Viscosity Estimation Utilizing Flow Velocity Field Measurements in a Rotating Magnetized Plasma

Shinji YOSHIMURA[1)] and Masayoshi Y. TANAKA[2)]

[1)] *National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292, Japan*

[2)]*Kyushu University, 6-1 Kasuga-koen, Kasuga 816-8580, Japan*

The importance of viscosity in determining plasma flow structures has been widely recognized. In laboratory plasmas, however, viscosity measurements have been seldom performed so far. In this paper we present and discuss an estimation method of effective plasma kinematic viscosity utilizing flow velocity field measurements. Imposing steady and axisymmetric conditions, we derive the expression for radial flow velocity from the azimuthal component of the ion fluid equation. The expression contains kinematic viscosity, vorticity of azimuthal rotation and its derivative, collision frequency, azimuthal flow velocity and ion cyclotron frequency. Therefore all quantities except the viscosity are given provided that the flow field can be measured. We applied this method to a rotating magnetized argon plasma produced by the Hyper-I device. The flow velocity field measurements were carried out using a directional Langmuir probe installed in a tilting motor drive unit. The inward ion flow in radial direction, which is not driven in collisionless inviscid plasmas, was clearly observed. As a result, we found the anomalous viscosity, the value of which is two orders of magnitude larger than the classical one.

Keywords: viscosity, flow, vorticity, fluid equation, directional Langmuir probe, Hyper-I device

1. Introduction

In various fields of plasma research such as fusion plasmas, space and astrophysical plasmas and laboratory plasmas, the importance of plasma flow has been widely recognized and many studies on plasma flow related phenomena have been performed extensively. In some cases, viscosity plays a crucial role in determining plasma flow structures. In torus plasmas, for example, poloidal plasma flow driven by the radial electric field produces toroidal plasma flow through the effect of viscosity. Anomalous viscosity has been found experimentally both in helical devices and tokamaks [1]. In addition, strong viscosity, the value of which is up to 10^8 times larger than predicted by classical collision theory, has been observed in magnetized pure-electron plasmas [2]. In spite of its importance, little attention has been paid to the viscosity of laboratory plasmas so far.

In a laboratory plasma, however, spontaneous formation of a stationary vortex with a density hole around the central axis, which is referred to as plasma hole [3-5], has been observed, where the viscosity has an essential role. The flow velocity field associated with the plasma hole exhibits a monopole sink vortex, and the vorticity distribution is identified as Burgers vortex [6], which is intrinsic in viscous fluids. Scale length of the Burgers vortex is determined by the kinematic viscosity and the inward convergent flow. The viscosity estimated from the size of the vortex is found to be up to four orders of magnitude larger than the classical value. Therefore we can conclude that the anomalous viscosity does occur in laboratory plasmas and its measurement is necessary for the deeper understanding of flow structure formation.

In this paper, we present and discuss an experimental estimation method using flow velocity field measurements and apply it to a rotating magnetized argon plasma as an example.

2. Radial flow of ion fluid with finite viscosity

The equation of motion of ion fluid with finite viscosity in an external constant magnetic field $\mathbf{B_0} = (0, 0, B_0)$ is written as

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} + (\boldsymbol{\upsilon} \cdot \nabla)\boldsymbol{\upsilon} = \frac{e}{M}\left[-\nabla\phi + \boldsymbol{\upsilon} \times \mathbf{B}_0\right] - \frac{1}{Mn}\nabla p + \nu\nabla^2\boldsymbol{\upsilon} + \frac{1}{3}\nu\nabla(\nabla\cdot\boldsymbol{\upsilon}) - \nu_c\boldsymbol{\upsilon} \qquad (1)$$

, where $\boldsymbol{\upsilon} = (\upsilon_r, \upsilon_\theta, \upsilon_z)$ is the ion flow velocity, n the ion density, M the ion mass, e the elementary charge, ϕ the plasma potential, p the pressure, ν the kinematic viscosity, and ν_c the collision frequency. Here we neglect the second viscosity. If the fluid may be regarded as incompressible, the term which contains $\nabla\cdot\boldsymbol{\upsilon}$ is vanished, and the equation becomes considerably

author's e-mail: yoshimura.shinji@LHD.nifs.ac.jp

simpler. In cylindrical polar coordinates r, θ, z, the azimuthal component of the fluid equation is given as follows [7].

$$\frac{\partial \upsilon_\theta}{\partial t} + (\mathbf{\upsilon} \cdot \nabla)\upsilon_\theta + \frac{\upsilon_r \upsilon_\theta}{r} = \frac{e}{M}\left[-\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \upsilon_r B_0\right]$$
$$-\frac{1}{Mn}\frac{1}{r}\frac{\partial p}{\partial \theta} + \nu\left[\Delta \upsilon_\theta + \frac{2}{r^2}\frac{\partial \upsilon_r}{\partial \theta} - \frac{\upsilon_\theta}{r^2}\right] - \nu_c \upsilon_\theta. \quad (2)$$

Imposing the steady ($d/dt \to 0$) and axisymmetric ($d/d\theta \to 0,\ d/dz \to 0$) condition into eq. (2), we have

$$\upsilon_r\left[\frac{\partial \upsilon_\theta}{\partial r} + \frac{\upsilon_\theta}{r}\right] = -\upsilon_r \Omega_i + \nu\left[\frac{\partial^2 \upsilon_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial \upsilon_\theta}{\partial r} - \frac{\upsilon_\theta}{r^2}\right] - \nu_c \upsilon_\theta \quad (3)$$

, where Ω_i is the ion cyclotron frequency. Note that the terms in the square bracket on the left and on the right are equivalent to the vorticity $\omega = (\nabla \times \mathbf{\upsilon})_z = (1/r)(\partial/\partial r)(r\upsilon_\theta)$ and its derivative, respectively. Finally, we have considerably simple expression for radial flow velocity as

$$\upsilon_r = \frac{\nu(\partial\omega/\partial r) - \nu_c \upsilon_\theta}{\omega + \Omega_i}. \quad (4)$$

It is obvious from eq. (4) that the radial flow is not driven in collisionless inviscid plasmas, which have axial symmetry. In other words, the existence of radial flow indicates the nonnegligible effect of finite viscosity and/or collisions. Eq. (4) contains kinematic viscosity, vorticity of azimuthal rotation and its derivative, collision frequency, azimuthal flow velocity and ion cyclotron frequency. Since all quantities except the viscosity are given when the flow field can be measured, we can estimate the magnitude of viscosity from eq. (4).

3. Experimental setup

The experiments were performed in the high density plasma experiment (Hyper-I) device [8] at National Institute for Fusion Science. The Hyper-I is a cylindrical plasma device (30 cm in diameter and 200 cm in length) with ten magnetic coils (Fig. 1). Plasmas are produced by electron cyclotron resonance (ECR) heating with a microwave of frequency 2.45 GHz. The magnetic field configuration is a so-called magnetic beach structure (1.2 kG at z=10 cm (measured from the microwave injection window), 875 G (ECR point) at z=130 cm). In the present experiment, the microwave input power was 7 kW and the operation pressure of an argon gas was 1.3×10^{-4} Torr. A Langmuir probe was used to measure the electron temperature and the electron density, typical values of which are 13 eV and $1\times10^{11}\ \mathrm{cm}^{-3}$, respectively.

The ion flow velocity measurements were carried out with a directional Langmuir probe (DLP) [9]. The DLP collects a directed ion flux through a small opening made on the side of insulator body. The Mach number of the flow velocity component at a certain angle θ is given by the difference between upstream ($I(\theta)$) and downstream ion currents ($I(\theta+\pi)$) as follows:

$$\frac{\upsilon(\theta)}{C_s} = K\frac{I(\theta+\pi) - I(\theta)}{I(\theta+\pi) + I(\theta)} \quad (5)$$

, where K is the calibration constant of the order of unity. We set $K = 1.9$ in the present experiments, which was justified by comparing the flow velocity measured with the DLP to the $\mathbf{E}\times\mathbf{B}$ drift velocity determined from the potential measurement with an emissive probe.

The DLP was installed in a tilting motor drive unit, which can vary the insertion angle up to ± 23 degrees. The schematic of the DLP measurement system is shown in fig. 2. Scanning simply in a straight line passing through the cross-sectional center of the device twice with collecting azimuthally-opposed DLP currents, we can obtain the continuous $\upsilon_\theta(r)$ profile. On the other hand, several steps are needed to measure $\upsilon_r(r)$ profile. At first, the insertion angle of the DLP is varied by tilting mechanism of the motor drive unit. Then the DLP position is set to the tangent point of the circumference of a circle of which radius is r_0 and the center is identical to the device center. By measuring radially-opposed DLP current, we have one data point of $\upsilon_r(r = r_0)$. Repeating this operation with changing insertion angles, discrete data set of $\upsilon_r(r)$ can be obtained.

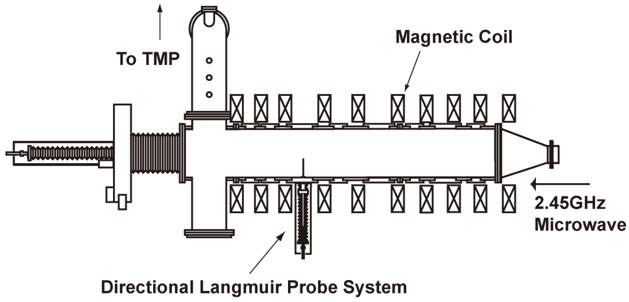

Fig. 1. Schematic of the Hyper-I device.

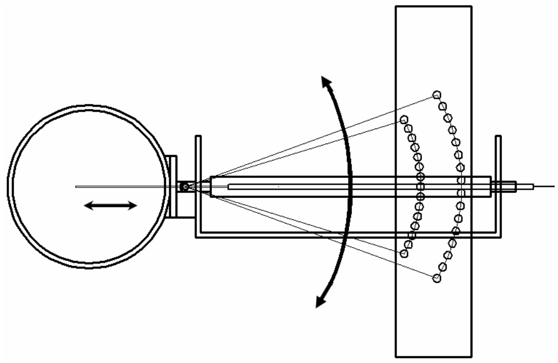

Fig. 2. Schematic of the DLP measurement system.

4. Results and Discussions

To demonstrate the validity of the viscosity estimation method proposed in Sec. 2, a plasma which exhibits a good axial symmetry is needed. Density and space potential profiles of the argon plasma we studied in the present experiments are shown in fig. 3(a) and (b), respectively. It is confirmed that both profiles, which are hollow in the centers, show good symmetry. The electron temperature is 13 eV, which is approximately constant over the experimental region.

The hollow potential profile shows that an inward electric field exists in this plasma, which drives counterclockwise $\mathbf{E}\times\mathbf{B}$ rotation. Figure 4 shows the azimuthal flow velocity profiles, where the open circles connected with solid lines indicate the velocities obtained by the DLP and the filled circles denote the $\mathbf{E}\times\mathbf{B}$ velocities calculated from the potential profile. Both velocities, which are normalized to the ion sound speed Cs, show quite a good agreement, where the calibration factor K in the eq. (5) is set to 1.9. The azimuthal flow profile shows an axial symmetry. Rigid like rotation is found around the central axis, and the maximum Mach number is $\upsilon_\theta / Cs \sim 0.4$.

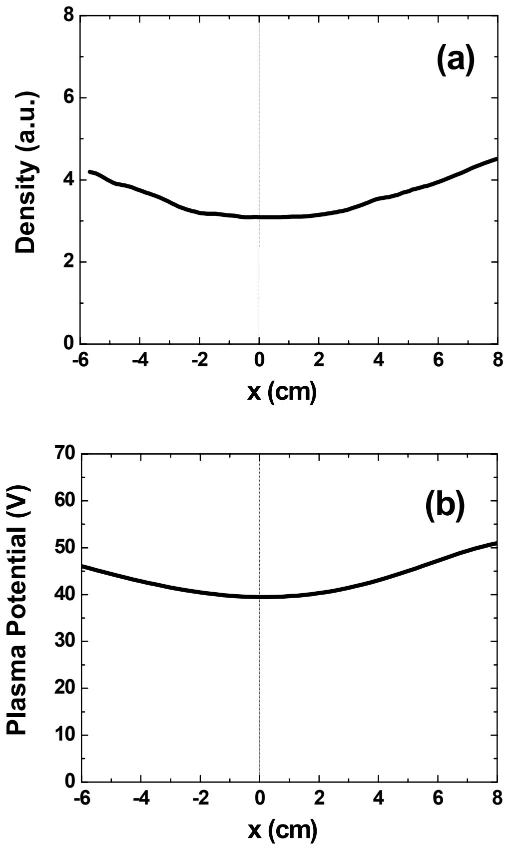

Fig. 3. (a) Density (ion saturation current) profile measured by Langmuir probe. (b) Plasma potential profile measured by emissive probe.

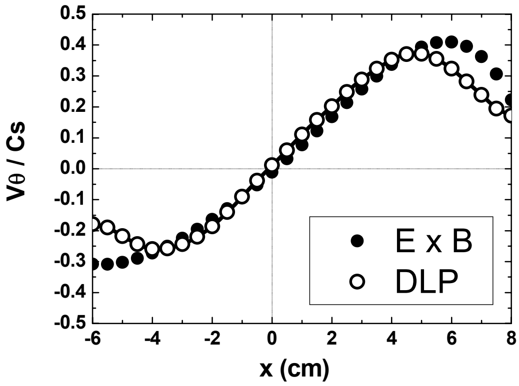

Fig. 4. Comparison between the azimuthal velocity measured with the DLP (open circle) and the E×B drift velocity determined from the potential profile (filled circle).

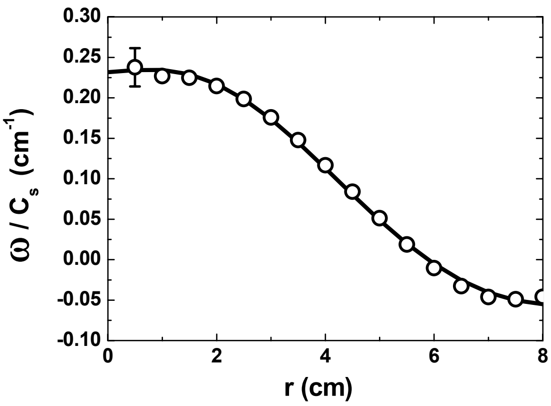

Fig. 5. Profile of the z-component of vorticity.

Once the azimuthal flow velocity profile is obtained, the vorticity profile can be easily calculated when the axisymmetric condition is satisfied. The vorticity profile is shown in fig. 5, where open circles, which denote the experimental data, are depicted with fitted curve (solid line). When we evaluate the radial flow velocity, the derivative of this fitted curve is substituted to eq. (4).

The ion cyclotron frequency is determined by the magnetic field strength (900 G) and the ion mass (40 u), so that we have $\Omega_i = 2.16\times10^5\ \mathrm{s}^{-1}$. Adopting the charge exchange with neutrals as the dominant collisional process, the collision frequency is given $\nu_c \sim 3\times10^3\ \mathrm{s}^{-1}$, where the cross section of the charge exchange collision for low energy argon is given by Sheldon [10]. Now we have all quantities except the kinematic viscosity on the right of eq. (4).

Figure 6 shows the comparison between the radial flow velocity measured with the DLP system (open circle) and the eq. (4) (solid curve, where anomalous kinematic viscosity $\nu = 2\times10^5\ \mathrm{cm^2/s}$ is assumed). In this figure, the inward flow, which implies the finite

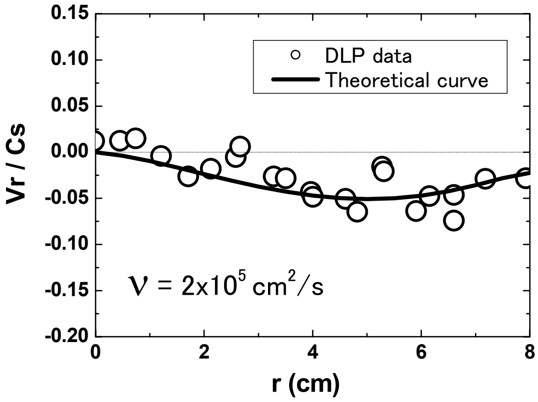

Fig. 6. Comparison between the radial flow velocity measured by the DLP (open circle) and theoretical curve given by eq. (4) with anomalous kinematic viscosity.

viscosity, is evident.

If the plasma may be regarded as inviscid, the radial flow velocity can be estimated as

$$\upsilon_r / C_s \sim (\nu_c / \omega + \Omega_i) \times (\upsilon_\theta / C_s) \sim 0.002.$$

This radial flow speed is considerably small and cannot explain the present experimental result. In contrast, the solid curve in fig. 6 well agrees with the DLP data and shows similar tendency. Note that the kinematic viscosity predicted by classical theory is $\nu_{cl} \sim 3\times10^3$ cm^2/s. The value of the viscosity we used in fitting curve is two orders of magnitude larger than the classical one, and therefore we can conclude that the anomalous viscosity was found in a laboratory magnetized plasma.

To make the presented estimation method more powerful, an accurate absolute velocity measurement system is required. Since there is an ambiguity in determining absolute velocity using the DLP or Mach probes, we are now preparing laser induced fluorescence (LIF) Doppler measurements for velocity field determination [11]. Detailed study of viscosity effects on flow structure formation utilizing LIF measurements will form our future work.

Acknowledgments

This work was partially supported by Japan Society for the Promotion of Science, Grant-in-Aid for Young Scientists (B) (No. 19740348) and by the National Institute for Fusion Science, a budgetary Grant-in-Aid NIFS07 ULPP103.

References

[1] K. Ida, Plasma Phys. Control. Fusion **40**, 1429 (1998).
[2] J. M. Kriesel and C. F. Driscoll, Phys. Rev. Lett. **87**, 135003 (2001).
[3] K. Nagaoka *et al*., Phys. Rev. Lett. **89**, 075001 (2002).
[4] S. Yoshimura *et al*., J. Plasma Fusion Res. Ser. **6**, 610 (2004).
[5] M. Y. Tanaka *et al*., IEEE Trans. Plasma Sci. **33**, 454 (2005).
[6] J. M. Burgers, Adv. Appl. Mech. **1**, 171 (1948), Sec. XV.
[7] The equation for ordinary viscous fluid (without collision term) is described in detail; L. D. Landau and E. M. Lifshitz, *Fluid Mechanics 2nd edition*, (Butterworth-Heinemann, Oxford, 1987), Chap. II.
[8] M. Y. Tanaka *et al*., Rev. Sci. Instrum. **69**, 980 (1998).
[9] K. Nagaoka *et al*., J. Phys. Soc. Jpn. **70**, 131 (2001).
[10] J. W. Sheldon, Phys. Rev. Lett. **8**, 64 (1962).
[11] A. Okamoto *et al*., J. Plasma Fusion Res. 80, 1003 (2004).