

# Study of neoclassical transport of LHD plasmas applying the DCOM/NNW neoclassical transport database

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## Abstract

A neoclassical transport database, DCOM/NNW, is developed including the configuration changes due to finite beta effect in LHD. The mono-energetic diffusion coefficients are evaluated based on the Monte Carlo method by DCOM code and the mono-energetic diffusion coefficient database is constructed using a neural network technique. Input parameters for the database are the collision frequency, the radial electric field, the minor radius and the plasma beta. The DCOM/NNW database is applied to finite beta plasma, and the neoclassical transport coefficients and the ambipolar radial electric field are evaluated in LHD.

## Keywords:

LHD, neoclassical transport, neural network, Monte Carlo method

## 1. Introduction

In helical systems, neoclassical transport is one of the important issues for sustaining high-temperature plasma. In particular, in the long-mean-free-path (LMFP) regime, the neoclassical transport coefficient increases as collision frequency decreases ( $1/\nu$  regime), and neoclassical transport plays an important role as well as anomalous transport by plasma turbulence.

Many studies have been carried out to evaluate the neoclassical transport coefficient analytically and numerically in helical systems. Among the studies, the Drift Kinetic Equation Solver (DKES)<sup>[1, 2]</sup> code has been commonly used for experimental data analyses<sup>[3, 4]</sup> and theoretical predictions.<sup>[5]</sup> However, in the LMFP regime, especially with finite beta, a large number of Fourier modes of the magnetic field must be used for distribution function and a convergence problem occurs.

On the other hand, the neoclassical transport coefficient has also been evaluated using the Monte Carlo method directly following particle orbits, where the mono-energetic diffusion coefficients are estimated by the radial diffusion of test particles.<sup>[6-8]</sup> This method has a good property in the LMFP regime except for its long calculation time. Thus, we have developed a Monte Carlo simulation code, the Diffusion Coefficient Calculator by Monte Carlo Method (DCOM) code,<sup>[9]</sup> which is optimized in

performance in the vector computer.

To evaluate the neoclassical diffusion coefficient of thermal plasma, we must take energy convolution into account. Therefore, it is necessary to interpolate discrete data using the DCOM. In a non-axisymmetric system, the diffusion coefficient shows complex behavior and strongly depends on collision frequency and radial electric field (e.g.,  $1/\nu$ ,  $\sqrt{\nu}$  and  $\nu$  regimes). The interpolation based on a traditional analytical theory has a problem with connected regions between two regimes.

The neural network (NNW)<sup>[10]</sup>, which has a strong nonlinearity and high fitting abilities for nonlinear phenomena, is applied in the study of fusion plasma and used in constructing the NNW database for neoclassical transport in TJ-II plasma.<sup>[11]</sup> Therefore, the NNW method is applied to the fitting of the diffusion coefficient of LHD, which shows a complex behavior in several collisional regimes, i.e.,  $\nu$ ,  $\sqrt{\nu}$ ,  $1/\nu$ , plateau, and P-S regimes. A multilayer perceptron NNW with only one hidden layer, generally known as MLP1, is used. The neoclassical transport database, DCOM/NNW<sup>[12]</sup>, has been constructed with input parameters  $r/a$ ,  $\nu^*$ , and  $G$  and  $D^*$  can be obtained as an output of the NNW, where  $\nu^*$  is the normalized collision frequency,  $G$  is the normalized radial electric field and  $D^*$  is the normalized diffusion coefficient respectively. We have constructed six database each of six LHD configurations with different values of the

magnetic axis shift in the major radius direction between  $R_{\text{axis}} = 3.45$  m and  $R_{\text{axis}} = 3.90$  m.

At construction of this database, we assumed only vacuum plasma and have not taken into account finite beta plasma. However, a relatively large Shafranov shift occurs in finite beta LHD plasmas and the magnetic field configuration becomes complex leading to rapid increase of the number of Fourier modes in Boozer coordinates.

In order to analyze transport of experimental plasma with finite beta, we have to take account into finite beta. Therefore, we apply effect of finite beta to the neoclassical database, DCOM/NNW. We evaluate the neoclassical transport coefficients and the ambipolar radial electric field using improved DCOM/NNW..

### 2. Construction of neoclassical transport database including finite beta effect

The Fourier spectrum of the magnetic field  $B_{m,n}$  in the Boozer coordinate as functions of normalized minor radius  $r/a$  in the  $R_{\text{axis}} = 3.75$  m configuration with  $\beta_0 = 0\%$  and  $\beta_0 = 3\%$  is shown in Fig.1. Here,  $B_{0,0}$  is the amplitude of the (0, 0) component of magnetic field strength at the magnetic axis, where (m, n) represents the Fourier component of magnetic field with the poloidal number, m, and the toroidal number, n. It is shown that the dominant magnetic field components in LHD with  $\beta_0=0\%$  plasma are (2,10) and (1,0); (2,10) corresponds to the main helical mode and (1,0) is the toroidicity. Figure 1(b) shows that the amplitude of (2,10) and (1,0) component decrease and a lot of other components increase by the effect of beta value, especially in the edge plasma, and the magnetic field configuration becomes complex.

In this study, we calculate a mono-energetic diffusion coefficient,  $D$ , by DCOM code. Using DCOM code, we can calculate the diffusion coefficient without convergence problem even if we assume a lot of Fourier modes of the magnetic field. We use 50 Fourier modes to evaluate the magnetic field in this research. To obtain mono-energetic diffusion coefficients, we assume the electron energy of  $1.0 \times 10^{-3}$  eV. The magnetic field is set to 3T at the magnetic axis. The test particles are monitored for several collision times until the diffusion coefficient is converged. Figure 2 shows the normalized diffusion coefficient  $D^*$  at  $r/a = 0.5$  in  $R_{\text{axis}} = 3.75$ m calculated using the DCOM code as a function of the normalized collision frequency  $\nu^*$

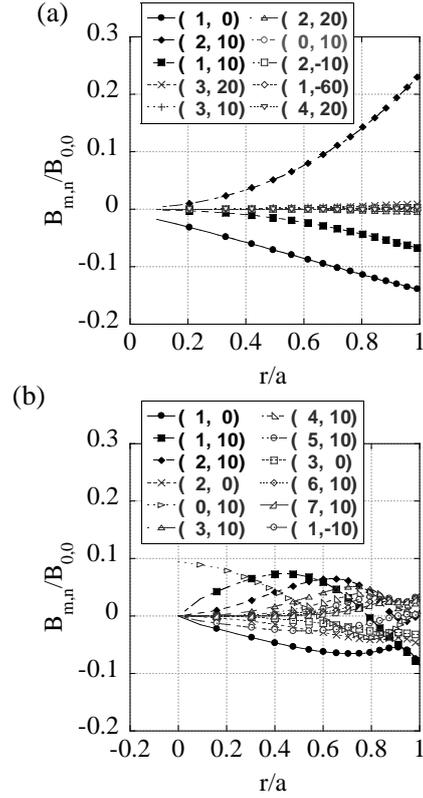


Fig. 1. Amplitude of dominant  $B_{m,n}$  component normalized by  $B_{0,0}$  ( $r/a$ ) as function of normalized minor radius  $r/a$  with (a)  $\beta_0=0\%$  and (b)  $\beta_0=3\%$ .

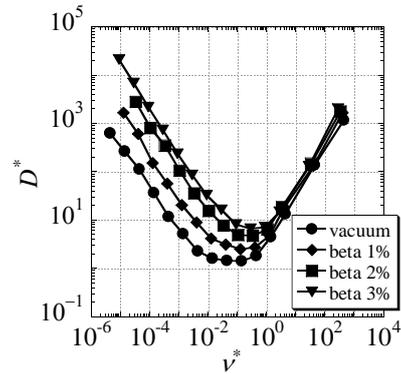


Fig. 2. Normalized monoenergetic diffusion coefficients as function of normalized collision frequency without radial electric field at  $r/a = 0.5$  in  $R_{\text{axis}} = 3.75$ m.

without radial electric field. Here, we normalize the collision frequency by  $\nu/R$ , and the diffusion coefficient by the tokamak plateau value in the mono-energetic case,  $D_p$ , as  $D_p = (\pi/16)(\nu^3/\iota R_{\text{axis}} \omega_c^2)$ , where  $\nu$ ,  $\iota$  and  $\omega_c$  are

the minor radius, the velocity of test particles, the rotational transform and cyclotron frequency of test particle, respectively. It is found that the normalized diffusion coefficients rise rapidly as the central plasma beta values increases from 0% to 3%.

In Fig. 3, the diffusion coefficient at  $r/a = 0.5$  are shown as function of  $\beta_0$  in each regime. We can see that the diffusion coefficients are increased exponentially with  $\beta_0$  in  $1/\nu$  regime. It is found that the plateau values are also increased by the beta value. On the other hand, in the P-S regime, the diffusion coefficients are almost independent with the value of  $\beta_0$ . Thus, it is necessary to take into account the effect of the beta value in order to accurately evaluate transport in high temperature plasma, in which the collision frequency is in the  $1/\nu$  or the plateau regime.

We add the beta values to the inputs of DCOM/NNW which is the neoclassical transport database. We consider two LHD configurations,  $R_{axis} = 3.60m$  and  $R_{axis} = 3.75m$ . We adjust the weights of NNW using the computational results of DCOM, which are called training data. As beta value, we calculate the diffusion coefficient with  $\beta_0 = 0\%$ ,  $\beta_0 = 1\%$ ,  $\beta_0 = 2\%$  and  $\beta_0 = 3\%$  for training data. In this research, we totally used 2688 training data for  $R_{axis} = 3.75m$  configuration using the DCOM with various collision frequencies, radial electric fields, minor radius and beta values. Also, 1777 training data are used for  $R_{axis} = 3.60m$  configuration.

The accuracy of the NNW depends on the number of hidden units. The error between training data and outputs of NNW decrease as the number of hidden unit increases. However, if we increase the number of hidden unit too much, overlearning occurs, which gives poor prediction for a new parameter data although the error is very small.

In Fig.4, it is shown that the sum of relative error as function of the number of hidden unit of NNW. In this research, we assume the number of hidden unit is set to 15 and mean relative error is about 12.3 %.

We can obtain  $D^*$  by using the improved NNW database for arbitrary  $\nu^*$ ,  $G$ ,  $r/a$  and newly,  $\beta_0$ . Figure 5 shows contour plot of  $D^*$ , obtained by the newly NNW, at  $r/a = 0.5$ ,  $G = 0.0$  in  $R_{axis} = 3.75m$ . The horizontal axis indicate  $\nu^*$  and the vertical axis is  $\beta_0$ . We can found that  $D^*$  dose not depend on the beta value in P-S regime. And  $D^*$  is increased in  $1/\nu$  and plateau regime as  $\beta_0$  increases. It is also seen that the plateau regime has narrowed

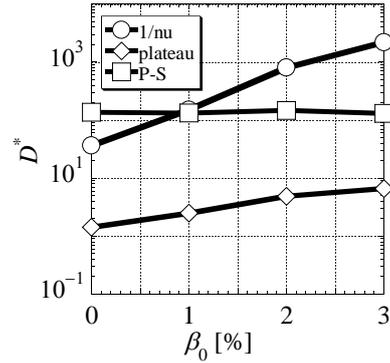


Fig. 3. Normalized diffusion coefficient as function of beta value,  $\beta_0$ , without radial electric field at  $r/a = 0.5$  in  $R_{axis} = 3.75m$ .

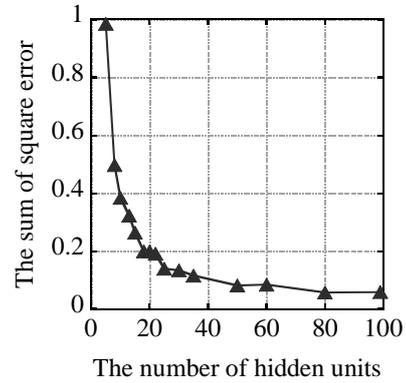


Fig. 4. The sum of relative error as function of number of hidden units of NNW.

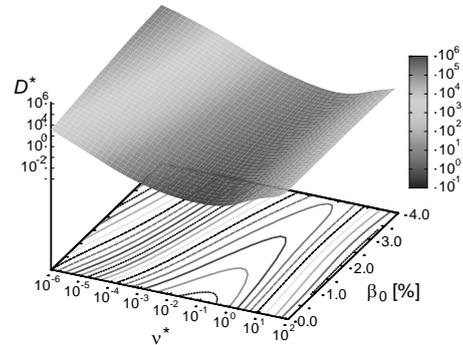


Fig. 5. The normalized monoenergetic diffusion coefficients,  $D^*$  as a function  $D^*(\nu^*, G, r/a, \beta_0)$  in  $R_{axis} = 3.75m$ , where  $G$  and  $r/a$  are constant ( $G=0.0$  and  $r/a = 0.5$ ), which are outputs of DCOM/NNW.

as  $\beta_0$  increases.

### 3. Neoclassical transport analysis

Next, we study the neoclassical transport by DCOM/NNW including the finite beta effect. In this analysis, we consider hydrogen plasma and  $R_{axis} = 3.75\text{m}$ . The assumed temperature and density profiles of electrons and ions are shown in Fig. 6, and the magnetic field is  $B=1.5\text{T}$  (#48584,  $t = 1.48\text{sec}$ ).

Figure 7(a) shows the ambipolar radial electric field as a function of  $r/a$ . There is only one root (ion root) with  $\beta_0=0.0\%$  and  $\beta_0=0.72\%$ . We obtain similar profiles of the electric fields in both cases. However, it is found that the diffusion coefficient,  $D_1$ , with  $\beta_0=0.0\%$  and with  $0.72\%$  are greatly different as shown in Fig. 7(b). In the case of  $\beta_0=0.72\%$ ,  $D_1$  increases to about three times that of  $\beta_0 = 0.0\%$ . The heat conductivity,  $D_3$ , also increases with an increase of  $\beta_0$ . In the case of  $\beta_0=0.72\%$ ,  $D_3$  increases to about four times that of  $\beta_0 = 0.0\%$ .

This results show that the inclusion of finite beta effect is necessary for the accurate evaluation of neoclassical transport.

### 3. Summary

We have extended the neoclassical transport database, DCOM/NNW, to include the effect of configuration changes due to the finite beta effect. In the finite beta plasma, the magnetic field becomes complex and the number of necessary Fourier mode increases. Using extended DCOM/NNW we can estimate neoclassical transport more accurately than that using the previous DCOM/NNW.

We have investigated the neoclassical transport and evaluate the ambipolar radial electric field in LHD using DCOM/NNW.

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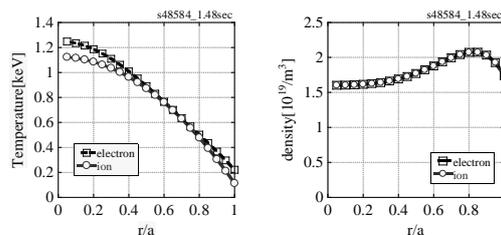


Fig. 6. Plasma temperature and particle density (#48584,  $t = 1.48\text{sec}$ ,  $\beta_0 = 0.72\%$ ).

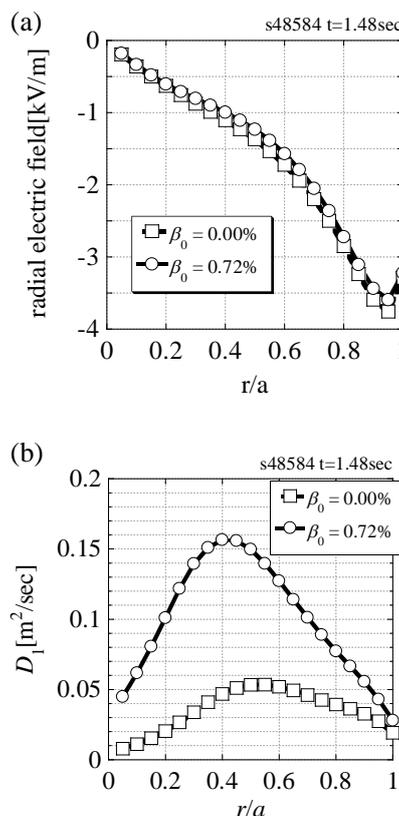


Fig. 7. (a) Radial electric field and (b) Diffusion coefficient as function of  $r/a$  with  $\beta_0=0.0\%$  and  $0.72\%$ .

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