Bispectral analysis of harmonic oscillations measured using beam emission spectroscopy and magnetic probes in CHS

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The coherent MHD oscillation which has the fundamental frequency of several kilohertz and its higher harmonics (harmonic oscillation: HO) has been observed in the compact helical system (CHS). HO consists of two pairs of harmonic series. One locates in the core region near $\iota = 0.5$ rational surface, the other locates in the edge region near $\iota = 1.0$ rational surface. In the present study, bispectral analysis is applied to the fluctuation data having HO measured using beam emission spectroscopy (BES) and magnetic probes. It has revealed that fundamental mode of HO in both the magnetic fluctuation and core density fluctuation have phase correlation with the harmonics including fundamental oscillation, while HO in edge density fluctuation does not have such phase correlation. Mode numbers of HOs are same between harmonic components having difference frequencies. It suggests that the generation of harmonics can not be interpreted simply as the mode coupling, even though bicoherence value is significant. The bicoherence value and relative amplitude of higher harmonics correlate with each other, which suggests that bicoherence means the degree of distortion of the signals.

Keywords: bispectral analysis, bicoherence, fluctuation, magnetohydrodynamics, edge harmonic oscillation,
compact helical system, beam emission spectroscopy, edge transport barrier

1. Introduction

Fluctuations of plasma parameters have been recognized to correlate with the properties of plasma confinement. Characteristics of the fluctuations includes the amplitude and level, frequency and wavenumber spectrum, spatial structure represented by radial locality or poloidal/toroidal mode number m/n, and so on. In addition to these characteristics, recently, attention has been paid to the nonlinear characteristics such as the correlation between spectral components having different frequencies. In order to investigate the nonlinear characteristics of the fluctuations, bispectal analysis is consider to be useful [1]. The bispectral analysis has been applied to several tokamaks [2], helicals [3] and other types of plasma confinement devices to investigate the interaction among turbulence, magnetohydrodynamic (MHD) oscillations, and zonal flows [4-6].

Recently, a coherent MHD mode which consists of the fundamental frequency of several kilohertz and its higher harmonic (harmonic oscillation: HO) [7,8] has been observed in the edge transport barrier (ETB) discharges of the compact helical system (CHS), which is a low-aspect-ratio, middle-sized heliotron (major radius = 1.0 m, minor radius = 0.2 m, toroidal period number = 8, polarity = 2) [7,9-13]. The characteristics of the oscillation have been investigated in the viewpoint of the

comparison with edge harmonic oscillation (EHO) [14-17], which is recognized to have a role enhancing the particle transport in the quiescent H-mode of tokamaks. Fluctuation measurement using the beam emission spectroscopy (BES) [18,19] and magnetic probe (MP) array [20] have revealed that HO in CHS consists of two pairs of harmonic series located in different radial position. One locates in the core region of normalized minor radius $\rho = 0.4 \sim 0.5$ near $\iota = 0.5$ rational surface (denoted as "HO (core))", the other locates in the edge region of $\rho = 0.9 \sim 1.0$ near $\iota = 1.0$ rational surface (denoted as "HO (edge)). All harmonic components of both HO (core) and HO (edge) have same mode numbers, namely, $m/n = -2/1$ for HO (core) and $m/n = -1/1$ for HO (edge), even though the frequency of pth harmonic is p times larger than that of 1st harmonic for both HO (core) and HO (edge), where p is positive integer [21].

In the present study, bispectral analysis is applied to the fluctuation data having HO measured using BES and MP in order to investigate the correlation between the harmonic components.

2. Experimental Setup

BES has been developed in CHS to simultaneously measure both local density fluctuations and gradients. The BES method detects emissions from the collisionally

excited neutral beam atoms (denoted as "beam emission") [22]. The spatial channels used in BES consist of 16 optical fibers with an object lense. In the present experiment, H_α emission from the neutral hydrogen atomic beam for heating in the intersection of the beam line and the sightline for each fiber channel was measured as the beam emission. One can select the arrangement of the fibers in the radial or poloidal direction to measure each structure of the fluctuations.

Magnetic probe array was also used to measure both the poloidal and toroidal structures of the magnetic fluctuations. The poloidal magnetic probe array consists of sixteen probes covering the poloidal angle $\Delta\theta$ = 235.5 deg. We define the absolute poloidal coordinate as θ = 0 deg. and 180 deg. at the midplane outside and inside torus, respectively [23]. Five toroidal magnetic probes covering the toroidal angle $\Delta\phi = \pi$ were located at every $\pi/4$ section at the major radius R = 1.2 m.

3. Data Analysis Method

When the wave having the frequency f is phase correlated with the spectral components with the frequencies of f_1 and f_2 satisfying the summation rule $f = f_1 \pm f_2$, we can use the bicoherence value as an indicater of phase correlation of these three waves [1]. Bicoherence $b^2(f_1, f_2)$ for the waves having the frequency f, f_1 and f_2 is defined as

$$b^2(f_1, f_2) = \frac{|< X(f_1)X(f_2)X*(f) >|^2}{< | X(f_1)X(f_2)|^2 >< | X(f)|^2 >} \quad (1)$$

where $X(f)$ is the Fourier transform of the time trace $x(t)$ and $<>$ means ensemble averaging.

Wave-wave coupling, namely which is the wave having the frequency f is generated by the coupling between the waves having the frequencies f_1 and f_2, is the representative case of which the bicoherence value becomes significant. However, the wave-wave coupling is not necessary the only mechanism responsible of the phase correlation revealed by bicoherence analysis. Summation rule about both frequency $f = f_1 \pm f_2$ and wavenumber $k = k_1 \pm k_2$ must be obeyed to generate the another wave by wave-wave coupling.

4. Experimental Results

Figure 1 shows Typical experimental conditions and temporal evolution of plasma parameters for a discharge in which the harmonic oscillation appears: (a) heating condition, (b) H_α intensity, (c) magnetic fluctuation, and (d) frequency spectrum of magnetic fluctuation. Plasma was initiated by electron cyclotron heating and was further heated by two neutral beam injection (NBI) systems.

Additional gas was puffed as a means to increase density. The standard magnetic field configuration of CHS was applied, in which the vacuum magnetic axis position R_{ax} was 92.1 cm from the torus center, the toroidal magnetic field strength at the magnetic axis B_{ax} was 0.95 T, and the magnitude of the quadrupole field B_q denoting the degree of cancellation of the intrinsic quadrupole component formed by the helical coils was -50 %. In the cases in which the heating power exceeds a certain threshold, ETB characterized by a sudden drop in the H_α intensity signal as shown in Fig. 1(b) is formed. It has been found that the line-averaged density, the edge density gradient, and the stored energy begin to increase at the ETB transition [7,9-13]. In the latter half of the phase having ETB, HO which consists of a 1st harmonic with the frequency of 4.0 kHz and a 2nd harmonic appears in the magnetic fluctuation as shown in Fig. 1(c) and (d). It is known that this oscillation is identical to HO (core) [21].

We applied the bispectral analysis to the fluctuation data having harmonic oscillation measured using the magnetic probes and BES. Figure 2 shows the bicoherence plane of (a) magnetic fluctuation, (b) core (ρ = 0.42) density fluctuation, and (c) edge (ρ = 0.95) density fluctuation. $b^2(f_1, f_2)$ of both the magnetic fluctuation and core density fluctuation have a clear peak having the frequencies (f_1, f_2) = (4.0 kHz, 4.0 kHz). This indicates the 1st component of HO (core) having the frequency 4.0 kHz is phase correlated with both the 1st component itself and the 2nd component having the frequency 8.0 kHz = $f_1 + f_2$, where $f_1 = f_2 = 4.0$ kHz. On the other hand, $b^2(f_1, f_2)$ edge density fluctuation does not have such a peak, even
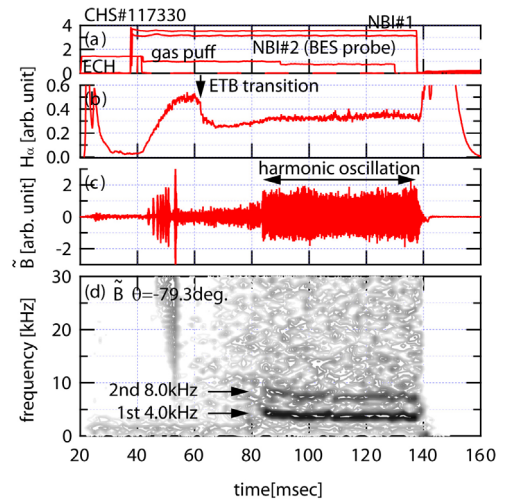

Fig.1 Typical experimental conditions and temporal evolution of plasma parameters for a discharge in which HO appears: (a) heating condition, (b) H_α intensity, (c) magnetic fluctuation, and (d) frequency spectrum of magnetic fluctuation.

though HO (edge) observed in BES signal consists of the 1st frequency of 4.0 kHz and its 2nd harmonic. It indicates that phase correlation between harmonic components seems to be different between HO (core) and HO (edge).

We have found that $b^2(f_1, f_2)$ which indicates the phase correlation between harmonic components has a poloidal asynmetry. Figure 3 shows the frequency spectrum of magnetic fluctuation having up to 3rd harmonic of HO. The 1st, 2nd, and 3rd frequency $f(1st)$, $f(2nd)$, and $f(3rd)$ is 3.0 kHz, 6.5 kHz, and 10.0 kHz, respectively. The bicoherence planes of magnetic fluctuations (a) outside torus and (b) inside torus are shown in Fig. 4. $b^2(f_1, f_2)$ outside torus has peaks with $(f_1, f_2) = (f(1st), f(1st))$, $(f(2nd),$

$f(1st))$, $(f(3rd), f(1st))$, $(f(2nd), f(2nd))$ and so on which indicates the phase correlation between each harmonic component. On the other hand, $b^2(f_1, f_2)$ inside torus also has peaks located at similar frequencies as those outside torus, however the values are small. Figure 5 shows (a) $b^2(f_1, f_2)$ at $(f_1, f_2) = (f(1st), f(1st))$ and $(f(2nd), f(1st))$ and (b) amplitudes of the 2nd $(f(2nd) = f(1st) + f(1st))$ and 3rd $(f(3rd) = f(2nd) + f(1st))$ harmonic of HO divided by that of the 1st harmonic with respect to poloidal angles. One can see that relative amplitudes of higher harmonics decrease with decreasing corresponding $b^2(f_1, f_2)$ in the poloidal direction.

We should pay attention to the physical interpretation of the result of the bispectral analysis. In the present study, $b^2(f_1, f_2)$ has peaks at $f(1st) + f(1st) = f(2nd)$ and $f(2nd) + f(1st) = f(3rd)$. However, the poloidal/toroidal mode number m/n does not satisfy the summation rule, namely, they show $m(1st) = m(2nd) = m(3rd) = -2$, $n(1st) = n(2nd) = n(3rd) = 1$ [23]. Therefore, generation of harmonics can not be interpreted simply as the mode coupling. $b^2(f_1, f_2)$ can be regarded as the degree of "distortion" of the signals in our case, because the poloidal asymmetry of $b^2(f_1, f_2)$ and relative amplitude of higher harmonics indicates that they have a positive correlation. Mechanism to distort signals has not been clarified yet, which remains as a future study.

5. Summary

HO having the fundamental frequency of 3~5 kHz and its higher harmonics has been observed in CHS. It consists of two pairs of harmonic series. One locates in core region near ι = 0.5 rational surface, the other locates in edge region near ι = 1.0 rational surface.

We applied the bispectral analysis to the fluctuation data having HO measured using BES and MP. It revealed that fundamental mode of HO in both the magnetic fluctuation and core density fluctuation have phase correlation with the harmonics including fundamental oscillation, while HO in edge density fluctuation does not have such phase correlation.

Generation of harmonics can not be interpreted simply as the mode coupling, even though bicoherence
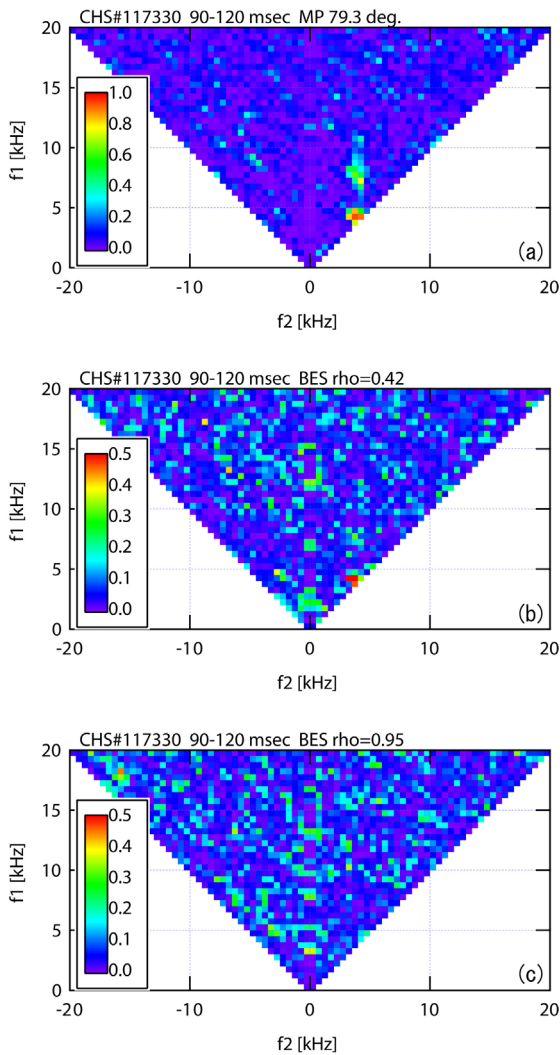

Fig.2 Bicoherence plane of (a) magnetic fluctuation, (b) core (ρ = 0.42) density fluctuation, and (c) edge (ρ = 0.95) density fluctuation. $b^2(f_1, f_2)$ of both the magnetic fluctuation and core density fluctuation have a clear peak having the frequencies $(f_1, f_2) = (4.0 \text{ kHz}, 4.0 \text{ kHz})$, while that of edge density fluctuation does not have such a peak.
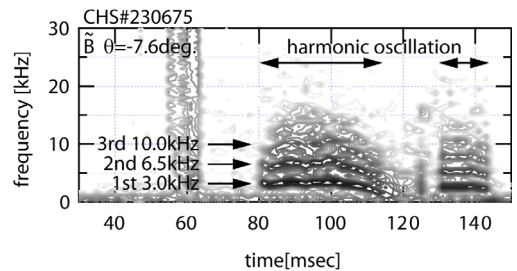

Fig.3 Frequency spectrum of magnetic fluctuation having up to 3rd harmonic of HO.

value $b^2(f_1, f_2)$ is significant, because mode numbers of HOs are same between harmonic components having difference frequencies. Correlation between $b^2(f_1, f_2)$ and relative amplitude of higher harmonics suggests that $b^2(f_1, f_2)$ means the degree of distortion of the signals.

The reason why the signals are distorted and all harmonic components have same mode numbers should be investigated in the future.

Acknowledgement

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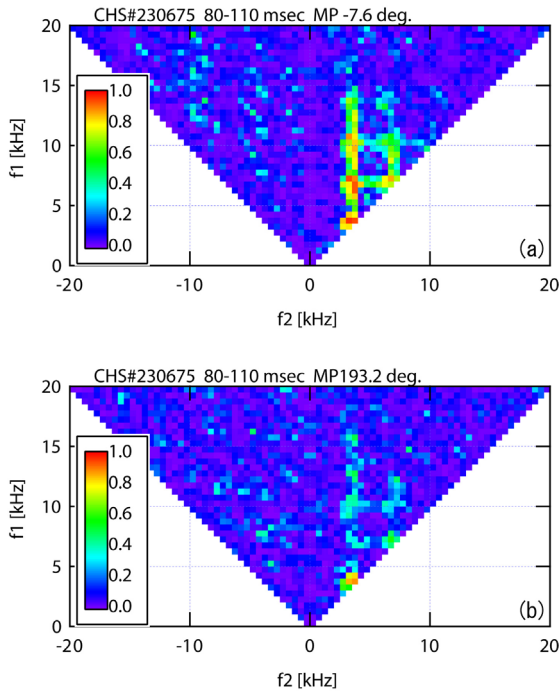[23]   S. Kado *et al.*, J. Nucl. Mat. **363-365**, p. 522 (2007)

Fig.4   Bicoherence plane of magnetic fluctuations (a) outside torus and (b) inside torus having frequencies of HO f(1st) = 3.0 kHz, f(2nd) = 6.5 kHz, and f(3rd) = 10.0 kHz. $b^2(f_1, f_2)$ outside torus has peaks with (f_1, f_2) = (f(1st), f(1st)), (f(2nd), f(1st)), (f(3rd), f(1st)), (f(2nd), f(2nd)) and so on which indicates the phase correlation between each harmonic component. $b^2(f_1, f_2)$ inside torus has peaks located at similar frequencies as those outside torus, however the values are small.
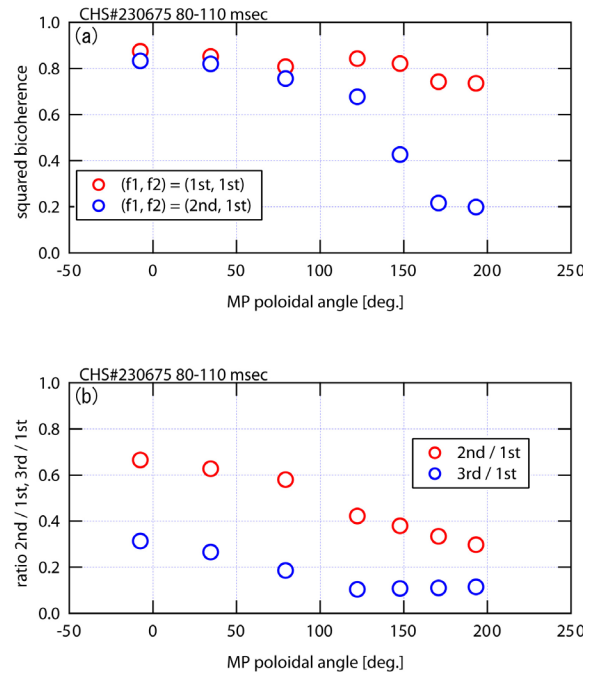

Fig.5   (a) $b^2(f_1, f_2)$ at (f_1, f_2) = (f(1st), f(1st)) and (f(2nd), f(1st)), (b) amplitudes of the 2nd (f(2nd) = f(1st) + f(1st)) and 3rd (f(3rd) = f(2nd) + f(1st)) harmonic of HO devided by that of the 1st harmonic with respect to poloidal angles. Relative amplitudes of harmonics decrease with decreasing corresponding $b^2(f_1, f_2)$ in the poloidal direction.