Instability of Thin-Walled Annular Beam in Dielectric-Loaded Cylindrical Waveguide

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Cherenkov and slow cyclotron instabilities driven by an axially injected electron beam in cylindrical waveguide are studied by using a new version of self-consistent liner theory considering three dimensional beam perturbations. There are three models of beam instability analysis, which are based on solid beam, infinitesimally thin annular beam, and thin-walled annular beam. Among these models, a model properly representing the often used actual annular electron beams is thin-walled annular beam. We develop a numerical code for a cylindrical waveguide with a thin-walled annular beam having a finite thickness. Our theory is valid for any beam velocity. We present eigen modes with plasma and beam. And instabilities driven by thin-walled annular beam in dielectric-loaded waveguide are examined.

Keywords: Thin-walled annular beam, low-frequency surface wave mode, high-frequency surface wave mode, Cherenkov instability, slow cyclotron instability

1. Introduction

The backward wave oscillator (BWO) or traveling wave tube (TWT) is one of the high power microwave sources and can be driven by an axially injected electron beam without initial perpendicular velocity. For relatively low power in some 10 kW level or less, Pierce type cathodes are commonly used and beam shape is approximated by a solid beam. In many high power experiments, cold cathodes are used and the shape of electron beam is a thin-walled annulus. A new version of self-consistent field theory considering three-dimensional beam perturbations are developed based on a solid beam [1-2]. For infinitesimally thin annular beam, the boundary is modulated due to the transverse modulation of annular surface. Analyses of infinitesimally thin annular beam need to be based on a different theory from thin-walled annular and solid beam. A pioneering work can be seen in Ref. [3]. Recently, instabilities of eigen mode due to the surface modulation are analyzed in Ref. [4], presenting a self-consistent field theory considering the moving surface modulation.

In this work, we develop a numerical code for a cylindrical waveguide with a thin-walled annular beam. Note that the annular beam thickness is finite. The boundary condition at the beam surface is different from the infinitesimally thin annular beam. Solid beam and thin-walled annular beam are based on the same boundary condition, but the number of boundary is different. Thin-walled annular beam has outside and inside surface, and solid beam has only outside surface. Our numerical codes are valid for any beam velocity v. We present the eigen modes for waveguide with plasma (v=0) and beam (v>0) modes.

The organization of this proceeding is as follows. In Sec. 2, we describe numerical method of thin-walled annular beam and solid beam. In Sec. 3, dispersion carves of thin-walled annular plasma and solid plasma are examined. In Sec. 4, we examine Cherenkov and slow cyclotron instabilities driven by thin-walled annular beam in dielectric-loaded waveguide. Discussion and conclusion of this paper are given in Sec. 5.

2. Numerical method

For slow-wave devices, the periodically corrugated slow-wave structure (SWS) is used [5, 6]. However, analyses of these devices are very complex. Therefore, we analyzed basic electromagnetic characteristics of cylindrical waveguide with straight wall. We consider a dielectric-loaded waveguide (Fig.1). The wall radius R_w=1.445 cm, dielectric radius R_d=0.85 cm. In the case of plasma (v=0), relative permittivity of dielectric ε_r=1.0. For analyzing beam interactions, ε_r=4.0. We used this model to examine basic characteristics of the electromagnetic wave and the beam interaction. For thin-walled annular beam, average beam radius R_a=0.75 cm, and beam thickness Δ_p=0.1 cm. For solid beam, beam outside radius R_b=0.8 cm.

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A guiding magnetic field $\mathbf{B_0}$ is applied uniformly in axial direction. An electron beam is propagating along the guiding magnetic field. The temporal and spatial phase factor of all perturbed quantities is assumed to be $\exp[i(k_z z+m\theta-\omega t)]$. Here, m is the azimuthal mode number and k_z is the axial wave number.

In the system with magnetized plasma or beam such as Fig.2, electromagnetic modes are the hybrid mode of transverse magnetic (TM) and transverse electric (TE) mode. Two letters of EH and HE is used, to designate the hybrid mode. In this paper, TM is dominant in EH mode and TE is dominant in HE mode.

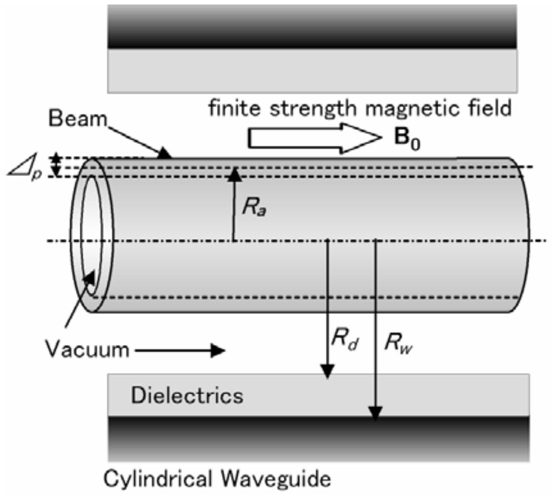

Fig.1 Model of thin-walled annular beam analysis.

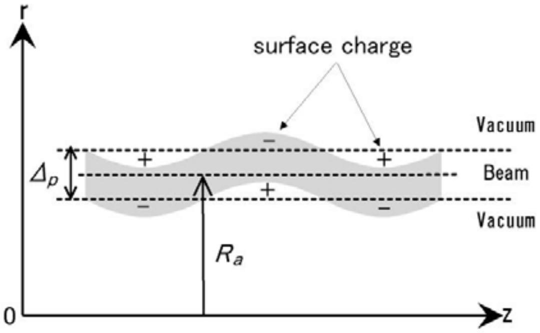

Fig. 2 Beam surface of thin-walled annular beam.

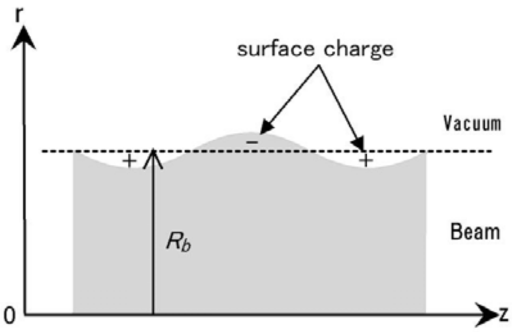

Fig. 3 Beam surface of solid beam.

An electron beam surface is modulated as beam is propagating. For thin-walled annular beam and solid beam, the transverse moderation appears as the surface electric charge at the fixed boundary as shown in Figs.2 and 3. Solid beam has one surface. Thin-walled annular beam has another surface inside beam because there is a vacuum region inside the beam.

3. Dispersion curves of plasma

We present results of thin-walled annular plasma and solid plasma analyses (ν=0). Here ω_p and Ω are plasma frequency and cyclotron frequency. In this section, $\omega_p=3\times10^{10}$ rad/s. Figure 4 shows the dispersion curves of thin-walled annular plasma and Fig. 5 shows the dispersion curves of solid plasma in the absence of magnetic field. Axisymmetric electromagnetic modes are the TM_{0n} and TE_{0n} mode. Here, n is any positive integer. Dispersion curves of thin-walled annular plasma shows two surface waves due to the inner and outer surface space charges: high- and low-frequency surface wave modes. Solid plasma has only one surface wave mode due to the outer surface space charge.

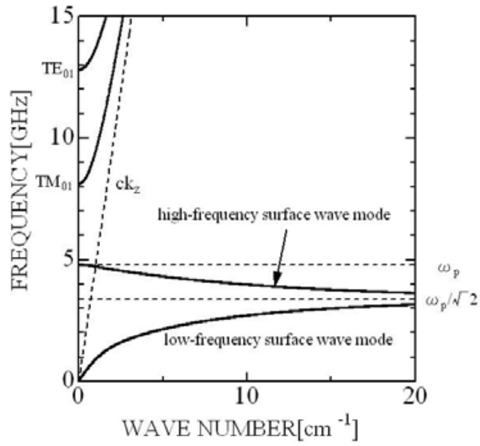

Fig. 4 Dispersion curves of thin-walled annular plasma in absence of magnetic field.

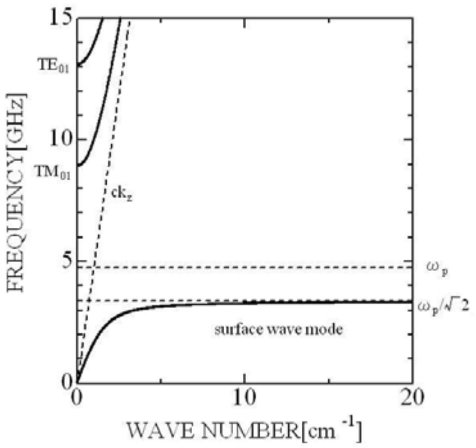

Fig. 5 Dispersion curves of solid plasma in absence of magnetic field.

The high-frequency surface wave mode is attributed to the inner boundary of thin-walled annular plasma (Fig.2). The low-frequency surface wave mode is attributed to the outer surface of annulus and corresponds to the surface wave mode of solid plasma. Their frequency is zero at k_z=0, increases with increasing k_z, and approaches an asymptotic limit as $k_z \to \infty$. This limit is $\omega_p/\sqrt{2}$. The frequency of high-frequency surface wave mode for the annulus is ω_p at k_z=0, decrease with increasing k_z, and approach an asymptotic limit as $k_z \to \infty$. This limit is also $\omega_p/\sqrt{2}$.

Te dispersion curves for the thin-walled annular and solid plasma are respectively shown in Figs. 6 and 7 at B_0=0.1 T ($\omega_p > \Omega$) and in Figs. 8 and 9 at B_0=0.2 T ($\Omega > \omega_p$). With the finite strength magnetic fields, electromagnetic modes are the hybrid mode of TM and TE modes: axisymmetric EH_{0n} and HE_{0n} modes in the figures. And cyclotron modes are appeared in addition to the space charge modes. For both of thin-walled annular and solid plasma, the frequencies of cyclotron mode are between the upper hybrid frequency $\omega_h=(\Omega^2+\omega_p^2)^{1/2}$ and Ω or ω_p, whichever is larger.

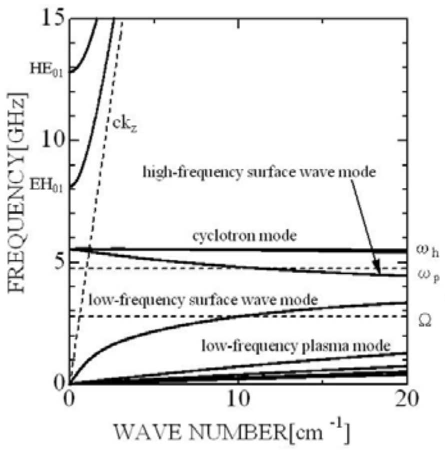

Fig. 6 Dispersion curves of thin-walled annular plasma with B_0=0.1T.

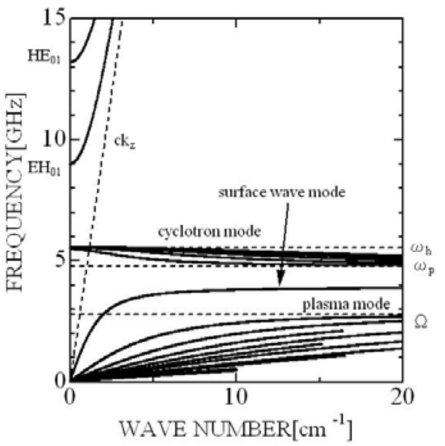

Fig. 7 Dispersion curves of solid plasma with B_0=0.1T.

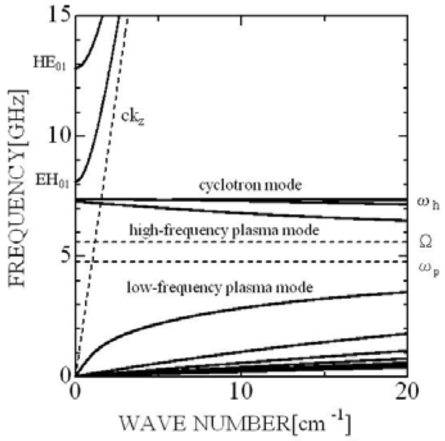

Fig. 8 Dispersion curves of thin-walled annular plasma with B_0=0.2 T.

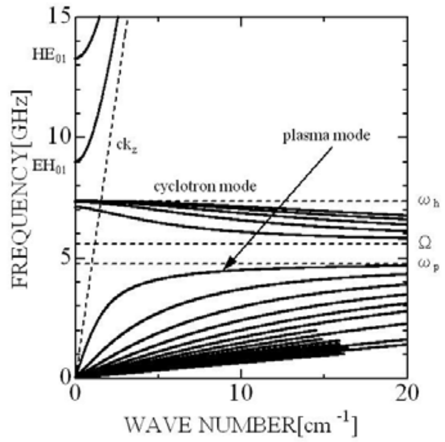

Fig. 9 Dispersion curves of solid plasma with B_0=0.2 T.

As for the space charge modes, surface wave modes exist as in the case of B_0=0 if $\Omega < \omega_p$. With increasing k_z, the frequencies of low-frequency surface wave mode and surface wave mode of solid plasma are increased and approach an asymptotic limit as $k_z \to \infty$. The limit is between ω_p and $\omega_p/\sqrt{2}$. The frequency of high-frequency surface wave mode is decreased with increasing k_z, and has the same asymptotic value as the low-frequency mode in the limit of $k_z \to \infty$

The presence of magnetic field yield plasma modes due to the volume space charge perturbation, those are denoted as plasma modes in Figs.6-9 [2]. The low-frequency plasma mode of thin-walled annular plasma and the solid plasma mode have frequencies that are zero at k_z=0, increase with increasing k_z, and approach an asymptotic limit as $k_z \to \infty$. This limit is ω_p or Ω whichever is smaller. The annular plasma has high-frequency plasma mode. Its frequency approaches an asymptotic limit between ω_h and Ω, as $k_z \to \infty$ with $\Omega > \omega_p$.

5. Instabilities driven by thin-walled annular beam

In this section, we examine instabilities driven by beam (v>0). Figure 10 shows the dispersion curves for a thin-walled annular beam with the energy 660 keV, beam current 2.3 kA and B_0=0.8 T. For an electron beam propagating along the direction of an axial magnetic field, there exist four beam modes. They are the fast and slow space charge modes and the fast and slow cyclotron modes. Slow space charge mode and slow cyclotron mode couple with both of EH_{01} and HE_{01} mode, resulting in the Cherenkov and slow cyclotron instabilities. Instabilities for EH_{01} mode are predominating, compared with those for HE_{01}mode.

Figure 11 shows the dependence of the temporal growth rate on beam thickness Δ_p. The beam outer radius is fixed to 0.8 cm, and beam inner radius has been changed with a fixed line charge density. The growth rate of Cherenkov instability increases by decreasing Δ_p.

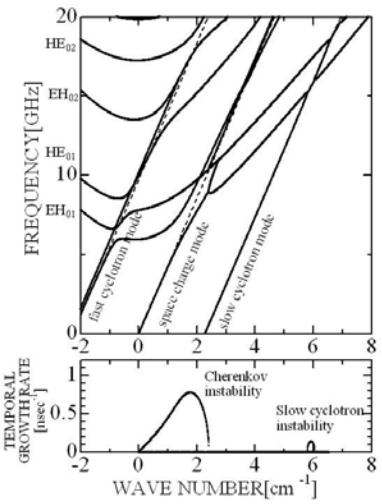

Fig. 10 Dispersion curves of thin-walled annular beam in dielectric-loaded waveguide with beam energy 660 keV, beam current 2.3 kA and B_0=0.8 T.

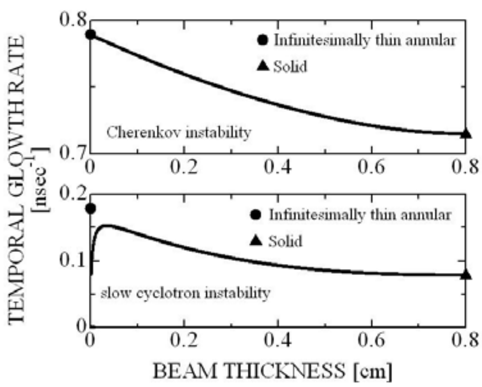

Fig. 11 The dependence of the temporal growth rate on beam thickness.

The growth rate of slow cyclotron instability also increases by decreasing the beam thickness. But, in the region of $\Delta_p < 0.035$ cm, the growth rate decreases.

In the limit that the beam inner radius is zero, the growth rate of Cherenkov and slow cyclotron instabilities of thin-walled annular beam approaches the growth rate of solid beam, ▲ in Fig.11. In the other limit that $\Delta_p \rightarrow 0$, the corresponding growth rates are those of an infinitesimally thin annular beam model with Δ_p=0 [2] and are depicted by ● in Fig.11. Two models based on finite and zero Δ_p give almost the same results for the Cherenkov instability. For slow cyclotron instability, the growth rates are different between two models. This might be caused by the difference of annulus structure: one has an internal structure between the inner and outer surface and the other is just a sheet without any internal structure.

5. Conclusion

We develop a numerical code for a cylindrical waveguide with a thin-walled annular beam considering three-dimensional beam perturbations, which is valid for any beam velocity v. Electromagnetic modes are the hybrid mode of TM and TE mode. Cyclotron and space charge modes corresponding to the solid case exist. Note that, for space charge modes, surface wave modes exist in addition to the volume wave mode if $\Omega < \omega_p$. As characteristic modes of annular case, the high-frequency space charge modes appear due to the inner surface. A thin-walled annular beam drives slow cyclotron as well as Cherenkov instability. The Cherenkov instability increases by decreasing Δ_p to zero. The slow cyclotron instability increases by decreasing Δ_p, reaches the maximum at a certain value of Δ_p and decreases in the limit of $\Delta_p \rightarrow 0$.

Acknowledgments

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