Self-similarly evolving, minimally dissipated, and dynamically stable self-organized states obtained by a universal theory

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With the use of a universal theory of self-organization, a novel set of simultaneous eigenvalue equations having dissipative terms are derived to find self-similarly evolving, minimally dissipated, and dynamically stable states of plasmas realized after relaxation and self-organization processes. Typical spatial profiles of plasma parameters, electric and magnetic fields and dissipative factors are presented, all of which are determined self-consistently with each other by physical laws and mutual relations among them, just as in experimental plasmas.

Keywords: self-organization, self-similarly evolving state, minimally dissipative state, dynamically stable, shear flow

1. Inroduction

By using model equations such as the Grad-Shafranov equation, stability of equilibrium configurations in a stationary state is analytically or numerically judged by using suitable stability criteria in the traditional algorithm to find stable states [1]. However, all dynamic quantities in any experimental systems are never stationary, but are continuously evolving along time. Reasonable judgment on experimentally stable states can be done only when the configurations have come into a phase of self-similarly evolving and dynamically stable states. A recent universal theory of self-organization for finding eigenvalue equations to obtain self-similarly evolving and dynamically stable states [2-7] has been shown to incorporate a previous theory [8] for obtaining the minimum dissipative state of magnetic energy, by means of which "the so-called Taylor state" [9] can be derived. The concept of selective decay together with that of helicity invariance in the traditional theories [9-11] is analytically proved in [4-6] to have theoretically unrelated with "relaxed states". The fusion plasma is known to be described for simulations by a set of charge, mass, momentum, and energy conservation laws, and Maxwell's equations to follow its dynamic evolution and to analyze relaxation processes and relaxed states. However, replacement of any element in the set by "the so-called helicity conservation law" [9-11] makes the dynamic evolution untraceable due to this nonphysical law. This fact also means that the all theoretical basis of the theories [9-11] has no theoretical and physical connection with simulation results [12] and experimental ones of relaxed plasmas [13, 14] observed and reported so far.

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2. Theories and numerical results

Applying a principle of the minimum change rate of global auto-correlations for a generalized dissipative dynamic open or closed systems with N dynamical quantities of M dimensional variables and using the variational calculus to the second variation, we obtain the following N simultaneous eigenvalue equations, which is used as a universal theory to find self-similarly evolving and dynamically stable self-organized states and is applicable to various dynamical systems [3, 7, 15];

$$\partial q_i^{\,j\#}\left[\xi^k\right]/\partial \xi^j = D_i^{\,j\#}[\boldsymbol{U}] = \tau_{i\mathrm{df}}{}^{-1}\Lambda_{im}\boldsymbol{U}_{im}[\xi^k{}_{k\neq j}]. \quad (1)$$

Here, $D_i^{\,j}[\,\boldsymbol{q}\,]$ represents dissipative or non-dissipative, linear or nonlinear operators for the change of a dynamical quantity q_i by a variable ξ^j. Applying Eq.(1) directly to all equations of mass, momentum and energy conservation laws and Maxwell's equations with the displacement current neglected for the two-fluid model of fully ionized, compressible and dissipative fusion plasmas and using usual normalization, we obtain the following set of simultaneous eigenvalue equations;

$$\partial \bar{n}_\mathrm{e}/\partial t = -\bar{\nabla}\cdot(\bar{n}_\mathrm{e}\bar{\boldsymbol{u}}_\mathrm{e}) = \Lambda_{n\mathrm{e}}\bar{n}_\mathrm{e}, \quad (2)$$

$$\partial \bar{n}_\mathrm{i}/\partial t = -\bar{\nabla}\cdot(\bar{n}_\mathrm{i}\bar{\boldsymbol{u}}_\mathrm{i}) = \Lambda_{n\mathrm{i}}\bar{n}_\mathrm{i}, \quad (3)$$

$$\begin{aligned}\partial \bar{\boldsymbol{u}}_\mathrm{e}/\partial t = &-(\bar{\boldsymbol{u}}_\mathrm{e}\cdot\bar{\nabla})\bar{\boldsymbol{u}}_\mathrm{e} - [\mathrm{f}_\mathrm{me1}\bar{\nabla}(\bar{n}_\mathrm{e}\bar{T}_\mathrm{e}) + \mathrm{f}_\mathrm{me2}\bar{\nabla}\cdot\bar{\boldsymbol{\Pi}}_\mathrm{e} + \mathrm{f}_\mathrm{me3}\bar{n}_\mathrm{e}(\bar{\boldsymbol{E}} \\ &+ \bar{\boldsymbol{u}}_\mathrm{e}\times\bar{\boldsymbol{B}}) - \mathrm{f}_\mathrm{me4}(\bar{n}_\mathrm{e} + Z\bar{n}_\mathrm{i})(0.5\bar{\eta}_\perp\bar{\boldsymbol{j}}_{//} + \bar{\eta}_\perp\bar{\boldsymbol{j}}_\perp)]/\bar{n}_\mathrm{e} = \Lambda_{u\mathrm{e}}\bar{\boldsymbol{u}}, \quad (4)\end{aligned}$$

$$\begin{aligned}\partial \bar{\boldsymbol{u}}_\mathrm{i}/\partial t = &-(\bar{\boldsymbol{u}}_\mathrm{i}\cdot\bar{\nabla})\bar{\boldsymbol{u}}_\mathrm{i} - [\mathrm{f}_\mathrm{mi1}\bar{\nabla}(\bar{n}_\mathrm{i}\bar{T}_\mathrm{i}) + \mathrm{f}_\mathrm{mi2}\bar{\nabla}\cdot\bar{\boldsymbol{\Pi}}_\mathrm{i} - \mathrm{f}_\mathrm{mi3}\bar{n}_\mathrm{i}(\bar{\boldsymbol{E}} \\ &+ \bar{\boldsymbol{u}}_\mathrm{i}\times\bar{\boldsymbol{B}}) + \mathrm{f}_\mathrm{mi4}(\bar{n}_\mathrm{e} + Z\bar{n}_\mathrm{i})(0.5\bar{\eta}_\perp\bar{\boldsymbol{j}}_{//} + \bar{\eta}_\perp\bar{\boldsymbol{j}}_\perp)]/\bar{n}_\mathrm{i} = \Lambda_{u\mathrm{i}}\bar{\boldsymbol{u}}_\mathrm{i}, \quad (5)\end{aligned}$$

$$\begin{aligned}\partial \bar{T}_e/\partial t = &(1-\gamma)\bar{T}_e\bar{\nabla}\cdot\bar{\boldsymbol{u}}_\mathrm{e} - \bar{\nabla}\bar{T}_e\cdot\bar{\boldsymbol{u}}_\mathrm{e} + \{\mathrm{f}_\mathrm{ene1}\sum_{,j}^{3}\bar{\boldsymbol{\Pi}}_{\mathrm{e}i,j}\,\partial\bar{u}_{\mathrm{e}i}/\partial\bar{x}_j \\ &+ \mathrm{f}_\mathrm{ene2}[\kappa_{\mathrm{e}//0}\bar{\nabla}\cdot(\bar{\kappa}_{\mathrm{e}//}\bar{\nabla}_{//}\bar{T}_\mathrm{e}) + \kappa_{\mathrm{e}\perp0}\bar{\nabla}\cdot(\bar{\kappa}_{\mathrm{e}\perp}\bar{\nabla}_\perp\bar{T}_\mathrm{e}) + \mathrm{f}_\mathrm{ene3}(Z\bar{\boldsymbol{u}}_\mathrm{i}/\bar{n}_\mathrm{e}\end{aligned}$$

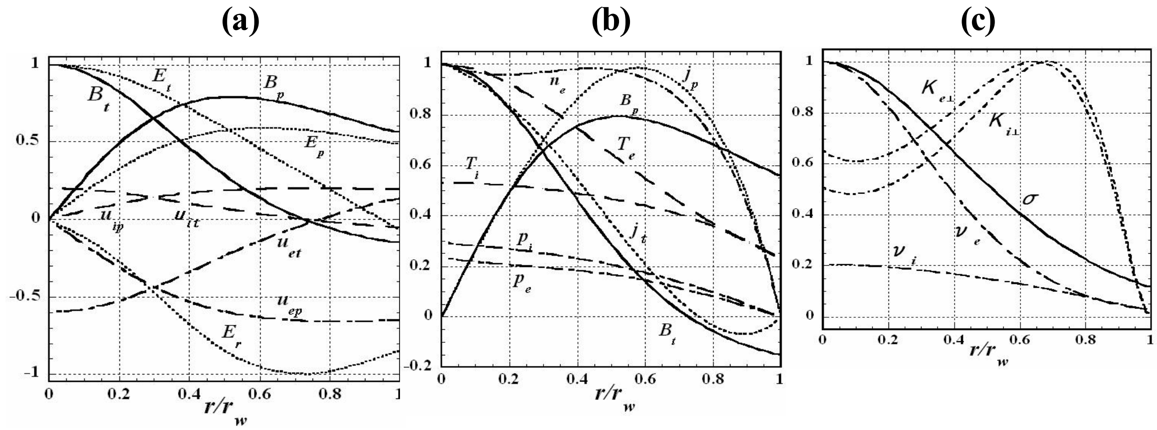

Fig.1 . Typical self-similarly evolving and dynamically stable self-organized configurations of the RFP in a simplified cylindrical.

$$+1)(\eta_{//}\bar{j}_{//}^{\,2}+\eta_{\perp}\bar{j}_{\perp}^{\,2})-\mathrm{f}_{\mathrm{ene4}}(\bar{T}_{\mathrm{e}}-\bar{T}_{\mathrm{i}})/\tau^{\varepsilon}{}_{\mathrm{ei}}\}/\bar{n}_{\mathrm{e}}=\Lambda_{pe}\bar{T}_{\mathrm{e}}, \qquad (6)$$

$$\partial\bar{T}_{\mathrm{i}}/\partial t=(1-\gamma)\bar{T}_{\mathrm{i}}\bar{\nabla}\cdot\bar{\boldsymbol{u}}_{\mathrm{i}}-\bar{\nabla}\bar{T}_{\mathrm{i}}\cdot\bar{\boldsymbol{u}}_{\mathrm{i}}+\{\mathrm{f}_{\mathrm{ene1}}\sum\nolimits_{,j}^{3}\bar{\boldsymbol{\Pi}}_{\mathrm{i}i,j}\,\partial\bar{u}_{\mathrm{i}i}/\partial\bar{x}_{j}$$
$$+\mathrm{f}_{\mathrm{ene2}}[\kappa_{\mathrm{i}//0}\bar{\nabla}\cdot(\bar{\kappa}_{\mathrm{i}//}\bar{\nabla}_{//}\bar{T}_{\mathrm{i}})+\kappa_{\mathrm{i}\perp 0}\bar{\nabla}\cdot(\bar{\kappa}_{\mathrm{i}\perp}\bar{\nabla}_{\perp}\bar{T}_{\mathrm{i}})$$
$$-\mathrm{f}_{\mathrm{ene4}}(\bar{T}_{\mathrm{e}}-\bar{T}_{\mathrm{i}})/\tau^{\varepsilon}{}_{\mathrm{ei}}\}/\bar{n}_{\mathrm{i}}=\Lambda_{pi}\bar{T}_{\mathrm{i}}, \qquad (7)$$

$$\partial\boldsymbol{B}/\partial t=-\bar{\nabla}\times\bar{\boldsymbol{E}}=\Lambda_{\mathrm{B}}\bar{\boldsymbol{B}}, \qquad (8)$$

$$\bar{\nabla}\times\bar{\boldsymbol{B}}=\bar{\boldsymbol{j}}, \quad (9) \quad \bar{\nabla}\cdot\bar{\boldsymbol{E}}=\mathrm{f}_{\mathrm{ce}}(Z\bar{n}_{\mathrm{i}}-\bar{n}_{\mathrm{e}}), \quad (10) \quad \bar{\nabla}\cdot\bar{\boldsymbol{B}}=0, \quad (11)$$

$$\bar{\boldsymbol{j}}\equiv\mathrm{f}_{\mathrm{cj}}(Z\bar{n}_{\mathrm{i}}\bar{\boldsymbol{u}}_{\mathrm{i}}-\bar{n}_{\mathrm{e}}\bar{\boldsymbol{u}}_{\mathrm{e}})+\bar{\boldsymbol{j}}_{//\mathrm{bs}}, \qquad (12)$$

where $\mathbf{j}_{//bs}$ is the bootstrap current, (f_{me1}, f_{me2}, f_{me3}, f_{me4}), (f_{mi1}, f_{mi2}, f_{mi3}, f_{mi4}) and (f_{ene1}, f_{ene2}, f_{ene3}, f_{ene4}), are factors by normalization for the conservation laws of electron and ion momentums and energy, respectively, and f_{ce} and f_{cj} are those for Eqs. (10) and (12), respectively. Using dominant terms in the limiting case of uniform conductivity σ and negligible viscosity ν and thermal conductivity κ, Eqs. (1) - (11) lead to the so-called Taylor state, just the same as in [7]. In general cases, however, these equations can be applicable to finite beta confinement systems of the Tokamak, the reversed field pinch (RFP), the field reversed configuration (FRC), and so on. Using the cylindrical model for simplicity and the 4 rank and 4th order Runge Kutta method under suitable boundary conditions on measurable quantities by referring to experimental data [14], we have numerically solved central terms = right-hand sides of Eqs. (1) - (11) to get self-similarly evolving and dynamically stable self-organized configurations of the RFP. A typical result is shown in Figs. 1(a), 1(b) and 1(c), where all physical quantities and dissipative factors are shown by their symbols. It is seen from the data profiles that all physical quantities are related self-consistently with each other, i.e. , $\kappa_{e\perp}$ and $\kappa_{i\perp}$ are determined by n_e, n_i, T_e, T_i, and B, and T_e and T_i are determined by $\kappa_{e\perp}$, $\kappa_{i\perp}$, σ, and j, and so on, to lead to negligibly small current density at the boundary wall like as in experimental plasmas. We also find from profiles of u_{ep}, u_{ip}, u_{et} and u_{it} in Fig. 1(a) that there exists the shear flow which depends on the profile of ν , i.e. , mainly on that of T_i, and would stabilize of the self-organized RFP plasma.

2. Concluding remarks

We have derived a novel set of simultaneous eigenvalue equations for finding self-similarly evolving, minimally dissipated and dynamically stable states realized after relaxation and self-organization processes [cf. Eqs.(1) - (12)]. The set of simultaneous equations is applicable to all type of magnetically confined fusion plasmas. Solving numerically the set of equations in the cylindrical model, we have shown typical self-similarly evolving and dynamically stable self-organized configurations of the RFP plasma including a lot of spatial information on related physical quantities useful for detailed experimental investigation. It should be emphasized that all physical quantities of interest are self-consistently determined by physical laws and mutual relations among them.

From the present universal theory of self-organization, we should recognize a clear fact that a paradigm shift from the traditional concept of stationary stability to more real concept of dynamical stability is definitely necessary for faster development of fusion science and technology in order to avoid near future energy crisis by shortening distance theories and experiments.

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