Effects of Coulomb Collisions on the Toroidal Spin-up of a Field-Reversed Configuration

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A direct conversion process from the magnetic flux to the kinetic angular momentum of plasma ions is a possible mechanism of a toroidal spin-up of a field-reversed configuration (FRC) [T. Takahashi *et al.*, Plasma Fusion Res. **2**, 008 (2007)]. For quantitative discussion, the toroidal rotation velocity of a FRC plasma which is resistively decaying is calculated numerically. The ion-ion pitch-angle scattering is reproduced by a Monte Carlo method. The obtained toroidal rotation velocity at the field-null is found to increase linearly up to $0.3v_{\mathrm{A0}}$, where v_{A0} is the Alfvén velocity defined by the external magnetic field and the ion density at the field-null just after the formation. The result is reasonable in a view of a time evolution of the toroidal flow velocity by the Doppler shift measurement at the Nihon University compact torus experiment-III device. It is found that a classical particle loss has an insignificant effect on the spin-up of a FRC.

Keywords: field-reversed configuration, toroidal spin-up, n=2 rotational instability, pitch-angle scattering, end loss, classical diffusion

1. Inroduction

The rotational instability with the toroidal mode number n=2 of a field-reversed configuration (FRC) plasma is a global instability that is most often observed experimentally [1-5]. The FRC current just after formation is primarily carried by electrons, while ions are approximately at rest [6]. The ions, however, gradually gain angular momentum in the ion diamagnetic direction before the onset of the n=2 rotational instability. Rotation of the FRC plasma has been often explained by selective loss of ions [7-9], end-shorting [10-13], or both [14]. Recent computer simulation shows that a resistive decay of the internal flux cause selective loss of ions and resultant spin-up of the FRC plasma [15, 16]. The work is valuable in that contribution of the flux decay to the spin-up of a FRC is firstly proposed. However, is a selective loss of ions necessary to explain toroidal spin-up of a FRC plasma?

Recently, we have proposed an another possible spin-up mechanism, which is direct conversion from the magnetic flux to the kinetic angular momentum [17]. Comparison between experiment and calculation with a simple model will show validity of our explanation.

Suppose a FRC plasma is axisymmetric. This assumption is valid until the rotational instability is triggered. In a collisionless plasma, the canonical angular momentum $P_\theta = mv_\theta r + q\psi(r,z)$ of every particle is conserved, where m, q are the mass and charge, respectively, v_θ is the toroidal velocity component, and $\psi(r,z)$ is the poloidal flux function. If the poloidal flux decays due to resistivity and toroidal axisymmetry still holds, then

$$m\Delta(v_\theta r) = -q\Delta\psi \ . \qquad (1)$$

Equation (1) shows that when the poloidal flux decays, every ion gains angular momentum in the ion diamagnetic direction. Generally, the separatrix radius decreases during the decay phase. If the guiding center r is also decreased, the toroidal velocity v_θ is further increased.

Another explanation for FRC plasma rotation can be given from a viewpoint of particle trajectories; it is not essentially different from discussion above. In a FRC plasma, small-gyroradius drift orbits, figure-8 orbits, and betatron orbits are three possible types of trajectories [18, 19]. In contrast to the betatron particles, the small-gyroradius drift particles and the figure-8 particles have smaller angular momentum. If the poloidal flux decays, the figure-8 particles can change abruptly to the betatron particles due to the increase of the Larmor radius. The transition of trajectory type results in the increment of the toroidal angular momentum.

The inductive electric field is also a possible cause of toroidal spin-up. The betatron particles move around the field-null. The toroidal electric field always accelerates the betatron ions in the ion diamagnetic direction. Here, the radial $\mathbf{E}\times\mathbf{B}$ drift motion contributes to the betatron oscillation, and therefore the guiding center is fixed at the

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equilibrium position at which the centrifugal force of the betatron motion and the Lorentz force are balanced.

Coulomb collisions of ions, however, break conservation of P_θ. The conversion process given above may not work in this case. In the present paper, effects of the ion-ion pitch-angle scattering on the spin-up of a FRC plasma are investigated. We assume that the plasma ions are fully thermalized, and therefore the slowing-down collisions are neglected here.

2. Calculation Model

To investigate the time evolution of toroidal flow velocity of a FRC plasma, plasma ions are traced numerically. The fields are resistively decaying, and they are modeled as the poloidal flux decay given by

$$\frac{\partial \psi}{\partial t} = -r\eta J_\theta . \qquad (2)$$

Here, the electric resistivity η equals $f_\mathrm{A}\eta_\mathrm{cl}$, where f_A is the anomaly factor and η_cl is the classical resistivity. The flux lifetime is controlled by the parameter f_A. By integrating Eq. (2) by means of the Runge-Kutta method, the flux function ψ at a calculation point is found. The electromagnetic fields are then written by the obtained ψ as

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times \left(\frac{\psi}{r}\mathbf{e}_\theta \right),$$

$$\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B}, \quad \mathbf{E} = \eta \mathbf{J} . \qquad (3)$$

For a detailed calculation, we often use $\mathbf{E} = \eta\mathbf{J} - \mathbf{u}_\mathrm{e} \times \mathbf{B} - \nabla p_\mathrm{e} / (en_\mathrm{e})$. When ions are at rest and $\mathbf{J} \times \mathbf{B} = \nabla p_\mathrm{e} + \nabla p_\mathrm{i}$ is satisfied, then $\mathbf{E} \approx \eta\mathbf{J} + \nabla p_\mathrm{i} / (en_\mathrm{e})$. The azimuthal electric drift due to the ion pressure gradient is in paramagnetic direction. Therefore, it is impossible to explain ion spin-up by the electric drift due to the ion pressure gradient. The electric field due to both the Lorentz force acting on electrons and the electron pressure gradient is neglected in calculating orbits of ions. Initially, ions are loaded in the r-z plane uniformly. The ion as a super-particle is weighted by the non-shifted Maxwellian distribution, and therefore the ions are initially at rest. The ion temperature is set to be uniform inside the separatrix, and it decreases with the flux function exponentially in open field region. The initial electron density is calculated to satisfy the Grad-Shafranov equation. The equation of motion is solved for ions in $\mathbf{B}$ and $\mathbf{E}$ fields given by Eq. (3), and the ion density and toroidal flow velocity are obtained by a particle in cell method at each calculation time step.

The ion-ion pitch-angle scattering is reproduced by a Monte Carlo method [20]. The pitch-angle is calculated as

$$\lambda_\mathrm{n} = \lambda_\mathrm{o}(1 - \nu_\mathrm{d}\tau) \pm \left[(1 - \lambda_\mathrm{o}^2)\nu_\mathrm{d}\tau \right]^{1/2},$$

$$\lambda = \frac{v_{//}}{v} = \cos\theta_p , \quad -1 \le \cos\theta_p \le 1 . \qquad (4)$$

Here, ν_d is the deflection collision frequency and τ is the time interval of random number generation. The subscripts n and o mean new and old variables. We choose either sign equally by the uniform random number at the sign ±.

Calculation results are compared with the experiment data measured on the NUCTE (Nihon University Compact Torus Experiment)-III device. Typical experiment parameters of the NUCTE-III device are the external magnetic field B_ex=0.4 T, the coil radius $r_\mathrm{c} = 0.17$ m, and the typical plasma parameters are the separatrix radius $r_\mathrm{s} = 0.05$ m, the separatrix length $\ell_\mathrm{s} = 0.40$ m, the ion temperature $T_\mathrm{i} \approx 100$ eV, the electron temperature $T_\mathrm{e} \approx 100$ eV and the density at the field-null $n_0 = 2.0 \times 10^{21}$ m^{-3}. A measured flux Φ at the field-null is shown in Fig. 1, where the flux Φ is equal to $2\pi\psi$. Gradual decrease of the flux can be seen after completion of FRC formation at about 8 μs. Decrement of the flux reaches 0.32 mWb by 50 μs, which corresponds to $35t_\mathrm{A0}$ after formation, where $t_\mathrm{A0} \equiv r_\mathrm{c} / v_\mathrm{A0}$ and $v_\mathrm{A0} \equiv B_\mathrm{ex} / \sqrt{\mu_0 m_\mathrm{i} n_0}$. When f_A is 10, decrement of the calculated flux for $35t_\mathrm{A0}$ is estimated by $\Delta\Phi = 2\pi(\Delta\psi) \approx 2\pi(0.01|\psi_\mathrm{w}|) \approx 0.31\,\mathrm{mWb}$; it is comparable to the NUCTE-III experiment. Therefore, orbits of ions are calculated at f_A of 10, and then the resultant toroidal flow velocity is compared with the NUCTE-III experiment.

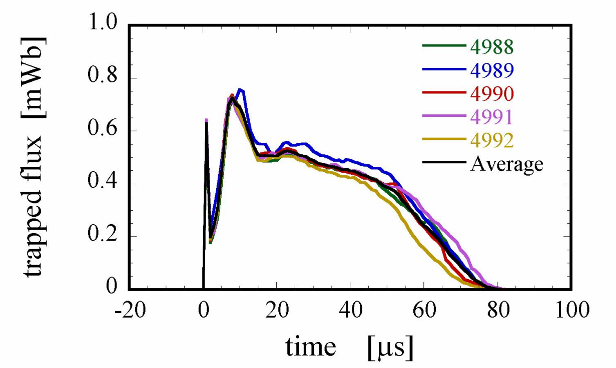

Fig. 1 The time evolution of the maximum poloidal flux measured at the NUCTE-III device.

3. Results and Discussion

To study an effect of flux decay on toroidal spin-up of a FRC plasma, orbits of ions are calculated for the presence of and absence of flux decay. The calculated time evolution of the toroidal flow velocity normalized by v_A0 is shown in Fig. 2, where flux decay is present for Fig. 2(a) and is absent for Fig. 2(b). Here, the flow velocities at the separatrix (the solid line) and at the field-null o-point (the dotted line) are presented separately. Gradual increases are found at both the separatrix and o-point in Fig. 2(a), whereas no increase can be seen in Fig. 2(b). By the time of $35t_{\mathrm{A0}}$, the FRC plasma is found to rotate with the toroidal velocity up to $0.35v_{\mathrm{A0}}$ at the separatrix and up to $0.25v_{\mathrm{A0}}$ at the o-point.

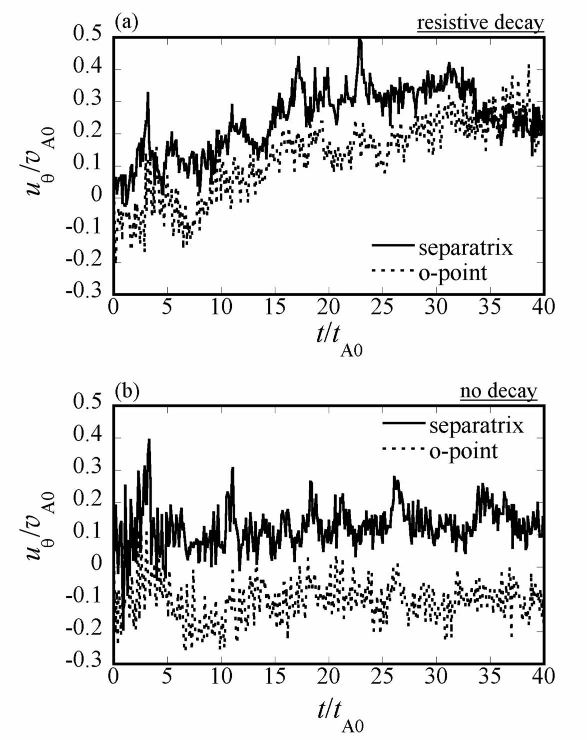

Fig. 2 The toroidal flow velocity evolution at the separatrix (the solid line) and at the field-null o-point (the dotted line). (a) The poloidal flux decays due to resistivity. (b) The flux decay is absent.

For comparison, the toroidal velocity measured by the Doppler shift of the impurity line $C^{4+}$is shown in Fig. 3. The measured velocity can be read as the average along the line of sight, whose distance of closest approach from the geometric axis is 4.5 cm; it locates initially between the separatrix and o-point. As with Fig. 2(a), an almost linear increase is found by the experimental result. Therefore, a qualitative agreement can be observed between experimental and computational results of the toroidal rotation velocity. Because $v_{\mathrm{A0}} = 138\ \mathrm{km/s}$, the velocity of 20 km/s corresponds to about $0.15v_{\mathrm{A0}}$ .A toroidal flow can be observed even for the absence of flux decay as shown in Fig. 2(b), although ions are assumed to obey the non-shifted Maxwellian distribution. The ion diamagnetic drift and absence of the radial and axial electric field may play a role of the rotation velocity in Fig. 2(b).

Comparison of the time evolution of toroidal flow velocity at the separatrix between the presence of and absence of end-loss ions are made as shown in Fig. 4. The axial direction of motion is reversed at the axial end of the calculation region for the case w/o end loss. Therefore, in this case no ions suffer from the end loss. Calculation of orbit stops for the case with end loss, when ions pass through the axial end. It is found that there is little difference between two cases, because near the separatrix few thermal ions are lost by $35t_{\mathrm{A0}}$ even in the decaying plasma. Therefore, it appears the effect of the ion loss on spin-up of FRC plasma is negligible.

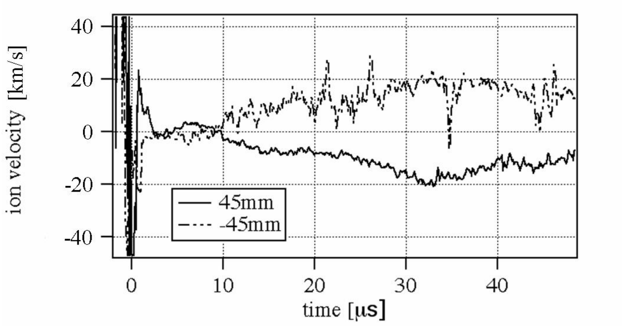

Fig. 3 The time evolution of toroidal velocity measured by the Doppler shift of a spectral line of impurity carbon at NUCTE-III device in Nihon University.

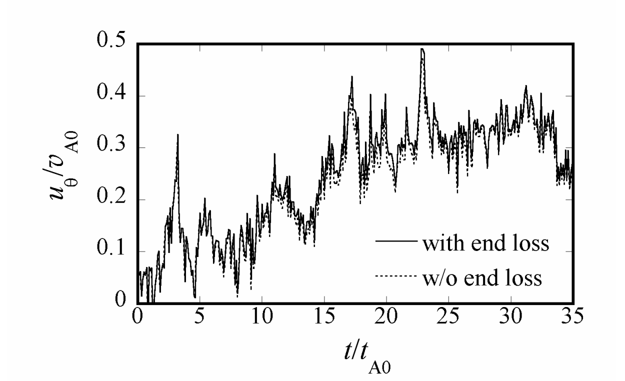

Fig. 4 Comparison of the time evolution of toroidal flow velocity at the separatrix between the presence of and absence of end-loss ions. The axial direction of motion is reversed at the axial end of the calculation region for the case w/o end loss (the dotted line). Calculation of orbit stops for the case with end loss (the solid line), when ions pass through the axial end.

When the ion-ion pitch-angle scattering is considered, the particle loss fraction gradually increases with time due to the classical transport process. The particle loss may enhance the spin-up of a FRC plasma, we compare the rotation velocity between the collisional case and the collisionless case. The time evolution of the particle loss fraction that is the ratio of the number of end-loss ions to the number of all ions inside the confinement region is shown in Fig. 5. Although the flux decays, it is found that ions are lost only before $20t_{\mathrm{A0}}$ for the collisionless case. About 10 % difference between the collisionless and collisional case is observed until $35t_{\mathrm{A0}}$, and it may cause a significant difference in the toroidal rotation velocity.

Comparison of the toroidal velocity between the collisionless and collisional case is shown in Fig. 6.

Higher fluctuation level is found for the collisionless case, and reduction of its level for the collisional case is due to the viscosity. Except time fluctuation, however, insignificant difference is observed. Therefore, the particle loss by the classical transport process is not be responsible for the spin-up of a FRC plasma.

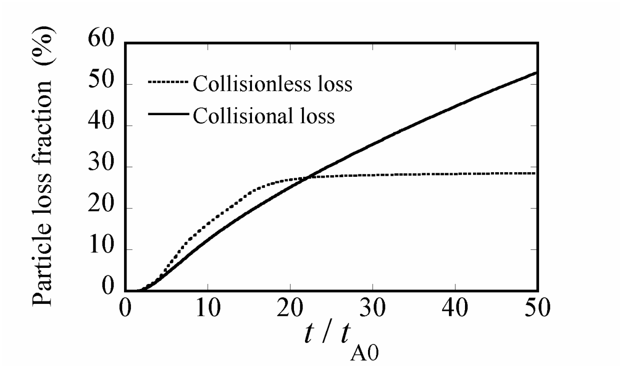

Fig. 5 The time evolution of the particle loss fraction. The ion-ion pitch angle scattering is taken into account for the solid line and is not taken into account for the dotted line.

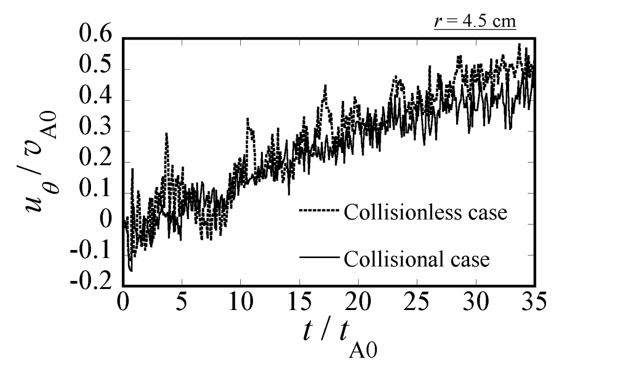

Fig. 6 The time evolution of the toroidal rotation velocity at the radial position of 4.5 cm. The solid and dotted lines are drawn as Fig. 5.

4. Summary

It has been shown that a FRC plasma can spin up without any loss of ions because of the direct conversion process from the poloidal flux to the kinetic angular momentum of ions. Abrupt change of the type of ion trajectory, such as the betatron, figure-8 and small-gyroradius drift orbits, is possible cause of spin-up of a FRC plasma. The rotation velocity is calculated, and the results are found to be in qualitative agreement with the experiment at the NUCTE-III device.

In this paper, Coulomb collisions that break conservation of the canonical angular momentum have also been considered. If collisionality increases, the loss of flux is not always converted to the angular momentum. Insignificant difference of the toroidal velocity, however, is found between the collisional and collisionless case. Therefore, the conversion process still plays an important role of the spin-up of a FRC plasma. The particle loss due to classical transport has been found to have a small effect on the spin-up process. Though the toroidal flow affects the electromagnetic fields and the ion motion, this effect is neglected in the present letter and is left as a subject for future study.

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