. Nezlin, G.P. Chernikov. 1995.

$$\vec{\alpha} \equiv \mathrm{rot}\vec{V}_e$$

$$d_t\left(\frac{\vec{\alpha} - \vec{\omega}_{ce}}{n_e}\right) = \frac{1}{n_e}\left(\left(\vec{\alpha} - \vec{\omega}_{ce}\right)\vec{\nabla}\right)\vec{V}$$

We have derived without any approaches a non-linear voctorial eq., describing vortical electron dynamics.

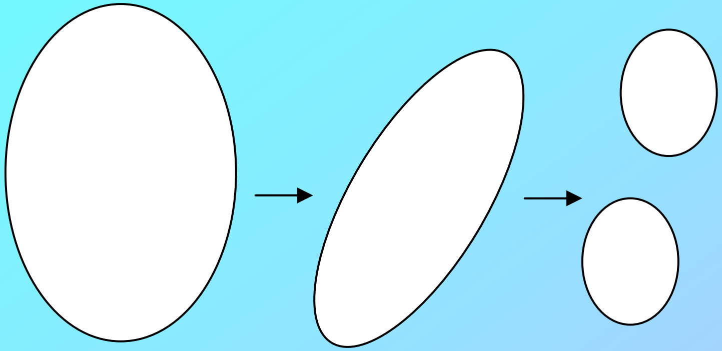
$$\boxed{\delta r = -\frac{1}{\omega_{ceo}\partial_r\left(n_{oe}/\omega_{ce}\right)}\delta n_e}$$

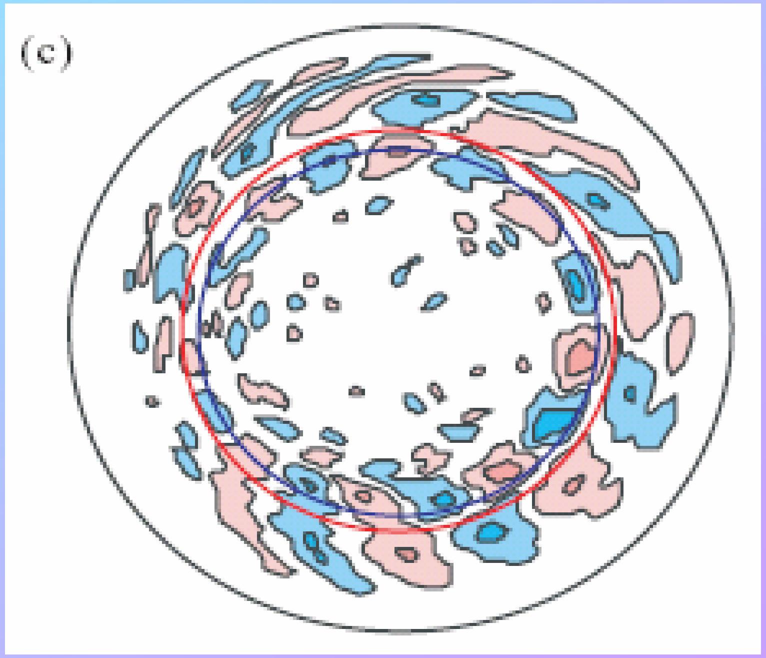
$$\delta r \propto \frac{1}{\partial_r n_{oe}}$$

It helps abrupt plasma density profile and ITB formation.

Amplitude of vortex saturation

Vortex is excited up to amplitude, at which layers, trapped by it, during γ^{-1} are shifted relative to each other due to shear $\delta r_v \partial_r \omega_{\theta 0}\big|_{r=r_v}$ on the angle not larger $2\pi/\ell_\theta$





$$\frac{\delta r_v \partial_r \omega_{\theta 0}\big|_{r=r_v}}{2\pi/\ell_\theta} \le \gamma$$

$$\left| e\phi_o - \frac{\delta p_{e0}}{n_e(r_v)} \right| = \left(\frac{\gamma\pi}{\ell_\theta} \right)^2 \frac{r_v m_e \omega_{ce}}{2\left| \partial_r \omega_{\theta o} \big|_{r=r_v} \right|}$$

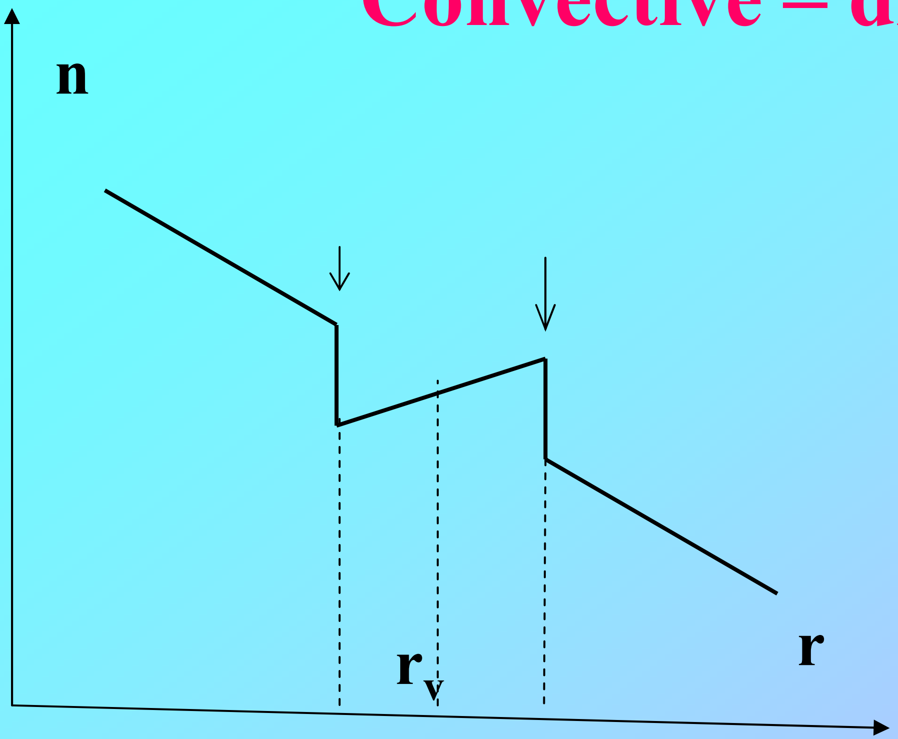
$$\left| e\phi_o - \frac{\delta p_{e0}}{n_e(r_v)} \right| \propto \frac{1}{\left| \partial_r \omega_{\theta o} \big|_{r=r_v} \right|}$$

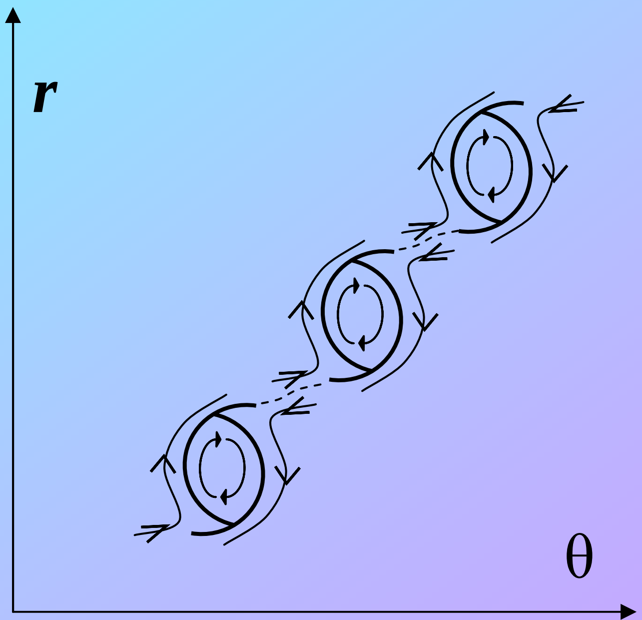
It promotes ITB formation.

Decrease of level of fluctuations at ITB formation has been observed, for example, in

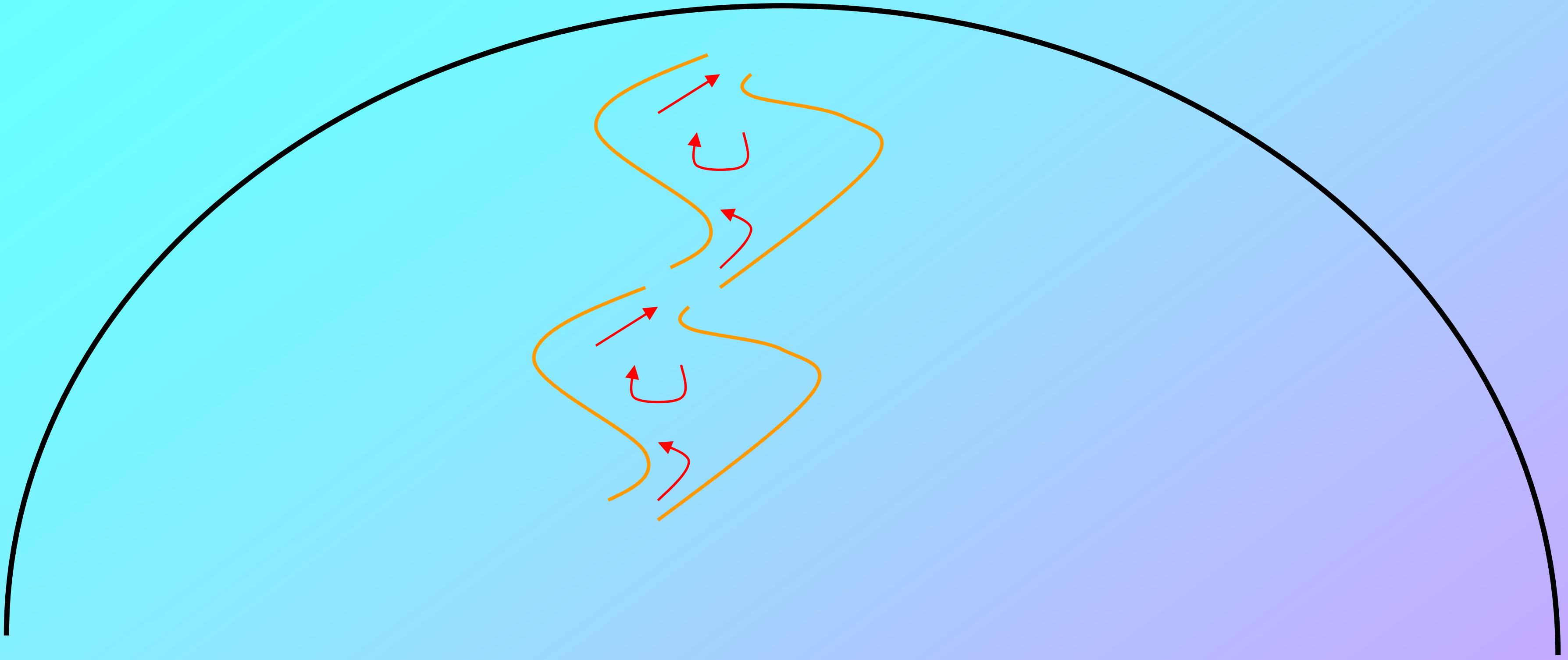
E.D.Volkov et al. 2003.

Convective – diffusion equation

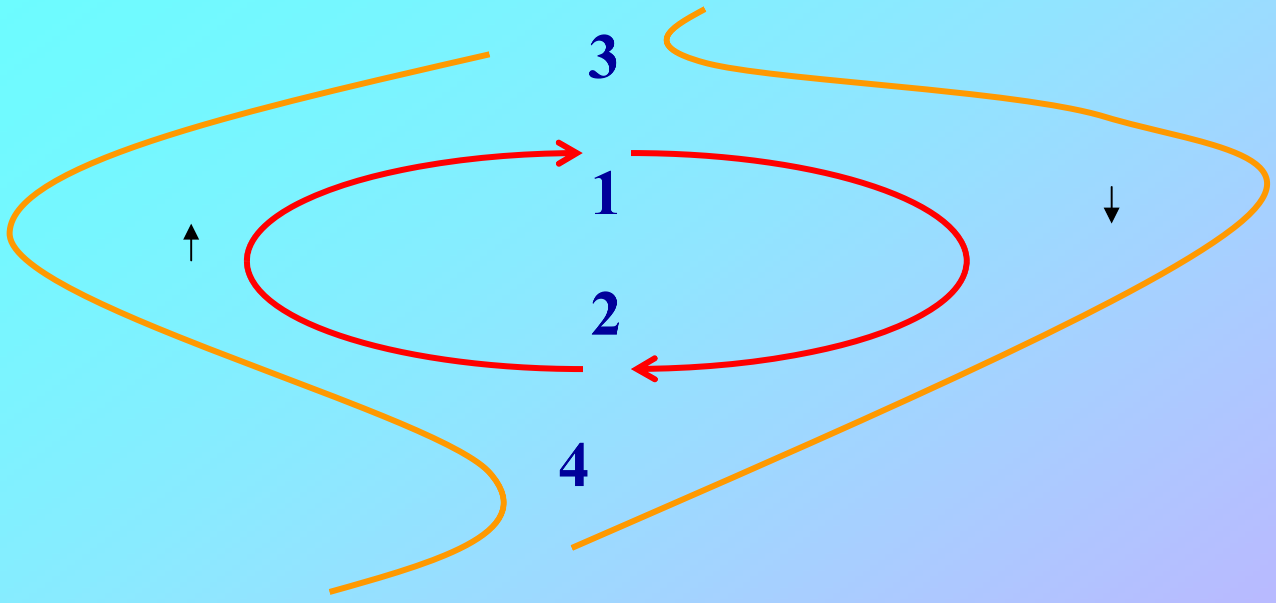




At large amplitudes, when frequency Ω_r of the electron oscillations in vortex, becomes large $\Omega_r > \gamma$ in vicinity of cell borders n_e jumps are formed, where γ becomes large. Therefore at large amplitudes the instability is developed for ordering of vortices and for lattice formation of vortices.



Lattice of vortices



Inside vortex ordered convective electron movement. However, they are effected by environmental vortex fields and fluctuations and amplitudes are not stationary.

Instead of average $n_{oe}(t,r)$, which does not take into account correlations, we use four electron densities $n_{ke}(t,r)$ averaged on small-scale oscillations: $n_{1e}(t,r)$, $n_{2e}(t,r)$, $n_{3e}(t,r)$, $n_{4e}(t,r)$.

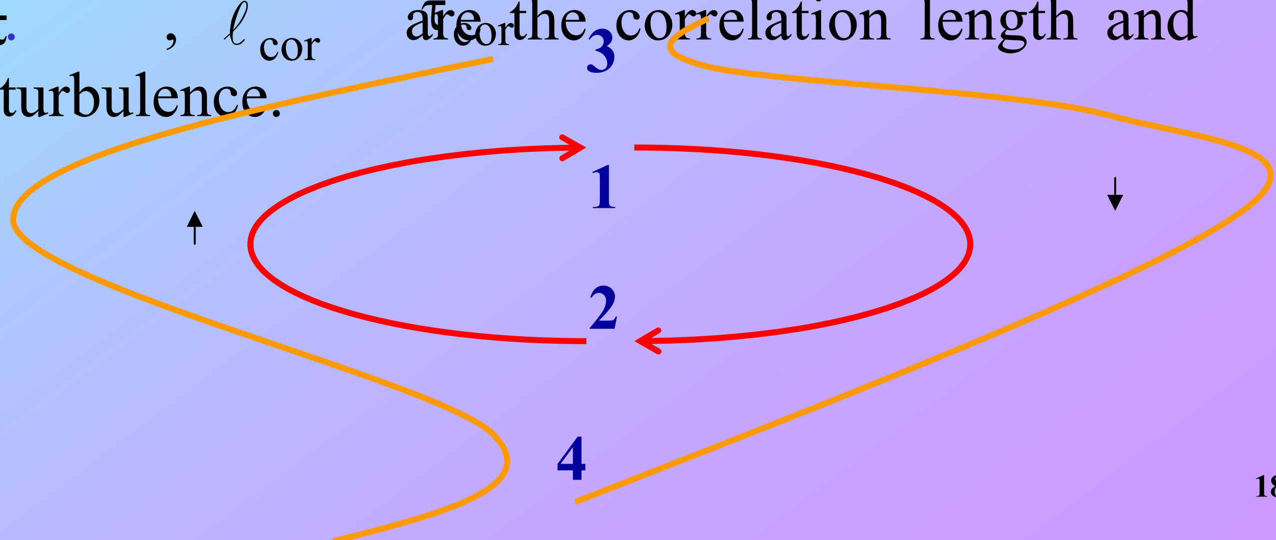
In vortex following processes are realized:

•plateau formation on $n_e(r)$ due to difference of angular speeds.

•due to jump formation on $n_e(r)$ accelerated diffusion in regions 1 and 2 and an exchange by electrons between regions 1 and 3 (factor α), and also between regions 2 and 4.

•effect of fluctuations, growth of amplitudes.

•adjacent vortices form integrated border. Particles in space between individual cell borders and integrated border move in radial direction from vortex to vortex for the distance min$\{\ell_{cor}, \delta r_v \tau_{cor} \Omega_r / \pi\}$. ℓ_{cor}, τ_{cor} are the correlation length and time of vortical turbulence.



$$n_1(t+\tau, r) = (1-\alpha)n_2(t, r) + \alpha\beta n_3(t, r)$$

$$n_2(t+\tau, r) = (1-\alpha)n_1(t, r) + \alpha\beta n_4(t, r)$$

$$n_3(t+\tau, r) = \alpha n_1(t, r) + \beta(1-\alpha)n_3(t, r-\delta r_v) + 0.5(1-\beta)[n_3 + n_4]$$

$$n_4(t+\tau, r) = \alpha n_2(t, r) + \beta(1-\alpha)n_4(t, r+\delta r_v) + 0.5(1-\beta)[n_3 + n_4]$$

Here β is factor of convective exchange of vortices by electrons. β is determined by ratio of area with convective electron dynamics, located between individual vortex borders and integrated borders to all area, located between individual vortex borders and integrated borders of adjacent vortices.

Entering $\bar{n} = (n_3 + n_4)/2,$ $\delta n = n_3 - n_4,$

$$\bar{N} = (n_1 + n_2)/2, \qquad \delta N = n_1 - n_2,$$

one can derive

$$\tau\partial_t \bar{n} = \alpha(\bar{N} - \beta\bar{n}) - (\beta/2)(1-\alpha)\delta r_v \partial_r \delta n,$$

$$\tau\partial_t \delta n + [1 - \beta(1-\alpha)]\delta n = \alpha\delta N - 2\beta(1-\alpha)\delta r_v \partial_r \bar{n},$$

$$\tau\partial_t \bar{N} = \alpha(\beta\bar{n} - \bar{N}), \qquad \tau\partial_t \delta N + (2-\alpha)\delta N = \alpha\beta\delta n$$

One can see that introduction $\bar{n}$ is similar to average $n_{oe}(t,r)$ but with taking into account correlations.

From these eq.s we have similar to

A.S. Bakai. 1978.

following convective – diffusion equation

$$\tau^2\partial_t^2\delta n + \tau\partial_t\left[\left(1-\beta\left(1-\alpha\right)\right)\delta n - \alpha\delta N\right] =$$
$$= -2\beta\left(1-\alpha\right)\delta r_v\partial_r\left[\alpha\left(\overline{N}-\beta\overline{n}\right) - \frac{\beta}{2}\left(1-\alpha\right)\delta r_v\partial_r\delta n\right]$$

As β is proportional to $\left(\delta r_v - \Delta\right)/\delta r_v$ then at $\delta r_v < \Delta$ we have $\beta = 0$ and there is no strong anomalous radial transport because vortices exchange by particles disappears.

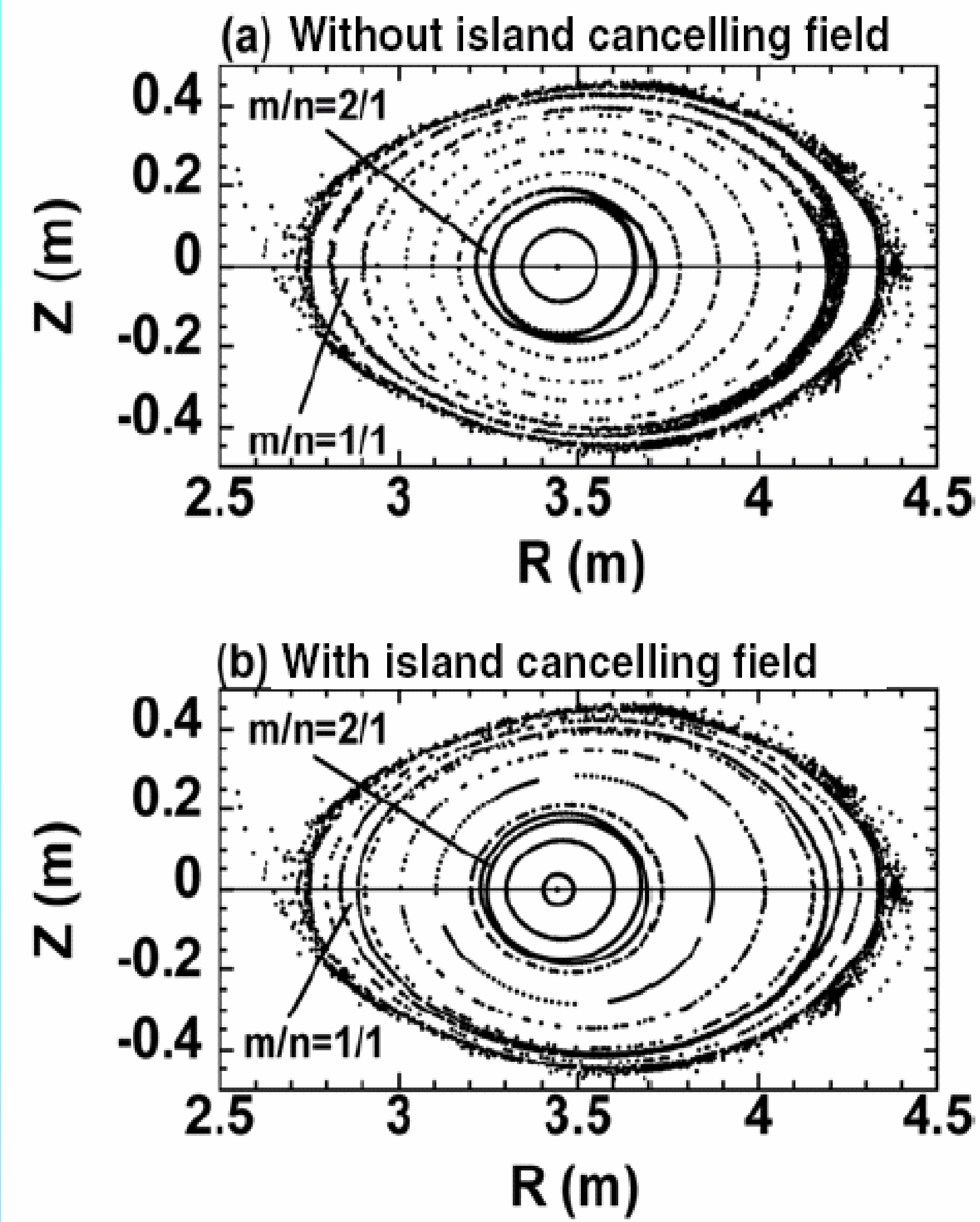
Shear formation due to electron heating near rational surface with poloidal chain of islands

In this part we discuss the angular velocity shear $\partial_r \omega_\theta \neq 0$ formation by magnetic islands. Not self-consistent islands but islands due to non-ideal construction or so-called natural islands. Experiments

T. Shimozuma et al. Nucl. Fusion **45,** 1396 (2005).

E.D.Volkov et al. Czech. J. Phys. **53**, 887 (2003).

show that narrow magnetic islands can improve plasma confinement.



Calculated results of flux surfaces with natural islands

T. Shimozuma et al. 2005.

The better confinement in experiments with several chain of magnetic islands due to sufficient heating

E.D.Volkov et al. 2003

and in Large Helical Device with neutral beam injection and with additional electron cyclotron heating, strongly focused on rational surface m/n = 2/1 with magnetic islands

T. Shimozuma et al. 2005.

There are many investigations on magnetic island formation

F. Porcelli et al. 2004.
M Ottaviani et al. 2004.

and their effect on nuclear fusion plasma

K. Ida et al. 2002.
E.D.Volkov et al. 2003
T. Shimozuma et al. 2005.

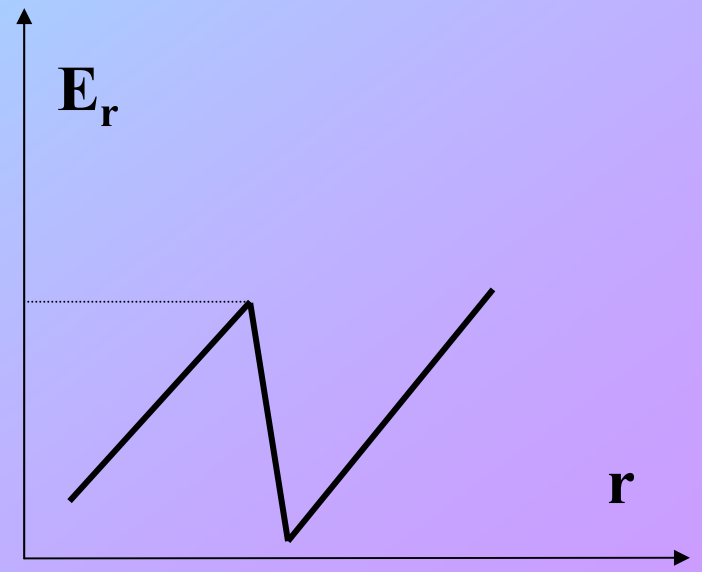
If at plasma electron heating near low order rational surface with poloidal chain of narrow magnetic islands

$$\boxed{2\pi r_{tor} \nu_e < V_{the}^{(hot)}}$$

then on the island dimension the radial distribution of the electric field $E_r(r)$ changes strongly and in plasma cross-section the strong shear $\partial_r \omega_{\theta o}$ is formed.

See experiment

E.D.Volkov et al. 2003.



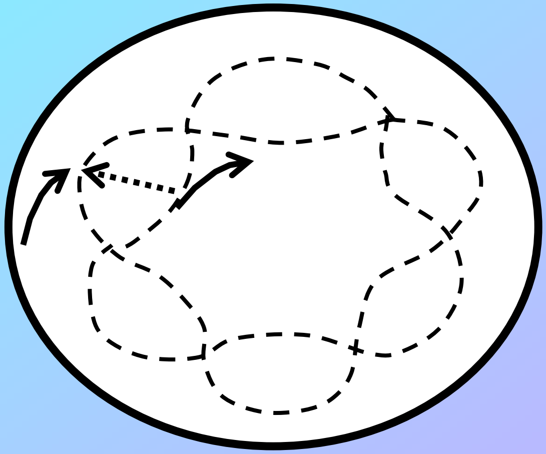
We suppose, that on wide interval $0 < r < r_m$ electrical field E_r in the case of ITB absence is proportional to $E_r \propto r$

$$E_r = -2\pi e N_0 r \qquad 0 < r < r_m \qquad N_0 \equiv n_e - n_i$$

It means, that there is no shear $\partial_r \omega_{\theta o}$. Then oscillations can be excited, which result in anomalous radial transport. Anomalous transport leads to easier electron follow for ions and to smooth plasma parameter distribution on radius.

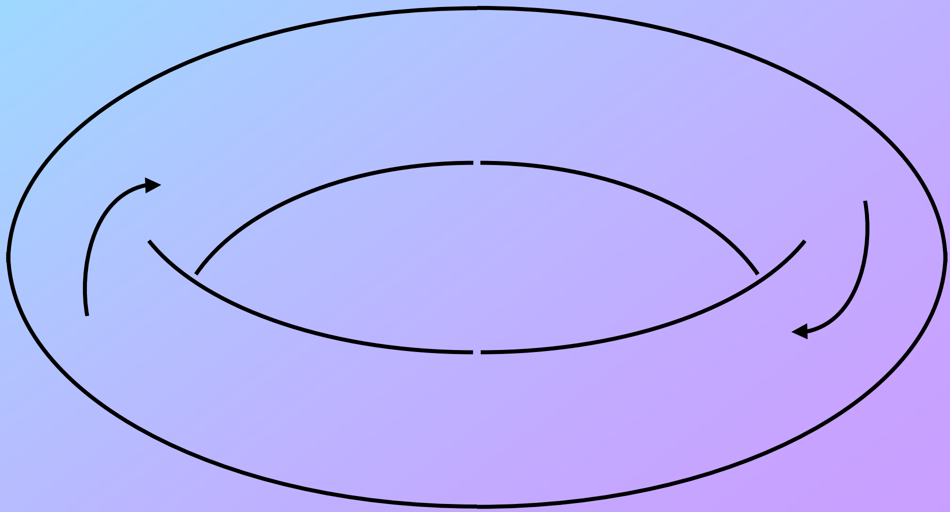
On plasma cross-section several chains of islands can exist

E.D.Volkov et al. 2003.



We for simple case consider influence of one poloidal chain on shear formation.

We consider rational surface with small numbers, because important property of this surface is appeared, when plasma is heated sufficiently that its electrons perform several rotation around toroidal surface during free pass time $2\pi r_{tor} \nu_e < V_{the}^{(hot)}$



At sufficient plasma electron heating $2\pi r_{tor}\nu_e < V_{the}^{(hot)}$ near rational surface electron transport through island changes from slow collisional to quick one collisionless. Quick transport is realized by such way that electrons miss island.

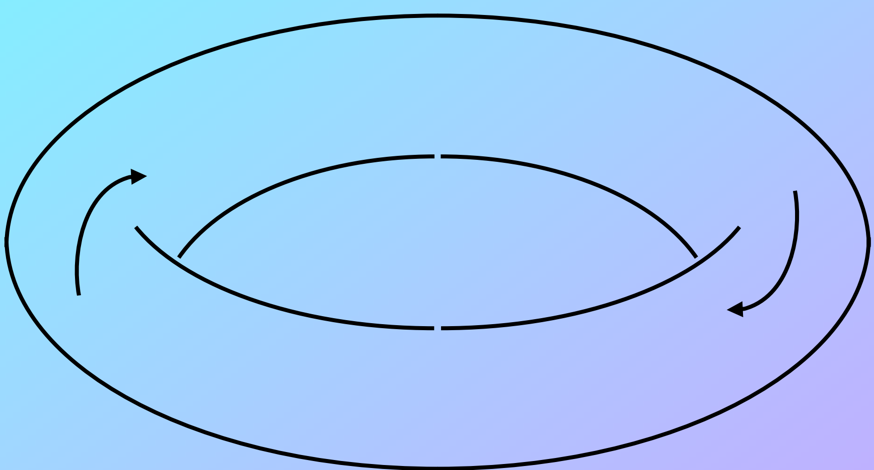
Electron moves along r with $V_{0r} = -(eE_{or} + \partial_r p_{0e}/n_e)(\nu_{ef} + \nu_e)/m_e\omega_{ce}^2$

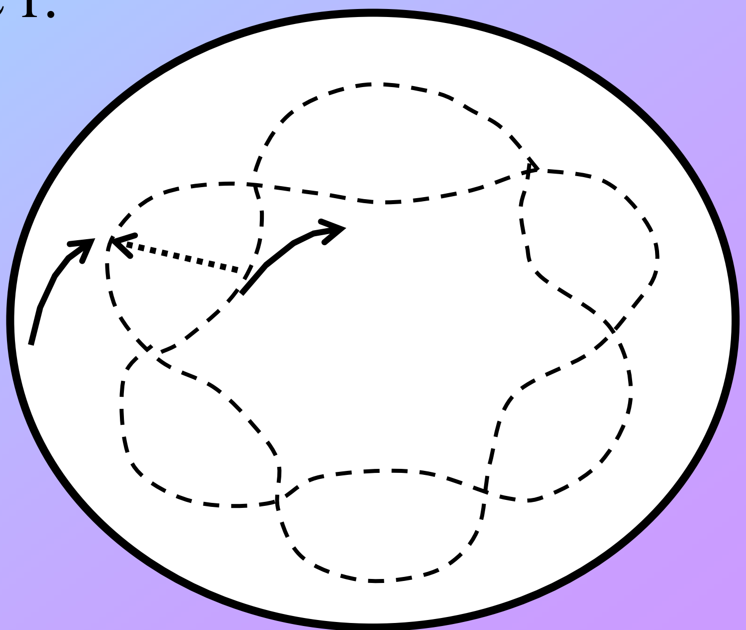
When electron reaches island, it propagates collisionally through island in case $2\pi r_{tor}\nu_e >> V_{the}^{(hot)}$

But in case $2\pi r_{tor}\nu_e << V_{the}^{(hot)}$

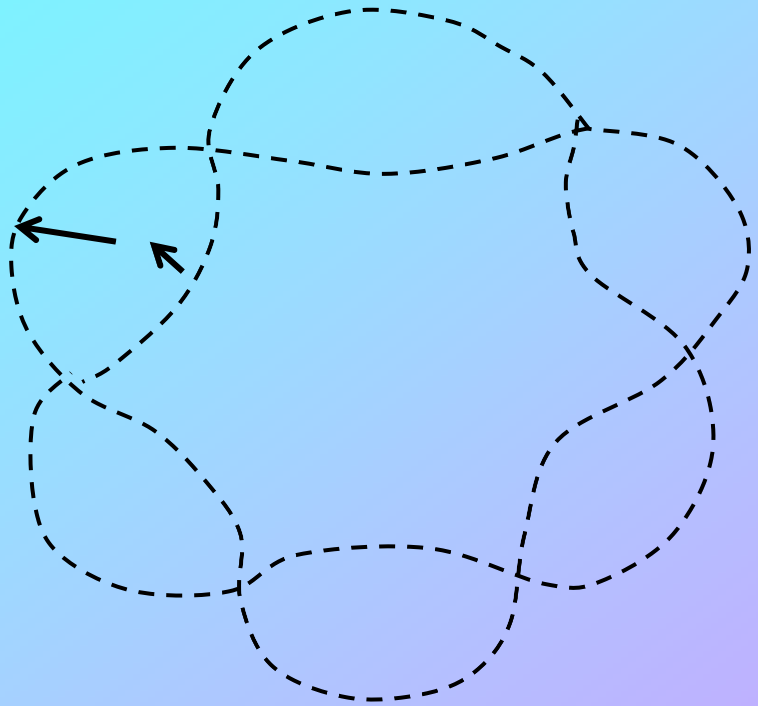
electron without collision quickly, during time $2\pi r_{tor}/V_{the}^{(hot)}$

get on second boundary of island. After that electron again can slow propagate with V_{0r} in direction of large r.





Part of trapped electrons leave island



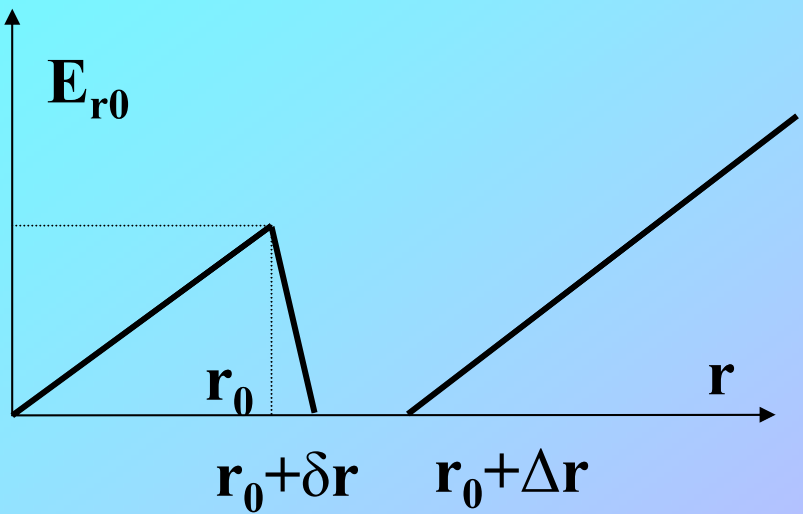
with velocity

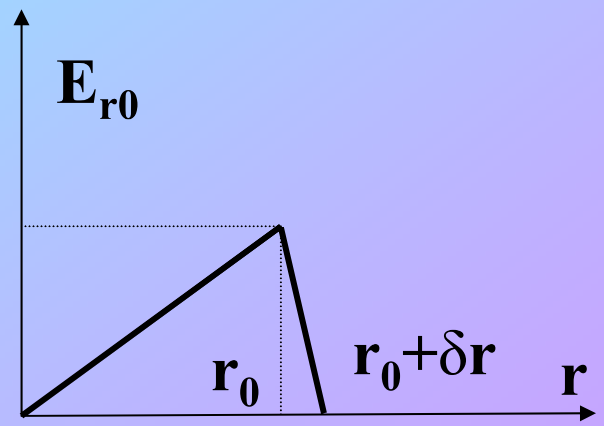
$$V_\ell \approx \Delta r \partial_r V_{er}\Big|_{r=r_0} \approx \frac{\left(\nu_{ef}+\nu_e\right)}{m_e\omega_{ce}^2}\frac{\Delta r}{r_0} eE_{r0}(r)\Big|_{r\approx r_0, t=0}$$

Island missing by electrons and trapped electrons leaving the island lead to appearance of uncompensated ion volume charge

$$\delta n \ll n_0$$

in island and to shear.





with strong turbulence on

$$r > r_0 + \Delta r$$

Using approximation of poloidal chain of narrow magnetic islands as azimuth symmetrical narrow layer we have

$$E_{r0} \approx -2\pi e \begin{cases} N_a r, 0 \le r \le r_0 \\ N_a\, r_0^2 / r - \delta n\left(r - r_0^2 / r\right), r_0 \le r \le r_0 + \delta r \\ 0, r \ge r_0 + \delta r \end{cases}$$

$$\begin{cases} 0 \le r \le r_0 \text{ small plasma polarization} \\ r_0 \le r \le r_0 + \delta r \text{ ion volume charge} \end{cases}$$

E_{r0}≈0 at $r=r_0+\delta r$, $\delta r<\Delta r$. Δr is island width. we have $N_a \approx \delta n \dfrac{2\Delta r}{r_0}$

Density of uncompensated ion volume charge $N_0 << \delta n << n_{0e}$

L is width of region with essential E_{r0}.

Island can be narrow $r_0 >> \Delta r > \delta r$ for essential shear formation

$$\boxed{\mathbf{(e\Delta\varphi/T_i)(r^2{}_{di}/L\delta r_{sep})<1}}$$

Let us estimate shear

$$S \equiv \frac{r^2 \partial_r (E_r / r)}{E_r\big|_{\text{without TB}}}$$

The shear is large for region of narrow magnetic islands $r_0 \le r \le r_0 + \delta r$

$$\boxed{S \approx (N_a / N_0)(r_0 / \delta r_{sep})} \qquad |S| >> 1$$

Shear of angle velocity

$$\partial_r \omega_{\theta 0} \qquad \omega_{\theta 0} = V_{\theta 0}/r \qquad V_{\theta 0} = \left(\frac{1}{m_e \omega_{He}}\right)\left(-eE_{r0} - \frac{\partial_r p_{0e}}{n_{0e}}\right)$$

$$S_\omega \equiv \frac{r\partial_r \omega_{\theta 0}}{\omega_{\theta 0}\big|_{\text{without TB}}} \qquad \omega_{\theta 0}\big|_{\text{without TB}} = \frac{\omega_{pe}^2 N_0}{2\omega_{He} n_0}$$

$$\boxed{S_\omega = -(N_a/N_0) r_0/\delta r_{sep} >> 1}$$

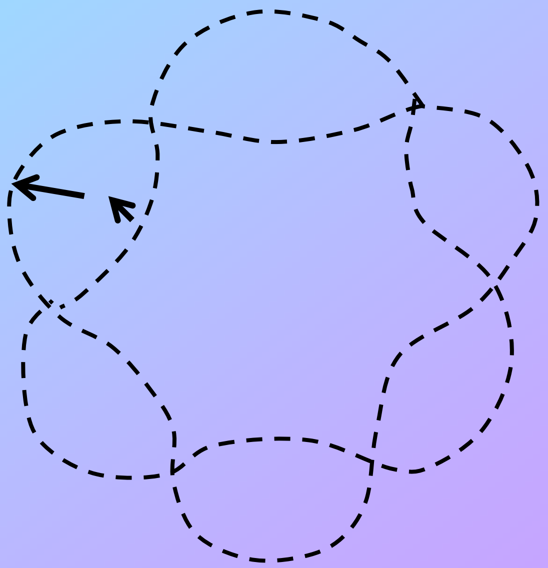
Time of shear formation

Electrons leave island with

$$V_\ell = V_{er}\Big|_{r=r_0+\Delta r/2} - V_{er}\Big|_{r=r_0-\Delta r/2} \approx \frac{(\nu_{ef}+\nu_e)}{m_e\omega_{ce}^2} e\Delta r \partial_r E_{r0}(r)\Big|_{r\approx r_0, t=0}$$

Electrons should shift on small radial distance

$$\delta r \approx \Delta r\, \delta n / n_{0e}$$



Shear is formed during short time for not very narrow islands

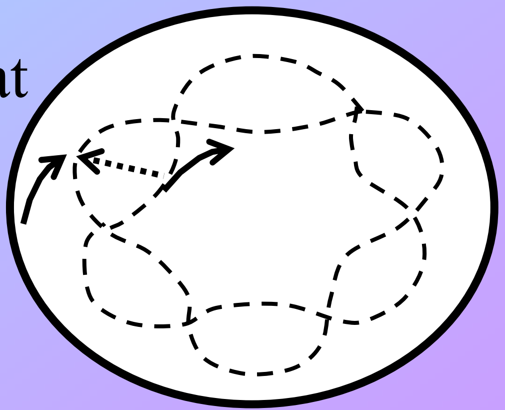
$$\tau_{TB} \approx (\nu_{ef}+\nu_e)^{-1}\left(\omega_{ce}^2 / 2\omega_{pe}^2\right)(r_0/\Delta r)$$

Conditions of shear formation by chain of magnetic islands in crossed fields

That in island uncompensated ion volume charge appears it is necessary $\Delta r > \rho_{ce}$.

Narrow islands $\Delta r << R$, though provide fast electron movement through Δr, strongly suppress transport in broad their neighborhood.

Uncompensated ion volume charge has appeared at



$$(\Delta r)^2 / D_{\perp} > (2\pi n r_{tor})^2 / D_{\parallel}$$

n is toroidal number.

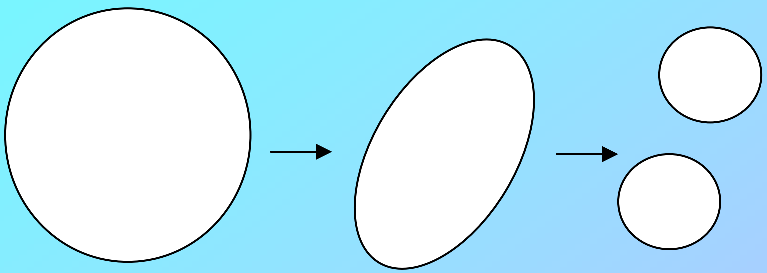
Let during free pass time electron has time to make q rotations around the torus. Then

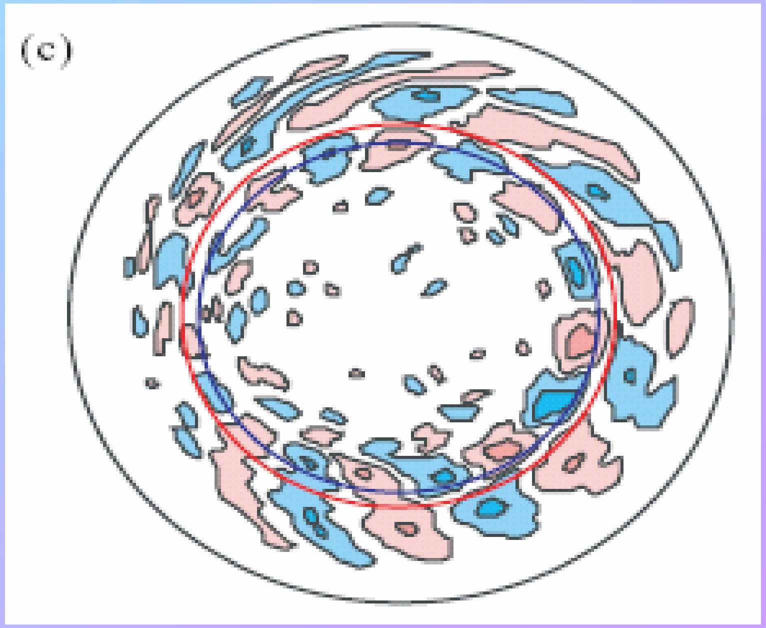
$$\boxed{\max\{\rho_{ce}, \frac{n\rho_{ce}}{q}\sqrt{1+\frac{\nu_{ef}}{\nu_{ei}}}\} < \Delta r}$$

Condition of anomalous transport damping by shear

J.W. Connor et al. 2004; R.C.Wolf. 2003; A. Fujisawa. 2003.

$$L\partial_r \omega_{\theta 0} > \gamma \qquad \Delta\varphi \approx 2\pi e N_0 L^2$$





$$\boxed{\frac{\Delta\varphi e}{T_i}\frac{\rho_{ci}^2}{L\delta r} > \frac{\gamma}{\omega_{ci}}}$$

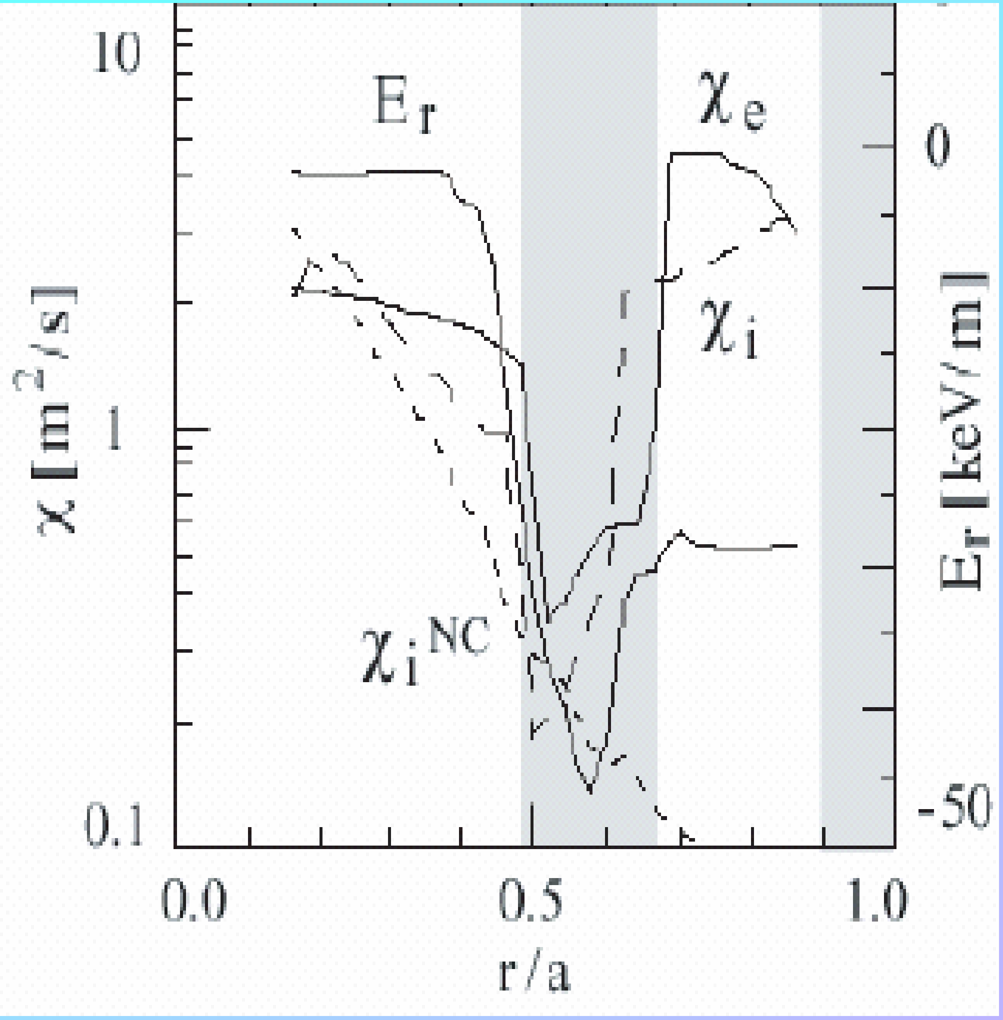
Shear can damp instabilities with growth rate $\gamma < \omega_{ci}$

We derive relative shear $S_\omega \doteqdot \dfrac{r\partial_r \omega_{\theta 0}}{\omega_{\theta 0}\big|_{\text{without TB}}}$

But absolute shear can be increased. In several experiments strong localization of region with $V_{0\theta} \neq 0$ has been observed. Radial width Δr_{sh} of area $V_{0\theta} \neq 0$ localization $\Delta r_{sh} = 1\text{cm}$ is observed. Shear $(\partial_r V_{0\theta})_{apr}$ can be increased in comparison with smooth case

$$(\partial_r V_{0\theta})_{smooth} \approx V_{0\theta}/R$$

strongly $(\partial_r V_{0\theta})_{apr} \approx V_{0\theta}/\Delta r_{sh} \approx (\partial_r V_{0\theta})_{smooth} R/\Delta r_{sh}$



ITB as a localized drop of ion and electron thermal conductivities. They decrease by factors of 10 to 20 within 5 cm. Also shown is the calculated neoclassical ion heat conductivity.

J.W. Connor et al. 2004

Another scenario of shear formation, when in region $r \le r_{tb}$

there is dense plasma with frequent collisions ν_e. Hence

$V_{0re} \approx -\dfrac{\partial_r p_{0e}(r)\nu_e}{n_e m_e \omega_{ce}^2}$ is sufficiently large and $E_{0r} \approx 0$ at $r \le r_{tb}$.

If turbulence and anomalous transport at $r \ge r_{tb} + \Delta r_{tb}$

is strong, again E_{0r} is small at $r \ge r_{tb} + \Delta r_{tb}$

If turbulence and anomalous transport at $r_{tb} \le r \le r_{tb} + \Delta r_{tb}$

are strongly damped, and plasma electrons here are collisionless, self-consistent strong shear is formed due to localized electric field E_{0r} formation at $r_{tb} \le r \le r_{tb} + \Delta r_{tb}$

as a double-layer-kind structure.