Analysis of Density Fluctuation Data Measured by Microwave Imaging Reflectometry

Z. B. Shi[1)], Y. Nagayama[2)], S. Yamaguchi[2)], Y. Hamada[2)]
[1)] *The Graduate University for Advanced Studies, Toki 509-5292, Japan*
[2)] *National Institute for Fusion Science, Toki 509-5292, Japan*

(Received 13 July, 2007 / Accepted 3 August, 2007)

The density fluctuation in the Large Helical Device (LHD) plasma has been investigated by using the microwave imaging reflectometry (MIR) system. The statistical properties of the fluctuation spectral on MIR signals are quantified by using the time-frequency analysis with ensemble averaging technique. Statistical analyses by using cross correlation spectrum and coherence spectrum reveals characteristics of MHD modes, such as mode numbers, during high power NBI heating.

Keywords: density fluctuation, Microwave Imaging Reflectometry, ensemble average

1. Introduction

Turbulence and instabilities has been considered to correlate with the properties of confinement, so that the spatial structure of the fluctuation continues to be the basic issue in the fluctuation study. The microwave imaging Reflectometry (MIR) has been applied to LHD [1]. The MIR is expected to be one of the most powerful diagnostics to investigate instabilities in plasmas, since it enables 2D/3D local measurement of density fluctuations [1-4]. This technology is based upon the reflection of microwave at the density-dependent cutoff layer, and the fluctuating phase of the reflected wave is dominated by the density fluctuation close to the cutoff layer. In fact, the reflecting signal has rich physical phenomena, which include the plasma turbulence and MHD instabilities.

Fluctuation signals often submerge in the strong background noises such as electronic noise and thermal noise, especially when the reflection surface is in the core plasma region. After the onset of the turbulence the spectrum becomes broad, and the intermittently burst of the large-scale turbulence eddy may cause the distortion of the spectrum. Therefore, it is hard to see something from the oscillation even in the frequency domain.

Many digital noise reduction methods have been developed in previous studies [5-7]. These methods use the statistical feature of the random noises that is its power spectral density is similar in any frequency band. The expected error rate of the ensemble average decreases monotonically as a function of the number of the data sets in the ensemble average. Therefore, the statistical analysis of a fluctuating quantity over a long period of time may be useful to pickup fluctuating signals.

This work presents the methods to quantify the statistical properties of the fluctuation spectra based on MIR signals. The analysis methods and the effects of ensemble average on the noise reduction in the spectrum are presented in section 2 and 3. In section 4, some example of our analysis will be applied to MIR data in LHD. Significant results are as follows: three types of the modes and turbulence are observed by using the ensemble technique; the mode numbers are obtained by the cross correlation technique. The turbulence shows an ion drift characteristic during high power NBI heating.

2. Spectral analysis methods

Fourier analysis is the most broadly used signal processing technique. In digital signal processing, we often face the necessity to separate the weak signals from a serious noise contaminated time series. The oscillation looks not very useful in many situations. However, if the time series is transformed into the frequency domain, the frequency spectrum will show the fluctuation power, frequency and primary phase. The Fourier transformation is given by

$$X(f)=\int_{-\infty}^{+\infty} w(t)x(t)e^{-j2\pi ft}dt \qquad (1)$$

where x(t) is the time series, $w(t)$ is the window function which is used to reduce the leakage of the sideband. Here, hanning window is used. In general the spectrum of density fluctuation changes with time, the short time FFT analysis is used to show the time evolution of the spectrum.

The cross-power spectral analysis is used to identify the two time series which have the similar spectral

---

E-mail: shi.zhongbing@nifs.ac.jp

properties. The cross-power spectrum between two time series x(t) and y(t) is defined as

$$G_{xy}(f) = Y(f)X(f)^* \qquad (2)$$

here * denotes the complex conjugate. $X(f)$ and $Y(f)$ is the discrete Fourier transforms of the time series $x(t)$ and $y(t)$, respectively. The auto-power spectrum is the same as taking the Fourier transform when use two identical time series, but the phase information is lost. The phase shift between two time series is:

$$\phi_{xy}(f) = \tan^{-1}\left\{\frac{\mathrm{Im}\left[G_{xy}(f)\right]}{\mathrm{Re}\left[G_{xy}(f)\right]}\right\} \qquad (3)$$

In order to obtain the phase shift whose value corresponds to a high correlation in the frequency domain, the coherence spectrum is introduced and it is defined by the cross-power spectrum normalized by the total power, as

$$\gamma_{xy}(f) = \frac{|< G_{xy}(f) >|}{\sqrt{< G_{xx}(f) >< G_{yy}(f) >}} \qquad (4)$$

where < > denotes ensemble average. The coherency is bounded between 0 and 1, and high value corresponds to high correlation, zero represents completely uncorrelated. The statistical confident level of coherence spectrum is determined by the number of the independent time series in the ensemble ($1/\sqrt{N}$).

The phase-frequency spectrum has a prominent advantage to show the dispersion relations of the MHD mode and turbulence with a distinct phase shift and propagation direction in a two dimensional figure. It can be obtained by two-point cross-correlation method.

$$S(\phi, f) = \left\langle | G_{xy}(f) | \, \delta\left(\phi_{xy}(f) - \phi\right)\right\rangle \qquad (5)$$

In the calculation the delta function is replaced by a rectangular window, and the width of the window is dependent on the number of the discrete sections in the value range of $\phi_{xy}(f)$. From phase shift, we can obtain the mode speed and the mode number by estimating the distance between two detecting points.

3. Ensemble averages

In many situations the signal from plasma is strongly contaminated by random noise. Sometimes its amplitude in the frequency domain is higher than the signal that we are interested in, causing difficult to get the useful information. Fortunately, by using the ensemble averaging technique in the frequency domain, the amplitude of the random jumps becomes an average power level in all frequency range and the peaks of the noise can be removed. The averaging has less influence on the mode whose amplitude doesn't change in the ensemble time. To show how the effect of the noise on signal in the frequency spectrum, a program with a test parameter composed of a sinusoid and a random function is developed. By changing the amplitude ratio of the random noise to signal (N/S), the ratio of the power spectra between the noise and the signal can be obtained. Figure 1 shows the ratio of the FFT amplitudes as a function of N/S. Here, the red star denotes the ratio of the amplitude spectra without ensemble and the black diamond is with ensemble. The ratio of FFT amplitudes changes as a linear function of N/S. The discriminating rate of the N/S is defined as it in the amplitude spectrum has decreased 1/e. With the ensemble technique the N/S increase to about 25, while it is only about 10 without ensemble. Therefore, the ensemble technique is an effective way to reduce noise. It requires the lifetime of the mode should be longer than the time window of FFT. If not, the signal might be distorted by averaging and new analysis method which has both high time and high frequency ability should be used, for example wavelet transforms [8-9].

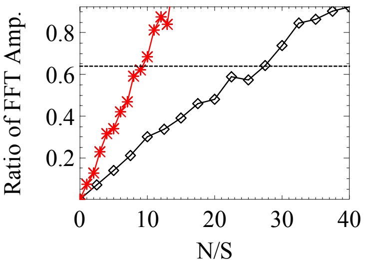

Fig. 1 The effects of the noise to signal (N/S) on the power spectrum. (Red star: without ensemble, Black diamond: with ensemble, dotted line: $1-e^{-1}$)

4. Analysis of MIR signals in LHD

At present MIR system on LHD has three antennas with 8.4cm spacing in toroidal and poloidal direction, and three probe beams with frequencies of 53, 66 and 69GHz in either O-mode or X-mode to illuminate the plasma [1]. The fluctuation signals are measured by the heterodyne receivers with the sampling frequency of 1 MHz. In this paper, we will present an analysis of one shot (75414) with the toroidal magnetic field of 1.5 T and the major radius of 3.6 m, heated by the co-injected NBI with the power of 2.5 MW and counter-injected NBI with the power of 1 MW between t=0.3 and 2.3 s, and the ECH with the total power of 1.2 MW is injected between t=1.4 and 2.0 s. X-mode is used for this shot. Therefore, the cutoff layer is determined by both the toroidal filed and the electron density. The electron density is obtained by the Thomson scattering with calibrating to the microwave interferometer. The normalized radius of the cutoff layer is about 0.1-0.3 during NBI heating.

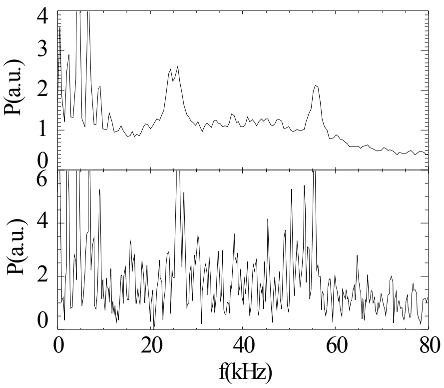

Fig. 2 Power spectrum at 1.6s (top: with ensemble averaging, 50 data sections with the time scale of 2ms each are used. bottom: without averaging and 4ms window is used.)

To extract the fluctuation information, FFT analysis with ensemble averaging technique is used. Figure 2 shows the power spectrum with/without ensemble averaging at 1.6 s. Without ensemble averaging, the MHD modes are concealed in the strong background fluctuation, leading to difficult to decide the mode frequency. With the ensemble averaging, the MHD modes clearly appear and the turbulence exhibits a broad spectrum with high coherency. Figure 3(a) shows the time-frequency spectrum. Three types of fluctuation appear in the MIR signals.

In the low frequency range, the density fluctuation has a fundamental frequency of 2.3 kHz and its higher harmonics. The frequency of this low frequency mode is much lower than that of the Alfvén eigen modes, and about 3 times higher than the electron diamagnetic frequency. It appears when turning on the NBI power and disappears after turning off the NBI power. It seems as if the onset of this mode depends strongly on the power of neutral beam. For the tangentially NBI heating, it is not easy to destabilize the fishbone instability. Indeed, the typical fluctuation of fishbone instability was not observed in magnetic probe signal. Therefore this mode is not the fishbone instability.

At *t*=0.9 s a mid-frequency mode (~23 kHz) appears when the plasma temperature increase to flat top. When turning on the ECH power, this frequency increases to 26 kHz and it disappears after turning off the ECH power. The frequency of this mode is close to the toroidal precession frequency of the resonant trapped fast ions [10].

When turning on the ECH power, a high frequency mode (~55 kHz) appears. This is in the range of Alfvén frequency. The mode frequency increases with time and it is up to 70 kHz at *t*=2.2 s. At *t*=2.0 s the turning-off of the ECH power causes no obviously effect on this mode. That means this mode may be related to the energetic ion mode but it is induced by the energetic electrons.

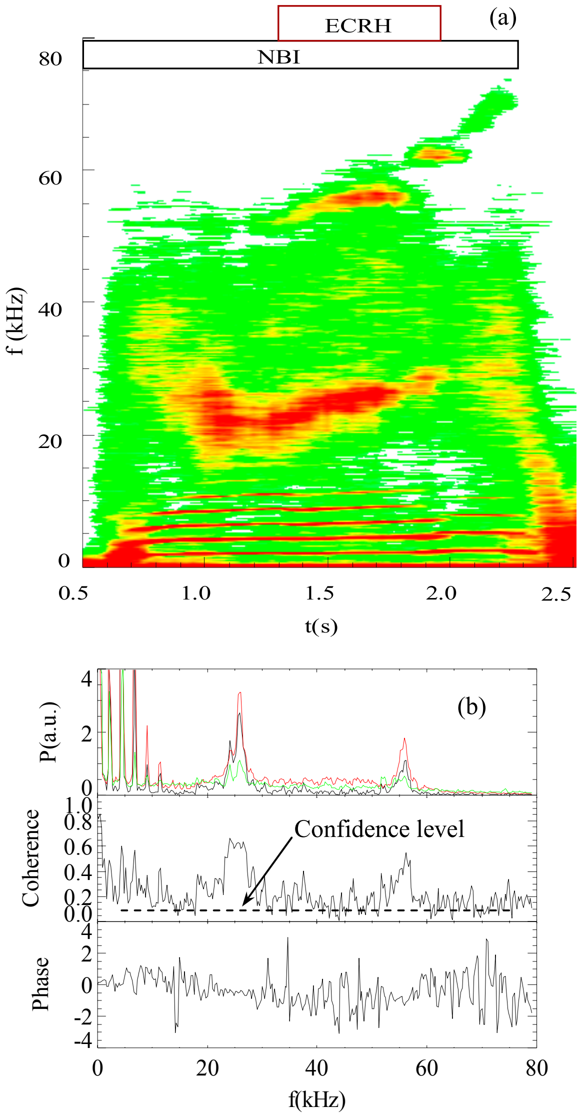

Fig. 3 (a) Time-frequency plot of FFT spectrum, (b) Cross-power spectrum (black: cross-spectrum, red and green: auto-spectrum), coherence and phase shift in the poloidal direction at t=1.6s.

When these modes appear, the cross-power spectrum is peaked and coherence become high, and the phase shift shows less jumps, as shown in figure 3(b). When calculating the cross-power and coherence spectra, FFT is done at every 200 data sections with the time period of 4 ms each. The overlap between the neighboring sections is the half of the time window. Before calculating the spectrum, the mean value and the linear trend have been removed from every time series.

Figure 4 shows the contour plot of the phase-frequency spectrum in the toroidal and poloidal direction at *t*=1.55 s. The high light color corresponds to the large amplitude. When the mode appears, it shows a wide spectral profile vs. the phase shift, maybe, this is because of the strong effects of the turbulence which may cause the distorted distribution of the spectrum. However, the exact phase shift can be obtained from the statistical profile of the phase spectrum. It should be the same as the phase shift in Fig. 3(b). The fluctuation is dominated by the frequency lower than 70 kHz and the turbulence propagates along the ion drift direction. The mode number of the 26 kHz is m=2/n=6, and the mode number of 56 kHz is m/n=4/7. The errors of mode numbers are about ±1.

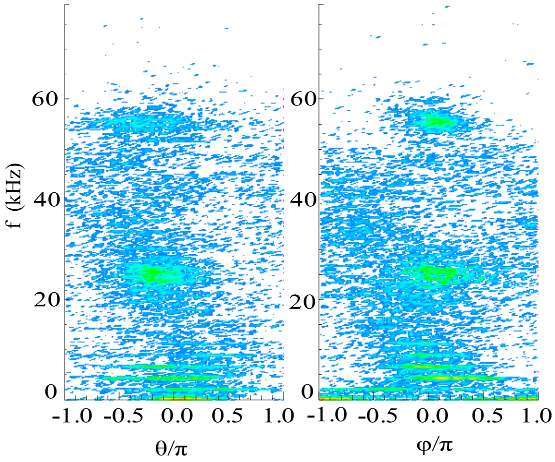

Fig. 4 The phase-frequency spectra in poloidal (left) and toroidal (right) directions at t=1.55s.

5. Summary

In summary, the analysis of plasma density fluctuation on LHD has been carried out based on MIR signals. The ensemble technique has been developed to reduce the noise effect in the spectrum analysis. This technique improves accuracy better than single data set when obtaining the statistical property of the fluctuation. Novel MHD modes and turbulence are observed during high power NBI heating. The mode numbers are obtained by the cross correlation technique. The turbulence shows an ion drift characteristic during high power NBI and ECH heating.

Acknowledgments:

This work is supported by National Institute of Natural Sciences (NIFS07KEIN0021) and by National Institute for Fusion Science (NIFS07ULPP525).

Reference:

[1] S. Yamaguchi, et al., Rev. Sci. Instrum. **77**, 10E930 (2006).
[2] R. Sabot, et al., Plasma Phys. Control. Fusion. **48**, B421 (2006)
[3] H. Park, et al., Rev. Sci. Instrum. **74**, 4239 (2003).
[4] E. Mazzucato, Rev. Sci. Instrum. **69**, 2201(1998).
[5] E. B. Hooper, Plasma Physics, **13,** 1 (1971)
[6] Ch. P. Ritz, et al., Rev. Sci. Instrum. **59**, 1739 (1988).
[7] G. S. Xu, et al., Physics of Plasmas. **13**, 102509 (2006)
[8] A. Grinsted, J. C. Moore, and S. Jevrejeva, Nonlinear Processes in Geophysics **11**,561(2004)
[9] V. P. Budaev, I. M. Pankratov, S. Takamura, etal., Nucl. Fusion **46**, S175 (2006)
[10] T. Kass, H. S. Bosch, F. Hoenen, et al., Nucl. Fusion **38**, 807 (1998)