Scaling Laws of Intermittent Plasma Turbulence in Edge of Fusion Devices

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The high-order structure functions have been analyzed to characterize the edge plasma intermittency in fusion devices. The scaling properties of edge turbulence have shown a strong deviation from a prediction of the Kolmogorov's K41 model. The turbulent fluctuations demonstrate a generalized scale invariance and log-Poisson statistics.

Keywords: edge plasma turbulence, intermittency, log-Poisson model, extended self-similarity, multifractal statistics

1. Inroduction

Incompressible hydrodynamic turbulence is described by Kolmogorov theory [1] (K41) which considers a hierarchical energy cascade and the energy scaling E(k) $\sim k^{-5/3}$. At large Reynolds numbers Re>1, statistical quasi-equilibrium of fluctuations is established in the process of energy transfer from large turbulent eddies towards the smaller ones in the inertial range l ($\eta \ll l \ll L$, where L is the largest eddy/ structure scale, η - dissipation Kolmogorov's scale). Investigation of self-similar turbulence properties resulted in multiplicative hierarchical cascade models [2]: the log-normal model K62, the β-model, and most favorable the log-Poisson model [3, 4]. To quantify whether boundary conditions influence the statistical properties in turbulence it was proposed multifractal formalism (see [2]). Intermittency leads to a local breaking of the turbulence homogeneity, when 'active' regions coexist with quasi-laminar ones. The intermittent fluctuations demonstrate non-Gaussian statistics, self-similarity and multifractality. The intermittency is observed in turbulent hydrodynamic fluids [3] and in turbulent magnetized plasmas (see e.g [5]).

The spectrum of magnetohydrodynamic (MHD) turbulence was first addressed by Iroshnikov and Kraichnan (IK) who considered the turbulent energy cascade affected by a magnetic field (see [6,7]). The IK model yields $E(k) \sim k^{-3/2}$ resulted from Alfven decorrelation effect. The validity of the two phenomenologies (K41 and IK) in MHD turbulence and plasma confined in fusion devices is still under a discussion. Numerical and experimental data indicate that in MHD turbulence the energy transfer occurs predominantly in the field-perpendicular direction [6]. This raises a question whether anisotropy is crucial for the energy cascade, and whether it changes the spectrum of turbulence. The two-dimensional direct numerical simulations (DNS) support the IK picture, while three-dimensional simulations and recent analytical results suggest K41 energy spectra (see [6]). A phenomenology of "intermediate" turbulence by Goldreich and Sridhar [7] (GS95) postulates a balance between K41 and IK energy cascades and accounts for the local anisotropy induced by a magnetic field. The GS95 assumes one-dimension filaments as the most dissipative structures, the same as in hydrodynamic turbulence. Other MHD turbulence models assume the singular structure shape of a current sheet [6,7].

There are numerous experimental observations of the magnetized plasma turbulence that share a lot of features of neutral fluid turbulence including many scales, the cascades, strong mixing, non-linear scalings and so on. Despite equations described neutral fluids and plasmas are different they have the same type of scale invariance (dilatation symmetry, namely, $x \to \lambda x$, $t \to \lambda^{1-h} t$, $\upsilon \to \lambda^{h} \upsilon$). This common dilatation symmetry is responsible for a common scaling property.

Most favorable log-Poisson model of intermittent turbulence consider anisotropic stochastic cascade and the generalized self-similarity implying the long-range correlations, which drive an anomalous transport, e.g. superdiffusion. An experimental analysis of the statistical moments in the frame of the log-Poisson model suggests a description of the transport processes in the real turbulent

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plasmas. Despite the large amount of experimental data that has been obtained in fusion devices, our understanding of the turbulence and diffusive transport process in magnetized plasmas is still rather limited. In this work we focus on the scale invariance property and scaling laws of the edge plasma turbulence in fusion devices and test analytical fits.

2. Experimental data

We analyze Langmuir probe signals of ion saturation current I_{sat} that is essentially plasma density. On Large Helical Device [9] I_{sat} was measured by 16 graphite dome-type electrodes (diameter of 1 mm separated by 6 mm) embedded in the divertor plate and by reciprocating probe. On JT-60U tokamak measurements (L-mode) in the SOL has been done by reciprocating Mach probes [10] installed at the low field side (LFS) mid-plane and just below the X-point .On T-10 tokamak [5] (R/r= 1.5 m / 0.4 m, I_p=200÷220 kA, B=2.2÷2.4 T) Langmuir probe (tungsten tips of 0.5 mm in a diameter and 3 mm in a length) I_{sat} was measured at SOL in a steady state of repetitive ohmic discharges with no MHD activity. In the linear divertor plasma simulator NAGDIS-II (n_e~ 10^{20} m^{-3} , T_e~ 1–3 eV , B= 0.25 T) attached and detached plasmas were investigated [11,5]. All signals are digitized with sampling rate 0.5 MHz in JT-60U and LHD and 1 MHz in other devices, the sample number 10^5 - 10^6. Signals have the intermittent behaviour typically observed in edge of fusion devices [5]. Power spectra exhibit a complicated frequency dependence without a trivial power-law behavior [5].

3. Generalized scale invariance

The classical approach [12] for an exploration of statistical features is an analysis of statistical moments (structure functions) $S_q(\tau)= \langle|X(t+\tau)-X(t)|^q\rangle$, where $\langle\ldots\rangle$- means an ensemble average of the time-dependent signal X(t). The structure function technique is equivalent to the detailed investigation of the probability distribution function of the turbulent fluctuation. The Kolmogorov theory K41 of isotropic turbulence infers Gaussian statistics for fluctuations. It predicts the structure function scaling $S_q(\tau)\sim\tau^{\zeta(q)}$, $\zeta(q)=q/3$, in the inertial range. Experimental investigations of the developed hydrodynamic turbulence demonstrate a departure of the high-order structure function scalings (q > 3) from the Kolmogorov K41 prediction due to the intermittency. The experimental structure functions typically shows a power law only over a limited (inertial) range $\eta << l << L$ (of ~10 μs in fig.1). Whereas the generalized self-similarity is registered in a broadened range extended to the dissipation scales [13,2]. A generalized scale invariance (extended self-similarity - ESS) was proposed in [13], and then considered in the log-Poisson model of turbulence [3,4]. Hidden statistical symmetries of the Navier-Stockes equations, hierarchy of moments, multifractality are behind the property of the ESS. The ESS infers a scaling $S_q(l)$~ $S_3(l)^{\zeta(q)/\zeta(3)}$ for the extended range $l \geq 5\eta$. All data from fusion devices, that we analyzed, demonstrate such ESS property(linear behavior in fig.2) over the time scales ~1

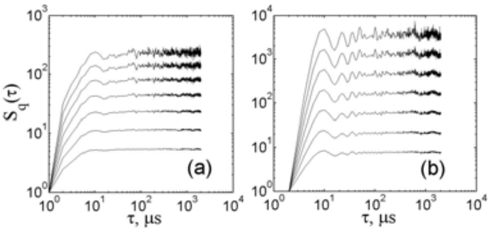

Fig.1. The structure functions $S_q(\tau)$ of high orders (q=2÷8 from bottom to top) vs. time scale τ. (a) LHD divertor probe #10 (b) LFS SOL JT-60U,shot#44421

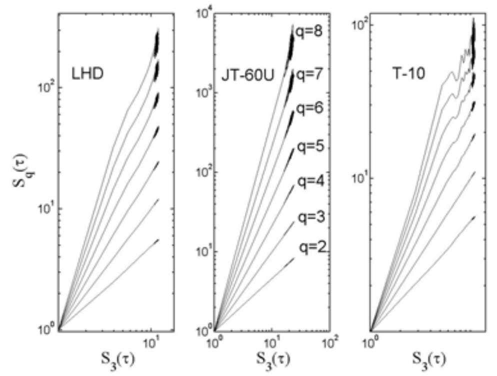

Fig.2. ESS plot of the structure function $S_q(\tau)$ of high orders (q=2÷8 from bottom to top) from the third-order one. (a) LHD divertor probe, (b) LFS SOL JT-60U,distance from separatrix 42mm (c) SOL T-10

msec substantially longer than an inertial range in fig.1. The ESS corresponds to the considering of the scaling in a turbulent cascade not with respect to the usual distance, but with respect to an effective scale defined by the third order moment of the velocity field.

4. The log-Poisson model of the turbulence

The scaling of the third-order moment can be deduced analytically ($\zeta(3)$ =1), therefore scaling of $\zeta(q)/\zeta(3)$ can be analyzed in experiments to improve the precision of the scaling estimation, especially at moderate Reynolds numbers assessed in experiments. It allows to obtain more accurate values of $\zeta(q)$ by using a property of the ESS plotting S_q as a function of S_3, fig. 2. We treat experimental scalings in the frame of log-Poisson turbulence model [4] predicted a scaling:

$$\zeta(\boldsymbol{q})=(1-\Delta)\frac{\boldsymbol{q}}{3}+\frac{\Delta}{1-\beta}\left[1-\beta^{\frac{q}{3}}\right] \quad (1).$$

It is based on the hypotheses of a "hidden symmetry" and a hierarchical structure of the moments of the energy dissipation. The logarithm of energy dissipation obeys the Poisson statistics (so-called the log-Poisson statistics) characterized by special scale-covariance properties. A hidden symmetry can be interpreted as a generalized scale covariance and β is a characteristic of the intermittency of the energy dissipation (β=1 for non-intermittent fully developed turbulence). The quantity ε_l^{∞} (associated with the most intermittent dissipative structures) has a divergent scaling $\varepsilon_l^{\infty} \sim l^{-\Delta}$, as $l \to 0$, where Δ is a parameter depending on the dimension of the dissipative structure. In an isotropic 3D turbulence $\Delta = \beta = 2/3$ which is obtained if the most dissipative structures are filaments. The ESS property is involved in the log-Poisson model. We use the wavelet transform modulus maxima method (WTMM) [5] to estimate $\zeta(q)$ from experimental signals. In fig.3, the

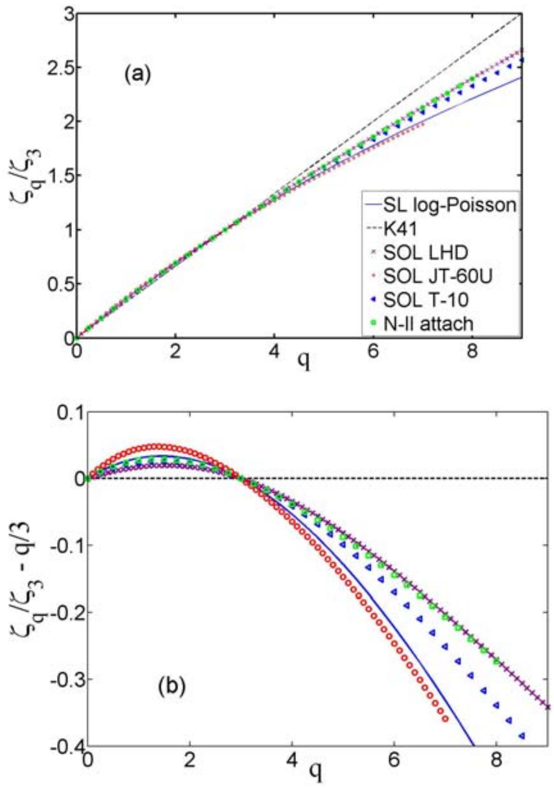

Fig.3. (a) Structure function scaling vs. order q. Kolmogorov K41 (a dashed line) and log-Poisson (a solid line, β=Δ=2/3) models and (b) the same for a departure of the scaling from the K41.

scalings $\zeta(q)/\zeta(3)$ are shown in the same plot with the scalings predicted by the K41 and the log-Poisson models. The scalings are anomalously deviated from the K41 scaling, fig.3b. Each experimental scaling could be fitted by (1) with adjusted parameters Δ and β. A solving of non-linear least-squares problem of fitting to (1) gives indexes in the range Δ =0.15 ÷ 0.8, β =0.25 ÷0.7. Some signals have non-intermittent behaviour (β=1). The observed range of Δ (between 1/3 and 2/3) can be interpreted [14] that the most intermittent dissipative structures are one-dimensional filament structures in these cases. Such dissipative structures have most likely not a trivial geometrical topology but a fractal one.

The log-Poisson model of 3D turbulence was modified in MHD case to account for the IK phenomenology [7]. To test IK model we plot in fig.4 a deviation of relative exponents $\zeta(q)/\zeta(4)$ from IK scaling q/4. The scaling of data in the vicinity of X-point in JT-60U is close to the IK indicating strong MHD turbulence property. The data from SOL are deviated strongly from IK scaling (see [6]). At the same time they are not fitted by the scaling for MHD log-Poisson model (see definition in [7]). It can be interpreted that IK phenomenology (two-dimension strong anisotropy) is not available for a treatment of the SOL plasma turbulence.

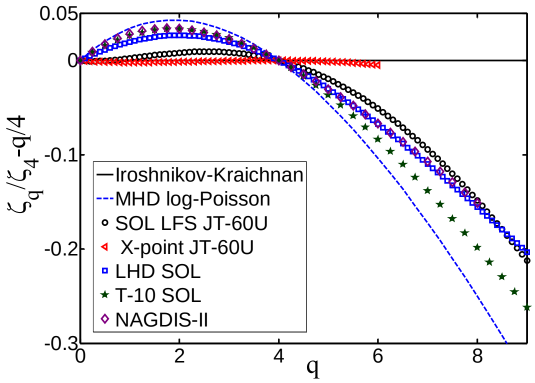

Fig.4. Deviation of scaling from Iroshnikov-Kraichnan scaling q/4 (solid line). MHD IK log-Poisson scaling (dashed line). Scaling in the vicinity of X-point JT-60U is close to IK.

5. Transport scaling laws

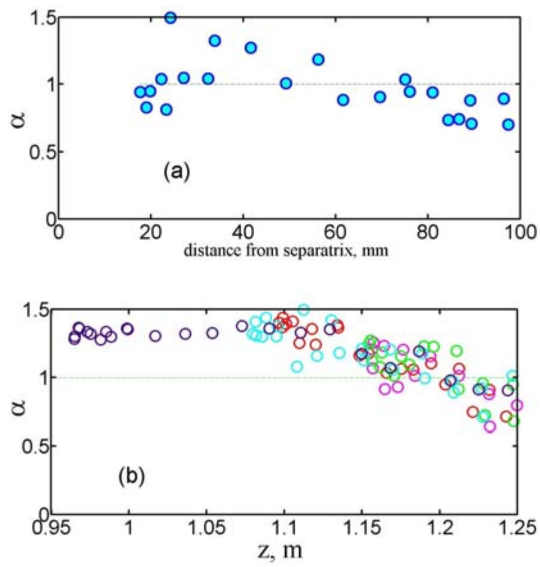

Fig.5 Diffusion scaling index (a) JT-60U LFS SOL (shot# 44421) (b) SOL LHD high β shots vs. vertical coordinate z, B=0.425 T

The statistical description of transport processes in fusion plasmas is an alternative approach to the traditional characterization of a transport based on the computation of effective transport diffusion coefficients.

The log-Poisson model could be used to estimate a transport scaling based on the self-similarity indexes β and Δ (1) that responsible for percolation effect in the turbulence. In a simplified approach [14], the diffusion scaling depends on the structure function scaling as $D \propto \tau^{K(-1)}$, the exponent $K(q)$ relates with the scaling of the high-order structure function $\zeta(q)$ as $K(q)=q-\zeta(3q)$. A displacement of particles across a magnetic field with time τ is scaled as $\langle \delta x^2 \rangle \propto D\tau \propto \tau^{\alpha}$ with an exponent $\alpha \propto 1+K(-1)$. This index was estimated from experimental

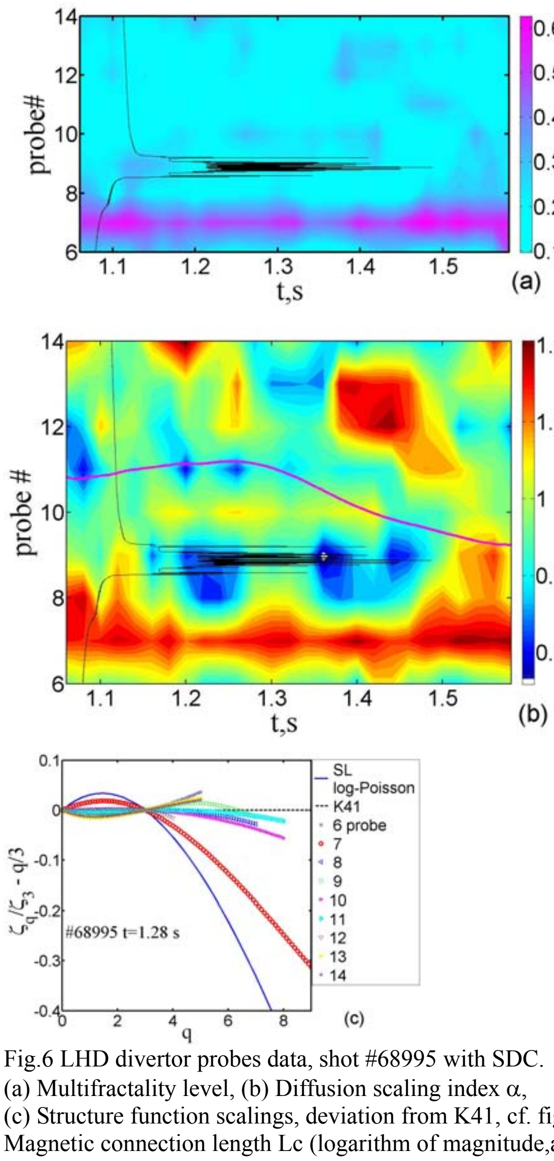

Fig.6 LHD divertor probes data, shot #68995 with SDC.
(a) Multifractality level, (b) Diffusion scaling index α,
(c) Structure function scalings, deviation from K41, cf. fig3a.
Magnetic connection length Lc (logarithm of magnitude,a.u.) in black line, and plasma storage energy evolution (magenta)

scalings. It varies with a radius in JT-60U SOL (fig.5) indicating superdiffusion process (α>1) at radial distance 20-50 mm where the direction of the parallel SOL flow changes upward to downward [10]. In LHD the natural island layers overlap and the stochastic field structure (a natural helical divertor) appears between the LCFS and the residual X-point. The magnetic field line with large connection length L_c reaches the ergodic layer surrounding the core plasma region. L_c varies from less than a few meters to over a few kilometers (see black line in fig.6a,b). Ion particle flux to the divertor plates [9] and multifractality level (deviation from Gaussian statistics, see a definition in [5]) follows the deposition profile of the magnetic field lines, fig.6a. The probe connected to the field line with a large L_c has a large ion particle flux. In LHD, an increasing of α>1 was observed in domains that are characterized by short connection length (probe # 7 in fig.6b). Scalings illustrate specific behaviour of #7 probe location compare with others that are closer to K41 scaling, fig.6c. This property is kept even at discharge evolution at super-dense core (SDC).

In conclusion, the statistical properties of the intermittent turbulence show a striking empirical similarity in the SOL plasma region of fusion devices. Scalings of the structure functions strongly deviate from the Kolmogorov's K41 theory prediction. The anomalous behavior of scaling is similar in the SOL plasma of helical device, tokamaks, and linear machine. Experimental scalings are close to the log-Poisson model. One-dimension filament structures are likely the most intermittent dissipative structures. The similar behavior of the scalings has been observed in the edge of fusion devices with different magnetic topology and heating. It supports a view that the edge plasma turbulence displays universality. By using self-similarity indexes, transport scaling indexes are estimated from percolation property of the turbulence with non-trivial self-similarity. The results of our study improve our understanding of intermittent turbulence in the edge of fusion devices.

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