Neoclassical viscosities in NCSX and QPS with few toroidal periods and low aspect ratios

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Previously reported benchmarking examples for the analytical formulas of neoclassical viscosities were made implicitly assuming applications in a future integrated simulation system for the LHD (Large Helical Device). Therefore the toroidal period numbers assumed there were mainly N=10. In this kind of calculation, however, an implicit (or sometimes explicit) assumption of ι/N<<1 is sometimes included. This assumption is included not only in simplified bounce averaged drift kinetic equations for ripple diffusions, but also in the equation before the averaging for non-bounce-averaged effects determining neoclassical parallel viscosity and the banana-plateau diffusions. To clarify the applicability of the analytical methods even for configurations with extremely low toroidal period numbers (required for low aspect ratios), we show here recent benchmarking examples in NCSX (National Compact Stellarator Experiment) with N=3 and QPS (Quasi-poloidal Stellarator) with N=2.

Keywords: neoclassical transport, neoclassical viscosity, moment equation approach, drift kinetic equation, non-symmetric toroidal plasmas, low-aspect ratio advanced stellarators

1. Inroduction

The moment equation approach for neoclassical transport [1,2] in non-symmetric toroidal plasmas had been developed mainly for neoclassical parallel flows and the associated parallel viscosity [3-6]. Even though it was shown in Ref.[7] that a consistent frame work including not only the flows but also radial diffusions (in other words, not only the parallel viscosity, but also poloidal and toroidal viscosities) can be constructed in this line of moment approach, methods to calculate all of required viscosity coefficients in general collisionality regimes in general toroidal configurations had not been shown. Motivated by design activities of advanced stellarators, a method to obtain the full neoclassical viscosity coefficients was developed in Ref.[8]. It was shown there that three mono-energetic viscosity coefficients M^*(parallel viscosity against flows), N^*(driving force for bootstrap currents), and L^*(radial diffusion) are required to describe the full neoclassical characteristics of general non-symmetric toroidal configurations. Since existing numerical methods such as variational methods and Monte Carlo methods for the drift kinetic equation described in the 3-D phase space (poloidal angle θ, toroidal angle ζ, pitch angle ξ) could be applicable [8], applications of the new theory were done for various types of advanced helical/stellarator configurations [9,10]. However, this step of the development of the moment approach was still in the "basic frame work". Even though there were many alternative methods for the L^* in the collisionless limit (aforementioned ripple diffusions), the other viscosity coefficients could be obtained only by the Drift Kinetic Equation Solver (DKES) code [8,9,10,11]. There are many demands for faster and easier estimation methods for the neoclassical quantities, in integrated simulation systems using iterative calculations of the equilibrium and the transport [12], in the configuration optimizations [9], and in experimental studies investigating dependences on configurations. Since the viscosities (or resulting neoclassical transport coefficients) are direct consequences of guiding center drift motions, evaluating them is an important part of understanding the characteristics of the designed magnetic configurations [9,10,13,14]. Even in tokamak experiments, neoclassical toroidal viscosity effects due to breaking the axisymmetry have recently been studied [15], and thus the framework of the moment approach for non-symmetric configurations and the methods to calculate the viscosity coefficients are now required for all studies of toroidal plasma confinements. In theories of axisymmetric tokamaks, simple analytical methods based on asymptotic expansions of the drift kinetic equations and connections of results of them are commonly used [1,16]. Since this approach using analytical methods will be required also in a future integrated simulation system for the LHD(Large Helical Device)[12], we had previously carried out derivations and benchmark tests of

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the analytical formulas for the three mono-energetic coefficients [17]. The previous benchmarking examples were made implicitly assuming applications in the LHD [a helical heliotron with major and minor radii of R_0=3.9m, a=0.6m, and magnetic field strength of B_0≤3T]. Therefore the toroidal period numbers assumed there were mainly N=10, and the assumed $B_{mn}^{\rm (Boozer)}$ [$B=\Sigma B_{mn}\cos(m\theta - nN\zeta)$] spectra didn't include $n\neq 0,1$. One open issue remaining there is the ripple-trapped/untrapped boundary layer. Even though there are many alternative calculating methods for the bounce-averaged motion of the ripple-trapped particles, this boundary layer causes coupling effects between the bounce-averaged motion of ripple-trapped particles and the non-bounce-averaged motion of untrapped particles (collisional detrapping/entrapping). In Ref.[17], a previous boundary layer theory by Shaing and Callen for rippled tokamaks [18] was applied with an extension to multi-helicity stellarators. One effect of this coupling, which was investigated also in Ref.[17], is a difference of N^* (or $G^{\rm (BS)} \equiv -\langle B^2\rangle N^*/M^*$) in the $1/\nu$ regime ($E_s/v\approx 0$) from the collisionless detrapping ν regime ($E_s/v\neq 0$, $\nu/v\to 0$) value given by Refs.[3,4]. However, we had not shown any benchmarking examples for the $1/\nu^{1/2}$ diffusion, which is another important effect discussed in Ref.[18]. Even for the N^* in the $1/\nu$ regime, the numerical examples for more general cases had not been shown. To investigate these effects in configurations including B_{mn} of $n\neq 0,1$ and with extremely low toroidal period numbers (required for low aspect ratios) giving larger ι/N, we show in this paper recent calculation examples in NCSX (National Compact Stellarator Experiment)[10,13] and QPS (Quasi-poloidal Stellarator) [9,10,14].

2. Calculations in NCSX

The NCSX is a quasi-axisymmetric (QA) toroidal system with N=3, R_0=1.4m, a=0.32m, and B_0≤2T. Here we consider the calculation on a flux surface with normalized toroidal flux of $(\psi/\psi_{\rm edge})^{1/2}$=0.51 (corresponding to r≅0.165m) in a standard configuration (NCSX-m50) with a finite beta of β=4% and a finite toroidal current of $I_{\rm p}$=178 kA as an example. The notations for the flux surface coordinates (mainly the Boozer coordinates) in Refs.[8,17] are followed and thus the radial derivatives of the poloidal and toroidal magnetic fluxes are χ' =0.1178T•m, ψ' =0.2513T•m, and covariant poloidal and toroidal components of the magnetic field are B_θ=0.0036T•m, B_ζ=2.3210T•m, respectively, on this flux surface. The $B_{mn}^{\rm (Boozer)}$ in a range of $0\le m\le 16$ and $|n|\le 11$ are used. As described in Ref.[18], the boundary layer structure is determined by a drift kinetic equation $(V_{//}-C_a^{\rm PAS})f_{a1}=0$, where $V_{//}\equiv$ $\mathbf{b}\bullet\nabla_{(\mu=\rm const)}$ is the linearized orbit propagator and $C_a^{\rm PAS}$ is the pitch-angle-scattering operator with the collision frequency of $\nu_{\rm D}^a$[8]. Since $V_{//} f_{a1}\neq 0$ for the non-trivial solution of this equation, the existing bounce- or ripple-averaging methods assuming $V_{//}f_{a1}=0$ is not appropriate for this analysis. Therefore we should use the bounce- or ripple-averaging methods to obtain $\partial f_{a1}/\partial\mu$ in the ripple-trapped pitch-angle range, which gives the boundary condition for the boundary layer analysis [18], together with the analytical solution for the boundary layer as complimentary methods. For this analytical solution, a model expression of the magnetic field strength $B/B_0=1+\varepsilon_{\rm T}(\theta)+\varepsilon_{\rm H}(\theta)\cos\{L\theta-N\zeta-\gamma(\theta)\}$ is required. We use here $\varepsilon_{\rm H}(\theta)=\{B_{\rm max}(\theta)-B_{\rm min}(\theta)\}/(2B_0)$ for each poloidal angle θ to define $\varepsilon_{\rm H}(\theta)$. This is a truncation of the Fourier series used by Todoroki [20] who expanded not the amplitude but the phase of $B/B_0-1-\varepsilon_{\rm T}(\theta)$.

Analogously, $1+\varepsilon_{\rm T}(\theta)$ is given by $1+\varepsilon_{\rm T}(\theta)=\{B_{\rm max}(\theta)+B_{\rm min}(\theta)\}/(2B_0)$. The residual ripple-well structure is distorted, or sometimes eliminated at $\theta\approx\pm\pi/2$, by finite rotational transform per toroidal period $(\chi'/\psi')/N$ in cases with small $\varepsilon_{\rm H}$. For this kind of situations, the effective ripple well depth $\delta_{\rm eff}$ and effective ripple well length correction α^* were introduced in the theory for rippled tokamaks [18]. We use also this technique with an extension to helical/stellarator configurations. The error of a well-known Shaing-Hokin formula [21] for the $1/\nu$ ripple diffusions in $\varepsilon_{\rm H}\to 0$ limits (for e.g., $\varepsilon_{\rm H}\le 0.01$), which was pointed out in Ref.[8], is strongly reduced by introducing this method. By using these notations, an expression for the $1/\nu^{1/2}$ diffusion coefficient in Ref.[18], which is a contribution of ripple-trapped pitch-angle range $0\le\kappa^2\le 1$ for $\kappa^2\equiv\{w-\mu B_0(1+\varepsilon_{\rm T}-\delta_{\rm eff})\}/(2\mu B_0\delta_{\rm eff})$, can be extended to a form including more general non-symmetric configurations as

$$L^*_{(-1/2)} = 2.92\frac{2}{\pi^2}\left(\frac{v}{\nu_{\rm D}^a}\right)^{1/2}\frac{2^{3/4}}{(\psi')^2}\left(\frac{V'}{4\pi^2}\right)^{1/2}\times \int_{-\pi}^{\pi}\frac{{\rm d}\theta}{2\pi}\,\delta_{\rm eff}^{3/4}\left(B_0\,\frac{\pi-2\sin^{-1}\alpha^*}{N\psi'-L\chi'-\chi'\partial\gamma/\partial\theta}\right)^{1/2}\times \left\{\left(\frac{\partial\varepsilon_{\rm T}}{\partial\theta}\right)^2-\sqrt{1-\alpha^{*2}}\,\frac{\partial\varepsilon_{\rm T}}{\partial\theta}\frac{\partial\varepsilon_{\rm H}}{\partial\theta}+\frac{2}{9}(1-\alpha^{*2})\left(\frac{\partial\varepsilon_{\rm H}}{\partial\theta}\right)^2\right\} \tag{1}$$

Here, for the aforementioned distribution function $\partial f_{a1}/\partial\mu$ in $0\le\kappa^2\le 1$ as the boundary condition, an analytical solution by Shaing and Hokin [21] is used to consider analytically the dependence on the magnetic configurations. From this form of $L^*_{(-1/2)}\propto\delta_{\rm eff}^{3/4}N^{-1/2}$, we can understand that this component of the diffusions can dominate over the $1/\nu$ diffusion of $L^*_{(-1)}\propto\delta_{\rm eff}^{3/2}N^0$

[21] only in configurations with small ripple amplitude $\delta_{\rm eff}$ and with small toroidal period numbers N, and therefore it will appear in QA configurations rather than the rippled tokamaks considered in Ref.[18]. In fact, previous numerical results in CHS-qa [23] with N=2 showed a clear $1/\nu^{1/2}$ dependence of L^* in a wide range of collisionality (ν/v).

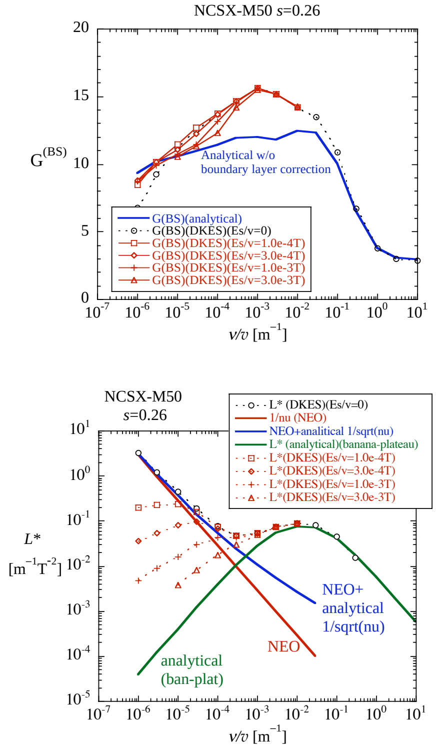

Fig.1 Mono-energetic viscosity coefficients in the NCSX given by the analytical methods (solid curves) and by the numerical method in the 3-D phase space (DKES) (open symbols). (a) the geometrical factor $G^{\rm (BS)} \equiv -\langle B^2 \rangle N^*/M^*$, (b) components of the diagonal diffusion L^*.

Figure 1(a) shows the mono-energetic viscosity coefficients N^* in the NCSX obtained by the analytical formulas [17] and those by the DKES [11] with the conversion formulas in Ref.[8]. The mono-energetic coefficient L^* ($E_s/v \approx 0$) is analytically given by sum of three components: (1) $L^*_{(-1)}$ given by appropriate bounce-averaging codes with field line integral methods, (2) $L^*_{(-1/2)}$ given by Eq.(1), and (3) contributions of non-bounce-averaged drifts given by Eq.(16) in Ref.[17] ($L^*_{\rm (banana\text{-}plateau)}$). We used here the NEO code [22] for the $L^*_{(-1)}$ in the NCSX, and Figure 1(b) shows these components $L^*_{(-1)}$, $L^*_{(-1)}+L^*_{(-1/2)}$, $L^*_{\rm (banana\text{-}plateau)}$, and the DKES results. The sum $L^*_{(-1)}+L^*_{(-1/2)}$ approximately predicts a deviation of the DKES from a pure $\propto 1/\nu$ scaling given by the bounce-averaging codes at $\nu/v<10^{-3}$m^{-1}. In these figures, we show also the dependences of the DKES results on the E×B drift parameter E_s/v. The N^* in NCSX is insensitive to E_s/v even in the range of $E_s/v \le 3\times10^{-3}$T since the $1/\nu$ diffusion of the ripple-trapped particles accompanying the boundary layer correction $N^*_{\rm (boundary)}$ in Eq.(14) in Ref.[17] is strongly reduced in this configuration. In spite of this reduction of $L^*_{(-1)}$ and accompanied $N^*_{\rm (boundary)}$, we can see another boundary layer effect in Fig.1(a). The N^* given by the DKES transiently becomes larger at $\nu/v\sim10^{-3}$m^{-1} compared with the analytical formula. This transient increase is peculiar to QA configurations where the $1/\nu^{1/2}$ component becomes comparable or dominates over the $1/\nu$ component in the radial diffusion, and thus was found also in CHS-qa [23]. Although this effect in the $1/\nu^{1/2}$ regime cannot be calculated by a method in Ref.[17] assuming a collisionless limit of the $1/\nu$ regime (This previous formula gives too small values for the QA configurations and thus is not included in Fig.1(a)), the transient increase, which is about 30% at most, will not be so important in the energy integrated coefficients.

3. Calculations in QPS

The QPS adopts a quasi-poloidal configuration reducing the radial drift of the trapped particles [19], with N=2, R_0=1m, a=0.3m, and $B_0 \le$1T. The parameters of the flux surface, where the calculation examples are made,

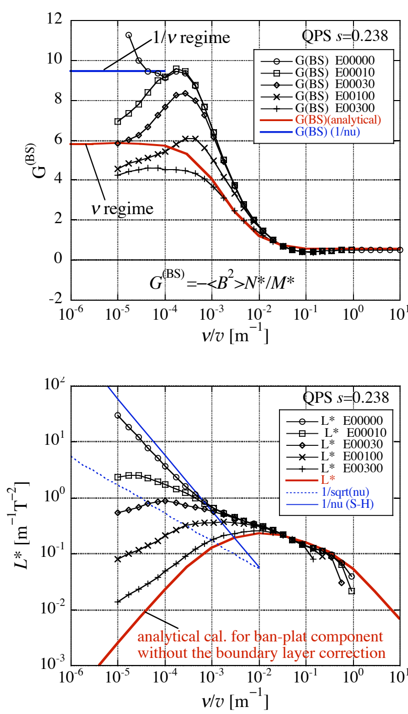

Fig.2 Mono-energetic viscosity coefficients in the QPS given by the analytical methods (solid curves) and by the DKES (lines with symbols). (a) the geometrical factor $G^{\rm (BS)} \equiv -\langle B^2 \rangle N^*/M^*$, (b) components of the diagonal diffusion L^*.

are $(\psi/\psi_{\rm edge})^{1/2}$=0.49 (corresponding to r≅0.14m), χ' =0.0275T•m, ψ' =0.1423T•m, B_θ=0, and B_ζ=1.1403T•m. The $B_{mn}^{\rm (Boozer)}$ ranges are $0\le m\le 20$ and $|n|\le 20$. Results in the QPS are shown in Fig.2. In Fig.2(b), the $1/\nu$ diffusion coefficient $L^*_{(1/\nu)}$ given by the Shaing-Hokin formula [21] including the minor modifications of B expression in Sec.2 is shown to confirm a validity of following discussions on the boundary layer correction. Even for $L^*_{(-1)}\propto\delta_{\rm eff}^{3/2}$, the Shaing-Hokin theory still retains an accuracy of factor 2. Therefore we can investigate characteristics of the boundary layer correction on the parallel viscosity $N^*_{\rm (boundary)}$ with a weaker dependence on $\delta_{\rm eff}$ and $\varepsilon_{\rm H}$ by the analytical method. As confirmed in Ref.[17], we have to interpret a previous "$1/\nu$ regime" formula for the parallel viscosity derived by Shaing,et al.[3,4] and the N^* connection formulae including it (red solid curve in Fig.2(a)) as expressions for strong E_s/v limit (i.e., the ν regime or the $\nu^{1/2}$ regime discussed later). The correct $1/\nu$ regime (E_s/v≈0) value is given by adding a boundary layer correction term $N^*_{\rm (boundary)}$ which was neglected in Refs.[3,4]. Although the calculation of $N^*_{\rm (boundary)}$ in the QPS ($\partial\varepsilon_{\rm H}/\partial\theta\approx 1.5\partial\varepsilon_{\rm T}/\partial\theta$ and $\theta_{\rm M}$=±2.4rad) requires some minor modifications for Eq.(14) in Ref.[17], they will be reported in a separated article. In Fig.2(a), we showed the $1/\nu$ regime asymptotic value of N^* given by $N^{*\rm (sym)}+N^{*\rm (asym)}+N^*_{\rm (boundary)}$ [17]. It approximately predicts the numerical result for a weak radial electric ranges of $E_s/v<1\times 10^{-4}$T by the DKES.

4. Concluding remarks

The mono-energetic neoclassical viscosity coefficients are investigated in two low aspect stellarator configurations with contrasting design concepts. For M^*, $N^{*\rm (asym)}$, (σ_{Xa}, $G_{Xa}^{\rm (asym)}$) defined in Ref.[17] due to pure non-bounce-averaged motions [3,4], the validity of the analytically approximated formulas [8,17] has been confirmed even in the NCSX and in the QPS. In the two configurations, there are contrasting effects of the ripple-trapped/untrapped boundary layer at κ^2≅1 causing coupling effects between the bounce-averaged motion of ripple-trapped particles and the non-bounce-averaged motion of untrapped particles. The $1/\nu^{1/2}$ ripple diffusion $L^*_{(-1/2)}$ in the QA configurations is peculiar to the configurations with small ripples. In the ripple-trapped pitch angle range $0\le\kappa^2\le 1$ in these configurations, the $1/\nu^{1/2}$ component of the perturbed distribution function is not negligible compared with the small $1/\nu$ component. However, their effects on the ripple-untrapped pitch-angle range $\kappa^2>1$ is not important. In contrast to this, the boundary layer affects on this range $\kappa^2>1$ in configurations without quasi-axisymmetry and make other boundary layer corrections on the viscosity coefficients; $N^*_{\rm (boundary)}$ appearing in the $1/\nu$ regime (E_s/v≈0) and also $L^*_{\rm (boundary)}$ near the collisionality regime boundary between $1/\nu$ and plateau regimes. Even though the integration constant in $0\le\kappa^2\le 1$ is negligible compared with a large $1/\nu$ component, boundary layer effects as a driving force of $\propto(\delta_{\rm eff})^{1/2}$ for the flows in $\kappa^2>1$ is not negligible for the $\propto\nu^0$ component of the distribution function in these configurations without quasi-axisymmetry.

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