Spectrum properties of Hall MHD turbulence

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Aiming to understand roles of the two fluid effect on plasma dynamics, direct numerical simulations of Hall MHD turbulence are carried out. A comparison between the numerical results of the Hall MHD turbulence to those of one-fluid MHD turbulence reveal that Hall term modifies small scale properties of MHD turbulence.

Keywords: MHD turbulence, Two fluid effect, Hall term, Energy spectrum, Vortex structure

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1 Introduction

In recent years, roles of two fluid effects have attracted attention in fusion plasmas [1, 2], as well as astrophysical plasmas [3, 4]. A two-fluid model contains various physical effects [5], and therefore exhibits rich phenomena. It may be sometimes expected that the two-fluid effects contribute to transfer the energy from larger scales to lower scales and enhance the dissipation. However, so far as the authors recognize, the two-fluid effects on the energy transfer among scales has not been studied sufficiently. One subject in which the energy transfer among the scales are well formulated and studied should be the isotropic turbulence of a neutral, incompressible fluid, as are seen in enormous number of works after the famous Kolmogorov's theory (see Ref. [6] and references therein) and MHD turbulence [7]. In studies of turbulence, the energy transfer among the scales are studied in the context of the Fourier energy spectrum and the energy transfer functions. Even in the long history of studies of isotropic turbulence, there remain some aspects of the energy transfer, such as the localness/non-localness, remains imperfect understandings. Compared to these preceding subjects, roles of the two-fluid effect in turbulence (whether it is fully developed or not) are not studied very well.

For studies of two-fluid effects among scales, a simpler model is more preferable so that the effects are distinguished easily from the other effects. Hall magnetohydrodynamics (Hall MHD) provides a minimal model which expresses two-fluid effects:

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla p + \boldsymbol{j}\times\boldsymbol{B} + \mu\nabla^2\boldsymbol{u}, \qquad (1)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times[(\boldsymbol{u}-\epsilon\boldsymbol{j})\times\boldsymbol{B}] + \eta\nabla^2 B, \qquad (2)$$

where $\boldsymbol{B}$ is the magnetic field (normalized by a representative value B_0, $\boldsymbol{j}$ $(=\nabla\times\boldsymbol{B})$ is the current (normalized by B_0/L_0; L_0 is the characteristic length), $\boldsymbol{u}$ is the velocity (normalized by the Alfvén speed $V_A = B_0/\sqrt{\mu_0 n_i M_i}$; μ_0 is the permeability of vacuum, M_i is the ion mass and n_i is the ion number density, which is assumed to be constant for simplicity), ν is the viscosity and η is the resistivity (normalized by $V_A L_0$), and p is the pressure (normalized by B_0^2/μ_0). The scale parameter $\varepsilon = l_i/L_0$ is called Hall parameter, where $l_i = \sqrt{M_i/\mu_0 n_i e^2}$ (e is an elementary charge) is the ion skin depth.

A excellent tool to study the energy spectrum is the shell model proposed by Yamada and Ohkitani [8] for a neutral fluid turbulence. The shell model approach mimics the dynamics of the basic equations as sparse and artificial Fourier mode couplings. Although detailed dynamics of the original equations are abandoned, it achieves very high Reynolds number, which direct numerical simulation (DNS) can not achieve even by the most powerful supercomputer of the recent years. Recently, one of the authors (D.H.) have developed a new shell model for Hall MHD, and performed numerical simulations both the Hall MHD case ($\varepsilon \neq 0$) and the single fluid MHD case ($\varepsilon = 0$). [9, 10] The shell model computations predict a modification of the energy spectrum by the Hall effects, suggesting the energy transfer from the large scales to small scales.

Based on these understandings, we conduct numerical studies of Hall MHD turbulence by means of the DNS of both MHD and Hall MHD equations in order to study the effects of Hall term on the energy transfer in isotropic turbulence. We first review the computational results of the shell model for MHD and Hall MHD turbulence very briefly. Then, some basic views in the two kinds of turbulence are studied. Summary appears in the final section.

2 Shell Model Computation Review

Here we review some numerical results reported in Refs. [9, 10] so that the background of our computational work is well understood. Refer to the references for the details of the model and the scaling of the energy spectra obtained by the computations.

The energy spectra for (a) the single fluid MHD (hereafter we simply refer to MHD) and (b) Hall MHD (for $\varepsilon = 10^{-2}$) are shown in Fig. 1. As is observed in Fig. 1,

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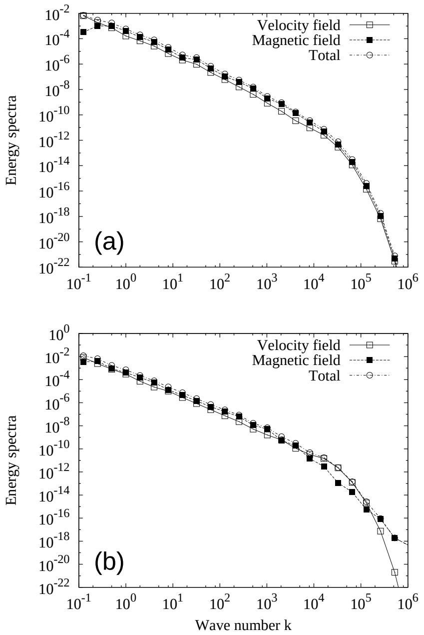

Fig. 1 Energy spectra obtained from the simulation of shell model. The MHD case is shown in the upper panel (a) and The Hall MHD case is done in the lower panel (b).

the scaling exponents of the magnetic field is significantly modified in very high wavenumbers. In the case of the MHD, the magnetic field spectrum is damped in the dissipation region in the same form as the velocity energy spectrum. However, in the Hall MHD spectrum, the magnetic field indicates $-7/3$ power-law in the small scale region. These contrast may reflect the contribution of the Hall term to the energy transfer in the wave number space.

3 Direct numerical simulation of MHD and Hall MHD turbulence

DNSs of the decaying MHD and Hall MHD turbulence are carried out for the $(2\pi)^3$ triple-periodic geometry. Spatial derivatives are approximated by the pseudo-spectrum method and the variables are marched into the time direction by the Runge-Kutta-Gill scheme. The aliasing error is removed by the 2/3-truncation method. The number of the grid points are $N^3 = 256^3$, so that the maximum wavenumber available in this simulation work is $k_{max} = 84$. In the case of Hall MHD case, Hall parameter is $\varepsilon = 1 \times 10^{-1}$. The dissipative coefficients are $\nu = \eta = 1 \times 10^{-2}$. Both the MHD and Hall MHD turbulence simulations start from the same initial conditions in which the velocity and magnetic fields have the energy spectrum roughly proportional to $k^2 exp(-(k/k_0)^2)$ (here $k_0 = 2$) and random phases. No external force is imposed so that the total energy of the velocity and the magnetic fields decays monotonically to time.

In DNS studies of fully developed turbulence, turbulent field is often characterized by the vorticity rather than the velocity field. It is partially because the vorticity field $\boldsymbol{\omega}$ is invariant to the Galilean transform and partially because the vorticity field represents small scale structures much more than the velocity field . For the same reason, the current density $\boldsymbol{j}$ is considered to be more suitable for studying the small scale properties than the magnetic field.

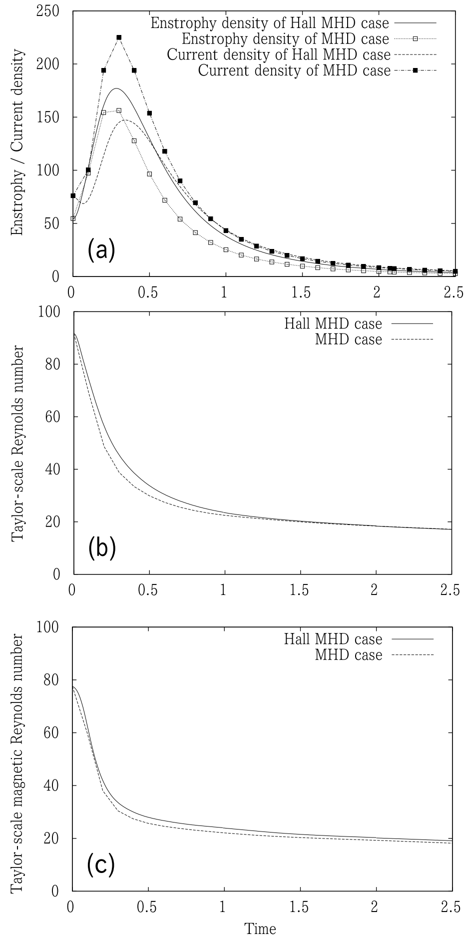

Fig. 2 Time series of the enstrophy density and the current density averaged over the computational volume is shown in the upper panel (a). Taylor scale Reynolds number Re_λ is shown in the middle panel (b), and Taylor scale magnetic Reynolds number is done in the lower panel (c).

In Fig. 2(a), time series of the enstrophy density $\left\langle |\boldsymbol{\omega}|^2/2 \right\rangle$ and the current density $\left\langle |\boldsymbol{j}|^2/2 \right\rangle$ are shown for the two kinds of turbulence, where $\langle\cdot\rangle$ denotes the volume average. A comparison on the enstrophy evolution shows that the enstrophy density is larger in the Hall MHD turbulence than in the MHD turbulence with the same ν and η. We also observe that the total current is larger in the MHD turbulence than in the Hall MHD turbulence. Another good index of fully developed turbulence is the Reynolds number Re_λ based on the Taylor micro-scale [6]. In Figs. 2(b) and (c), we see the time evolution of Re_λ and its counter part for the magnetic field, Re^b_λ for the two kinds of turbulence. We find in both Fig. 2(b) and (c) that the Hall MHD turbulence have larger micro-scale Reynolds numbers Re_λ

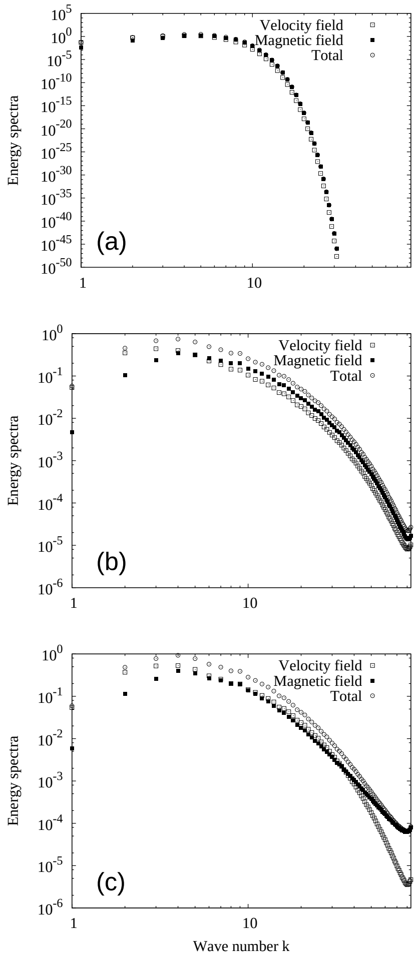

Fig. 3 Energy spectra in DNS of MHD and Hall MHD turbulence. The initial state is shown in the upper panel (a), the energy spectra of the MHD is done in the middle panel (b), and the energy spectra of the Hall MHD is done in the lower panel (c).

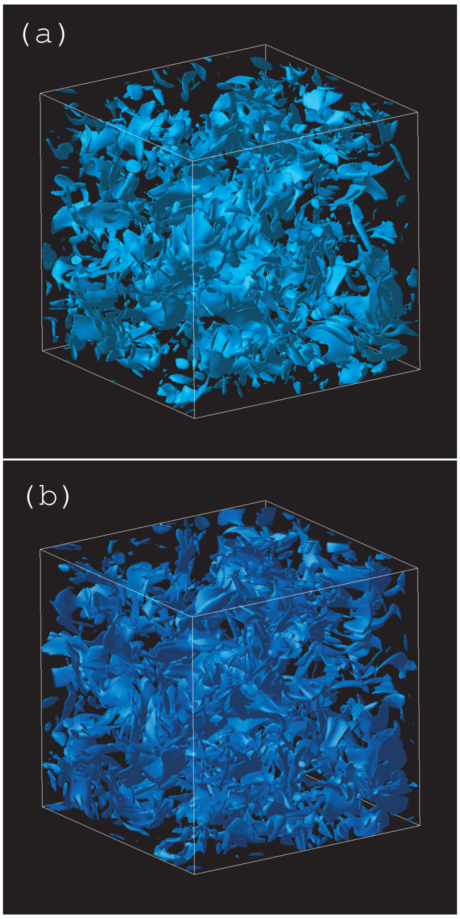

Fig. 4 Isosurfaces of the enstrophy density in (a) MHD and (b) Hall MHD turbulence.

and Re^b_λ than the MHD turbulence, although the difference between the two kinds of turbulence is smaller in (c) than in (b). These observations suggest that the small scales in the velocity (or the vorticity) field is more excited in the Hall MHD turbulence than in the MHD turbulence.

In Fig. 3(a) the energy spectra of the velocity and the magnetic field vectors at the initial time are shown. Since the energy is conserved as the total form $\boldsymbol{u}^2 + \boldsymbol{B}^2$ rather than the kinetic and magnetic energies separately in the ideal limit, we need to study not only the individual spectra u_k^2 and b_k^2 but also the total spectrum $u_k^2 + b_k^2$ as well. Figs. 3(b) and (c) are the plots of the energy spectra of the MHD and Hall MHD turbulence, respectively. By comparing Figs. 3(b) and (c), clear differences between the two kinds of turbulence are seen. The magnetic energy spectrum, denoted by the filled boxes, at the middle (near-peak) wavenumber region is larger in Fig. 3(b) than in Fig. 3(c), while it is opposite in the high wavenumber region. It explains the observations in Fig. 2 that the total current is larger in the MHD turbulence because of the dominance in the middle wavenumber region. In the middle wavenumber region (say, $10 < k < 30$), $b_k^2 > u_k^2$ in the MHD turbulence while $b_k^2 < u_k^2$ in the Hall MHD turbulence. In the highest wavenumber region $k > 40$, the difference between the MHD and Hall MHD turbulence becomes the clearest. The decay of b_k^2 obeys to rather a power-law than the exponential decay. It is similar to the earlier numerical results shown in Fig. 1 and considered to be the direct influence of the Hall term. The high wavenumber region of the velocity energy spectrum in the Hall MHD turbulence decays more rapidly than that in the MHD turbulence, as if compensating the slow decay of the magnetic spectrum b_k^2. The total spectrum $u_k^2 + b_k^2$ of the Hall MHD turbulence has also weaker amplitudes in the $10 \le k \le 50$ wavenumber region than that of the MHD turbulence, but becomes larger in the higher region $k > 50$.

In Figs. 4, isosurfaces of the enstrophy density of the MHD (a) and the Hall MHD turbulence (b) are shown. In Figs. 5, isosurfaces of the current density of the MHD (a) and the Hall MHD turbulence (b) are shown. The thresholds of the isosurfaces of the enstrophy density (current density) are the same between the MHD and the Hall MHD turbulence in Figs. 4 (Figs. 5), the volumes covered by the isosurfaces are clearly different between the two kinds of

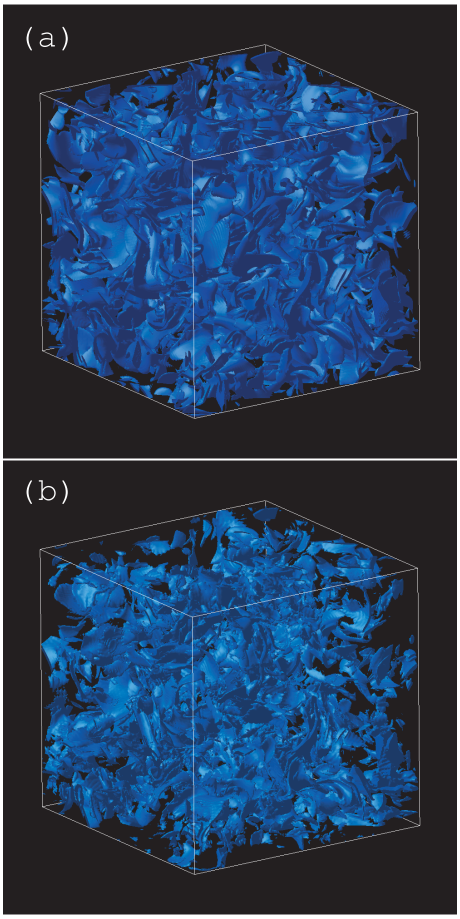

Fig. 5 Isosurfaces of the current density in (a) MHD and (b) Hall MHD turbulence.

turbulence.

4 Summary

We performed the direct numerical simulation of the both Hall MHD and MHD turbulence to study the role of the two fluid effect. Various data show clear difference between the MHD and the Hall MHD turbulence. Time series of the enstrophy, the current, and the Taylor scale Reynolds numbers show that small scale motions of the Hall MHD turbulence is more excited than those of the MHD turbulence. In the small scale, the energy spectra of the magnetic field in the Hall MHD turbulence is observed to be power-law decay, while that of the MHD turbulence is to be exponential decay.

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