Strong Shear Formation by Poloidal Chain of Magnetic Islands

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We estimate the electron angular velocity shear $\partial_r \omega_{\theta o}$ which can be formed due to plasma heating near the low order rational surface with poloidal chain of magnetic islands (MIs). We suppose that the electrons are heated sufficiently that they propagate without collisions around torus. Then the plasma electrons start to miss the MIs during their radial shift. This provides ion volume charge in some region of MIs. Then the essential shear is formed. The time of shear formation is small. The conditions on MI width are derived, at which it leads to strong shear. It is shown that even narrow MIs can lead to shear. Shear can damp instabilities with growth rate smaller than ion cyclotron frequency.

The spatial structures of some convective cells have been described. We derive inverse dependences of radial width of excited vortices on $\partial_r \omega_{\theta o}$ and on $\partial_r n_{0e}$. Amplitude of the electron radial oscillations is smaller for larger $\partial_r \omega_{\theta o}$ and $\partial_r n_{0e}$. These dependences promote abrupt plasma density profile and internal transport barrier.

Keywords: internal transport barrier, shear, magnetic island, vortex, rational surface, radial electrical field

1. Introduction

Now ITB formation is widely investigated (see [1-3]). Earlier the effect is considered, that shear of the electron angle velocity $\partial_r \omega_{\theta o}$ damps anomalous transport, separating the coherent ordered motion owing the relative movement of layers. ITB is formed for under-threshold shear. We consider one more effect of anomalous transport suppression in plasma, located in crossed magnetic $\vec{H}_0$ and electrical $\vec{E}_{0r}$ fields. Namely, the shape of vortices is described. The exact nonlinear eq., connecting the electron vorticity and density, is derived in vector view without any approximations. We derive inverse dependences of radial width of some excited vortices on the shear $\partial_r \omega_{\theta o}$ in crossed fields and on degree of steepness of the plasma density profile $\partial_r n_{0e}$. These dependences promote abrupt plasma density profile and ITB formation. The amplitude of the vortex saturation is inversely proportional to the shear. It also promotes ITB formation, suppressing the transport especially in the case of small magnetic shear. It is determined by that small magnetic shear leads to large spatial interval Δ between rational surfaces [4]. If radial correlation length of excited perturbations becomes less than Δ the radial transport could be suppressed. The convective-diffusion equation, describing transport of plasma particles in the field of lattice of overlapped vortices, is derived.

Formation [5, 6] and role (see [4, 7, 8]) of MIs in nuclear fusion plasma are investigated now intensively. In particular, their effect on ITB formation is very important. It is shown that due to sufficient plasma electron heating near the rational surface with the poloidal chain of MIs, that they become to propagate approximately without collisions around torus, electrons start to miss MIs and in consent with [7] the essential shear is formed. The time of shear formation is derived and shown to be small.

The conditions on MI width are derived, at which it leads to essential shear formation. The condition on MI width is known, at which it does not lead to anomalous transport on trapped particles. For that the island should be narrow. It is shown that even narrow island can lead to essential shear. One can note that several poloidal chain of islands are better for ITB formation as in [7].

It is shown that this shear can damp instabilities with growth rate γ smaller than ion cyclotron frequency $\gamma < \omega_{ci}$.

There are some effects, connected with shear. At strong anomalous transport, when it is determined by a streamer, formed by single wide vortex, which one

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overlaps all inhomogeneous area, the shear mince vortices. It leads to intermediate ITB formation. But in this case some anomalous transport remains after ITB formation [1]. At not strong anomalous transport, determined by lattice of vortices, the shear can spatially separate vortices. In this case the strong ITB is formed.

2. Spatial structure of vortical convective cells

Let us describe some chain on azimuth θ of the vortices in the plasma, located in crossed $\vec{H}_0$ and $\vec{E}_{r0}$ fields. Neglecting nonstationary and nonlinear on electric potential ϕ of vortices members, from electron motion eq. one can obtain at small $\delta r \equiv r - r_v$ deviations r from radial position of vortices r_v, eq., describing oscillatory dynamics of electrons in the field of the perturbation

$$(\delta r)^2 + 4\left(e\phi - \delta p_e/n_e(r_v)\right)\Big/ r_v m_e \omega_{ce}(r_v)\partial_r \omega_{\theta o}\Big|_{r=r_v} = C \,. \quad (1)$$

$$\omega_{\theta o} \equiv V_{\theta o}(r)/r$$

$$\vec{V}_{\theta 0} = -e\left[\vec{e}_z, \vec{E}_{ro}\right]\Big/ m_e\omega_{ce} - \left[\vec{e}_z, \vec{\nabla} p_{0e}\right]\Big/ n_e m_e \omega_{ce} \,,$$

δp_e is the electron density perturbation. (1) describes the radial electron oscillations $\delta r \equiv r - r_v$ through dependences of $\phi(\theta, r)$ and electron density perturbations $\delta n_e(\theta, r)$ on θ and r. Let us connect $e\phi - \delta p_e/n_e(r_v)$ with characteristic of electron vortical movement $\alpha \equiv \vec{e}_z \mathrm{rot}\vec{V}_e$. From electron eq. of motion we derive

$$\alpha/V_{thi}\rho_{ci} = e\Delta\varphi/T_e - \Delta n_e/n_e \quad (2)$$

For vortex with dimension ρ_{ci} we have $\alpha < \omega_{ci}$ and $e\varphi/T_e - \delta n_e/n_e \approx \alpha/\omega_{ci}$. One can see that the amplitude of the radial oscillations of the electrons for the given amplitudes of the electrical potential and density perturbation is smaller for larger shear $\partial_r \omega_{\theta o}$. The latter helps the ITB formation.

In crossed fields the vortices can be with phase velocity $V_{ph} \approx V_{\theta 0}$, as well as slow vortices $V_{ph} << V_{\theta 0}$, for example of Rossby kind. Let us derive the spatial structure of slow vortex. For this purpose one can derive similar to [9] without any approaches general nonlinear vectorial equation, describing vortical electron dynamics

$$d_t\left(\frac{\vec{\alpha} - \vec{\omega}_{ce}}{n_e}\right) = \frac{1}{n_e}\left(\left(\vec{\alpha} - \vec{\omega}_{ce}\right)\vec{\nabla}\right)\vec{V} \quad (3)$$

$\vec{\alpha} = \left[\vec{\nabla} \times \vec{V}\right]$, $d_t \equiv \partial_t + \left(\vec{V}\vec{\nabla}\right)$. From it one can obtain eq., describing the vortex of the small amplitude. Hence we derive for perturbation of electron trajectory in the field of the vortex

$$\delta r(\theta, r) = -\frac{1}{\omega_{ceo}\partial_r\left(n_{oe}/\omega_{ce}\right)}\delta n_e(\theta, r) \,. \quad (4)$$

From here one can see that at the same amplitude of plasma density perturbation δn_e the vortex is narrower in radial direction for larger $\partial_r\left(n_{oe}/\omega_{ce}\right)$, i.e. for more abrupt plasma density profile. The latter helps ITB formation.

The vortex is excited up to the amplitude, at which the layers, trapped by it, during excitation time γ^{-1} are shifted relative to each other due to shear on the angle not larger $2\pi/\ell_\theta$, i.e. $\delta r_v\left(\ell_\theta/2\pi\right)\partial_r\omega_{\theta 0}\Big|_{r=r_v} \le \gamma$. Here ℓ_θ is the azimuth wave number of excited vortices. From this expression we derive

$$\left|e\phi_o - \delta p_{e0}/n_e(r_v)\right| = \left(\gamma\pi/\ell_\theta\right)^2 r_v m_e \omega_{ce}\Big/ 2\left|\partial_r \omega_{\theta o}\right|_{r=r_v}\Big| \quad (5)$$

The amplitude of the vortex saturation is inversely proportional to $\partial_r \omega_{\theta 0}$. The decrease of level of fluctuations at ITB formation has been observed in [3]. It promotes ITB formation.

3. Convective – diffusion equation

Let us consider the finite amplitudes, when frequency Ω_r of the electron oscillations, forming the vortex, in its field becomes larger than the growth rate γ of the excitation $\Omega_r > \gamma$. Then in vicinity of the vortex borders the jumps on electron density profile $n_e(r)$ are formed. Hence, on these n_e jumps new cells with the greatest growth rates are excited. It results in ordering of convective cells. Therefore at achievement of the large amplitudes the instability is developed for ordering of cells similar investigated in [10]. Inside borders of a vortex ordered convective movement of the electrons occurs. However, they are influenced by environmental vortex fields and fluctuations. Also it is important that amplitudes of vortexes are not stationary. Instead of average $n_{oe}(t,r)$, which does not take into account correlations, we use four densities of the electrons $n_{ke}(t,r)$ average on small-scale oscillations: $n_{1e}(t,r)$ ($n_{2e}(t,r)$) is the average density of the electrons, located in region 1 in depth of a cell on $r>r_v$ (in region 2 in depth of a cell on $r<r_v$); $n_{3e}(t,r)$ ($n_{4e}(t,r)$) is the average density of the electrons, placed in region 3 near border of a cell on $r>r_v$ (in region 4 near border of a cell on $r<r_v$). The importance of use of different $n_{ke}(t,r)$ is also determined by that angular speeds of electron rotation inside a cell are different in dependence on distance from its axis. Also in central area of the convective cell the following processes are still realized: plateau formation on $n_e(r)$ due to difference of angular speeds of electron rotations; due to jump formation on $n_e(r)$ at the certain moments of time in the regions 1 and 2 there is an accelerated diffusion and an exchange by electrons between regions 1 and 3 (factor α), and also between regions 2 and 4. α is the factor of mixing also due to influence of fluctuations, growth of amplitudes, differences of characteristic times of the electrons.

But at ordering the adjacent cells form integrated border. The particle in space between individual cell border and integrated border move in radial direction from cell to cell for the distance min{ ℓ_{cor}, $\delta r_v \tau_{cor}\Omega_r/\pi$ }. ℓ_{cor}, τ_{cor} are the correlation length and time of vortical convective cell turbulence.

From the above we have approximately

$$n_1(t+\tau, r) = (1-\alpha)n_2(t, r) + \alpha\beta n_3(t, r)$$
$$n_2(t+\tau, r) = (1-\alpha)n_1(t, r) + \alpha\beta n_4(t, r) \quad (6)$$

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$$n_3(t+\tau,r)=\alpha n_1(t,r)+\beta(1-\alpha)n_3(t,r-\delta r_v)+ \\ +0.5(1-\beta)[n_3+n_4]$$

$$n_4(t+\tau,r)=\alpha n_2(t,r)+\beta(1-\alpha)n_4(t,r+\delta r_v)+ \\ +0.5(1-\beta)[n_3+n_4]$$

β is the factor of the convective exchange of cells by particles. The value of β is determined by ratio of the area with convective electron dynamics, located between individual cell borders and integrated borders to all area, located between individual cell borders and integrated borders of adjacent cells. From these equations, entering $\overline{n}=(n_3+n_4)/2$, $\delta n=n_3-n_4$, $\overline{N}=(n_1+n_2)/2$, $\delta N=n_1-n_2$, we derive

$$\begin{gathered}\tau\partial_t\overline{n}=\alpha(\overline{N}-\beta\overline{n})-(\beta/2)(1-\alpha)\delta r_v\partial_r\delta n \\ \tau\partial_t\delta n+[1-\beta(1-\alpha)]\delta n=\alpha\delta N-2\beta(1-\alpha)\delta r_v\partial_r\overline{n} \qquad (7) \\ \tau\partial_t\overline{N}=\alpha(\beta\overline{n}-\overline{N}),\ \ \tau\partial_t\delta N+(2-\alpha)\delta N=\alpha\beta\delta n\end{gathered}$$

One can see that introduction $\overline{n}$ is similar to average $n_{oe}(t,r)$ but with taking into account correlations. From these equations we have similar to [10] the following convective – diffusion equation

$$\tau^2\partial_t^2\delta n+\tau\partial_t[(1-\beta(1-\alpha))\delta n-\alpha\delta N]= \\ =-2\beta(1-\alpha)\delta r_v\partial_r\left[\alpha(\overline{N}-\beta\overline{n})-\frac{\beta}{2}(1-\alpha)\delta r_v\partial_r\delta n\right] \qquad (8)$$

As β is proportional to $(\delta r_v-\Delta)/\delta r_v$ then at $\delta r_v<\Delta$ we have $\beta=0$ and there is no convective radial transport because convective cell exchange by particles disappears.

4. Shear formation by magnetic islands

Let us consider the shear formation near the poloidal chain of narrow magnetic islands. The electrical field E_{r0} is approximately equal to zero on the axis of a plasma column. It receives maximum value inside a plasma column at $r=r_m$. We suppose, that on a some interval $0<r<r_m$ (r_0-δR<r<r_0+Δr+δR around chain of MIs, located in r=r_0+$\Delta r/2$ with radial width Δr, r_o is the lower border of island) on radius r the electrical field E_{r0} in the case of shear absence is proportional to $E_{r0}\propto r$. Then on this interval it is possible to present

$$E_{r0}=-2\pi eN_0 r\,,\ \ 0<r<r_m\,,\ \ N_0\equiv n_e-n_i\,. \qquad (9)$$

It means, that there is no shear $\omega_{\theta 0}\neq\omega_{\theta 0}(r)$. One can take into account effect of oscillations on electron transport by effective collision frequency ν_{ef} in diffusion coefficient $D_\perp\propto(\nu_e+\nu_{ef})$. ν_e is the electron collision frequency

On the plasma cross-section $0<r<R$ several chains of magnetic islands can exist [7]. But we for the simple case consider influence of one poloidal chain of islands on shear formation. We consider low order rational magnetic surface, because important property of this surface is appeared, when plasma is heated sufficiently that its electrons perform several rotation around toroidal surface during free pass time. The shear formation at

$$2\pi r_{tor}\nu_e<V_{the} \qquad (10)$$

[7] is considered near this surface. V_{the} is the electron thermal velocity, moving near this surface. According to [4, 7] we consider local plasma heating near this surface. The local heating leads to important effect: inequality (10) becomes to be satisfied more easier.

If condition (10) is satisfied the electrons starts to miss the island. The part of the electrons, located in the island, escapes its, up to reaching E_r to zero inside it, except for the layer of width

$$\delta r_{sep}\approx 2\pi n r_{tor}\sqrt{D_\perp/D_\parallel}\approx\rho_{ce}\sqrt{(\nu_e+\nu_{ef\perp})/(\nu_e+\nu_{ef\parallel})}$$

near r=r_0. $D_\perp$, $D_\parallel$ are the transversal and longitudinal diffusion coefficients, $\nu_{ef\perp}$ and $\nu_{ef\parallel}$ are the transversal and longitudinal effective collision frequencies. When part of the electrons escapes the magnetic island, in the region of the island the plasma ion volume charge n_i are not compensated by electrons on δn. δn can be determined from observed in experiment condition, that in the island $E_{r0}\approx 0$ is approximately established.

Let us consider shear formation near chain more in detail. In the case (10) the electron transport through MIs changes from slow collisional to quick one. Electrons miss MI. Though the radial size of MI is small the quick electron transport leads to appearance of strong uncompensated ion volume charge in MI and to the strong shear $\partial_r\omega_{\theta o}$.

That in the region of the magnetic island the uncompensated ion volume charge appears it is necessary that its width Δr should be larger $\Delta r>\rho_{ce}$ in comparison with the electron cyclotron radius.

Let us consider radial electron dynamics in small neighborhood of the chain of islands. The electrons move in radial direction in crossed $\vec{E}\times\vec{H}$ fields with velocity $V_{0r}=-(eE_{or}+\partial_r p_{0e}/n_e)(\nu_{ef}+\nu_e)/m_e\omega_{ce}^2$. When electron reaches with V_{0r} island from the side of small r, it propagates collisionally through island in the case $2\pi r_{tor}\nu_e>>V_{the}^{(hot)}$. But in the case $2\pi r_{tor}\nu_e<<V_{the}^{(hot)}$ electron without collision quickly, during the time $2\pi r_{tor}/V_{the}^{(hot)}$ get on the second boundary of the island. After that electron again slow propagates with velocity V_{0r} in the direction of large r.

Thus at $2\pi r_{tor}\nu_e<V_{the}^{(hot)}$ near the island the shear is formed. Using approximation of poloidal chain of narrow MIs as a azimuth symmetrical narrow layer we have for the radial electric field distribution

$$E_{r0}\approx-2\pi e\begin{cases}N_a r,\ 0\le r\le r_0 \\ N_a r_0^2/r-\delta n(r-r_0^2/r),\ r_0\le r\le r_0+\delta r_{sep} \\ 0,\ r\ge r_0+\delta r_{sep}\end{cases} \qquad (11)$$

If the field is small $E_{r0}\approx 0$ at $r=r_0+\delta r_{sep}$, we have $N_a\approx 2\delta n\delta r_{sep}/r_0$. One can see that the density of uncompensated plasma ion volume charge is relative large in the island $r_0>>\Delta r$ if $E_r\approx 0$ at $r=r_0+\delta r_{sep}$.

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One can use estimation $N_a \approx 4n_0(e\Delta\varphi/T_i)(r_{di}/L)^2$. L is the width of the region with essential E_r, $\Delta\varphi$ is the potential on L. One can conclude that the island can be narrow for essential shear formation

$$(e\Delta\varphi/T_i)(r^2_{di}/L\delta r_{sep})<1 . \qquad (12)$$

This inequality means that in real island the small ion volume charge uncompensation $\delta n << n_0$ is sufficient for essential shear formation.

Let us calculate shear of the electric field and normalize it on the electric field $E_r = -2\pi e N_0 r$ in shear absence $S \equiv \left(E_r\big|_{no\,TB}\right)^{-1} r^2 \partial_r (E_r/r)$. If $E_r\big|_{r=r_0+\delta r_{sep}} \approx 0$ we have

$$S = \begin{cases} (N_a/N_0)(r_0^2/r^2)(2 + r_0/\delta r_{sep}), \; r_0 \le r \le r_0 + \delta r_{sep} \\ 0, \; r \ge r_0 + \delta r_{sep} \end{cases} \qquad (13)$$

The shear is large for region of narrow magnetic islands $r_0 >> \Delta r > \delta r_{sep}$

$$S \approx (N_a/N_0)(r_0/\delta r_{sep}), \;\; r_0 \le r \le r_0 + \delta r_{sep} . \;\; |S| >> 1 \quad (14)$$

Really, in experiment [7] the decrease of correlation of plasma density fluctuations were observed.

Let us consider the shear of $\omega_{\theta 0} = V_{\theta 0}/r$

$$V_{\theta 0} = (m_e \omega_{He})^{-1}(-eE_{r0} - \partial_r p_{0e}/n_{0e}), \qquad (15)$$

as for ITB formation this shear $\partial_r \omega_{\theta 0}$ is more important. We determine the angular velocity shear

$$S_\omega \equiv \left(\omega_{\theta 0}\big|_{no\,TB}\right)^{-1} r \partial_r \omega_{\theta 0}, \;\; \omega_{\theta 0}\big|_{no\,TB} = (\omega_{pe}^2/\omega_{He})(N_0/2n_0)$$

Then we derive

$$S_\omega = -\begin{cases} (N_a/N_0)(r_0^2/r^2)(2 + r_0/\delta r_{sep}), \; r_0 \le r \le r_0 + \delta r_{sep} \\ 0, \; r \ge r_0 + \delta r_{sep} \end{cases}$$

Absolute value of the relative angular velocity shear is of order of $S_\omega = -(N_a/N_0) r_0/\delta r_{sep}$. But absolute shear can be increased. In several experiments the strong localization of the region with $V_{0\theta} \ne 0$ has been observed. As in experiments the radial width Δr_{sh} of area $V_{0\theta} \ne 0$ localization $\Delta r_{sh} = 1\text{cm}$ is observed, the shear $(\partial_r V_{0\theta})_{apr}$ can be increased in existing nuclear fusion installations in comparison with smooth case $(\partial_r V_{0\theta})_{smooth} \approx V_{0\theta}/R$ almost in 100 times $(\partial_r V_{0\theta})_{apr} \approx V_{0\theta}/\Delta r_{sh} \approx (\partial_r V_{0\theta})_{smooth} R/\Delta r_{sh}$.

The shear is formed during

$$\tau_{TB} \approx (\nu_{ef} + \nu_e)^{-1}(\omega_{ce}^2/2\omega_{pe}^2)(r_0/\Delta r) \qquad (16)$$

the short time for not very narrow MI.

Narrow islands, though provide a fast electron transport through island dimension Δr, strong suppress transport in broad their neighborhood. Really in experiment [4] $\Delta r << R$.

In the island, placed on rational surface with numbers n, m the uncompensated ion volume charge has appeared at $(\Delta r)^2/D_\perp > (2\pi n r_{tor})^2/D_\parallel$. r_{tor} is the toroidal radius. Hence the island should be wider

$$\max\{\rho_{ce}, 2\pi n r_{tor}\sqrt{D_\perp/D_\parallel}\} < \Delta r . \qquad (17)$$

$D_\perp/D_\parallel = (\nu_{ei} + \nu_{ef\perp})(\nu_{ei} + \nu_{ef\parallel})/\omega_{ce}^2 <<1$. But for ITB formation the island should be narrow [11]

$$\Delta r << R .$$

Let during free pass time the electron has time to make q rotations around the torus. Then

$$\max\{\rho_{ce}, (n\rho_{ce}/q)\sqrt{(1+\nu_{ef\perp}/\nu_{ei})(1+\nu_{ef\parallel}/\nu_{ei})}\} < \Delta r . \; (18)$$

From obtained expressions and from condition [1-3]

$$L\partial_r \omega_{\theta 0} > \gamma \qquad (19)$$

using

$$\Delta\varphi \approx 2\pi e N_a L^2 \qquad (20)$$

one can show

$$(\Delta\varphi e/T_i)(\rho_{ci}^2/L\delta r_{sep}) > \gamma/\omega_{ci} \qquad (21)$$

that shear can damp low-frequency instabilities with growth rate $\gamma < \omega_{ci}$.

5. Conclusions

It is shown that the amplitude of the radial electron oscillations is smaller or the vortex is narrower in radial direction for larger shear $\partial_r \omega_{\theta o}$ and for more abrupt plasma density profile in nuclear fusion plasma. The latter helps ITB formation. The amplitude of the vortex saturation is inversely proportional to $\partial_r \omega_{\theta o}$. It also promotes ITB formation. The convective-diffusion equation for electron transport has been derived.

It is shown that the value of the shear can be large. The shear is formed in consent with [7] due to plasma heating near the low order rational surface with poloidal chain of narrow magnetic islands. The plasma is heated up to such electron temperature that the electrons become to propagate approximately without collisions around torus. Small part of electrons leaves the island and the shear is formed. The time of shear formation is small. The conditions on island width is derived, at which it leads to shear formation. Even narrow island can lead to shear formation. The condition on island width is known, at which it does not lead to anomalous transport. This shear can damp instabilities with growth rate smaller than ion cyclotron frequency $\gamma < \omega_{ci}$.

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