Collisional transport of multi-ion-species plasmas in general non-symmetric toroidal configurations

S.Nishimura, H.Sugama, Y.Nakamura [a]

National Institute for Fusion Science, Oroshi-cho 322-6, Toki, 509-5292, Japan
[a] *Graduate School of Energy Science, Kyoto University, Gokasho, Uji, 611-0011, Japan*

A previous formulation of the neoclassical transport in helical/stellarator devices based on the moment equation approach is extended to allow the poloidal and toroidal variation of the densities and temperatures of $\delta n_a/n_a$, $\delta T_a/T_a < \delta B/B$. Since the transport of impurities with high collisionalities (so-called Pfirsch-Schlüter diffusions which are separated in our previous works) is determined by the local parallel force balance before the flux-surface averaging including these variations δn_a, δT_a an important purpose of this extension is to study radial profiles of the impurity density under the self-consistent ambipolar radial electric field E_r in plasmas containing electrons and main ions corresponding to the collisionless ($1/\nu$, ν, or banana) regimes or the plateau regime, and impurity ions in the Pfirsch-Schlüter regime. The Legendre-Laguerre expansion with orders of l=0,1 and j=0,1,2 is used for this local momentum balance to include the energy scattering collisions and the effect of the radial electric field in non-symmetric toroidal plasmas.

Keywords: neoclassical transport, multi-ion-species, impurity transport, moment equation approach, drift kinetic equation, non-symmetric toroidal plasmas

1. Introduction

Recently, various types of high-density operations are studied in helical/stellarator devices [1,2] and the neoclassical processes on the impurity transport in these high-density conditions also attract much attention [1,3]. Although codes based on the so-called moment equation approach [4] are used for this kind of studies in axisymmetric tokamaks, codes to handle multi-ion-species plasmas in the helical/stellarator devices are still under developments. Even though a method to obtain the neoclassical transport matrix in general multi-ion-species plasmas in general non-symmetric toroidal configurations had been shown [5], it handled only a part relating to the flux surface averaged part of the momentum balance $\langle \mathbf{B}\bullet\nabla\bullet\boldsymbol{\pi}_a\rangle - e_a\langle n_a\rangle\langle BE_{//}\rangle = \langle BF_{//a1}\rangle$. We had not shown any method to handle the poloidally and toroidally varying part (local structure) of the momentum balance and flows $n_a u_{//a}$, $q_{//a}$ before the flux-surface averaging. In this momentum balance, the densities n_a and temperatures T_a of each particle species (a = e^-, H^+, D^+, T^+, He^+, He^{2+}, …) are not flux surface quantities. Although the poloidal and toroidal variations of the potential are small because of a constraint by the total energy conservation to minimize the Joule loss $\mathbf{J}\bullet\mathbf{E}$, the density and temperature perturbations δn_a, δT_a are not limited by this constraint. Only the total pressure $\Sigma p_a = \Sigma n_a T_a$ can be the flux surface quantity in the MHD equilibrium. In a present study, we extend the stellarator moment equation approach [5] to allow the poloidal and toroidal variation of the densities and temperatures of $\delta n_a/n_a$, $\delta T_a/T_a < dB/B$. The theory for the "neoclassical transport of impurities"[6] in general toroidal plasmas including full parts of collisional diffusions and neoclassical parallel flows is completed by this extension. Since the parallel force balance including the variation δn_a, δT_a determines the transport of impurities with high collisionalities (so-called Pfirsch-Schlüter diffusions which are separated in our previous works), an important purpose of this extension is to study radial profiles of the impurity density (accumulation or shielded "hole") under the self-consistent ambipolar radial electric field E_r in plasmas containing electrons and main ions corresponding to the collisionless ($1/\nu$, ν, or banana) regimes or the plateau regime, and impurity ions in the Pfirsch-Schlüter(P-S) regime.

2. Moment Equations

Although the moment equations for the poloidally and toroidally varying part must be derived from the Vlasov-Fokker-Planck equation [7], we will report details of this derivation in separated articles. Also on the flux surface averaged part of momentum balance, which is basically unchanged from Ref.[5] even in the extension of the theory except replacing $u_{//a}$ by $n_a u_{//a}/\langle n_a\rangle$, the details will not be described here. We show here only the essential part of the results on the poloidally and toroidally varying part. By taking the Legendre-Laguerre

author's e-mail: nishimura.shin@lhd.nifs.ac.jp

moments of the orders of l=0,1 and j=0,1,2 of a part of the linearized drift kinetic equation (LDKE) after separating σ_{Xa} as a source of the flux surface averaged part of the momentum balance [5], we obtain following equations. Hereafter, notations in Ref.[5] are followed. Furthermore we concentrate in this paper only in the local momentum balance in which the flux surface averaged components of flows and forces are subtracted. The particle and momentum conservation laws, which are the Legendre-Laguerre moments of the LDKE with orders of l=0 and j=0,1,2, are given by

$$\langle p_a\rangle\begin{bmatrix}1&0&0\\0&\frac{5}{2}&0\\0&0&\frac{35}{8}\end{bmatrix}\nabla\bullet\begin{bmatrix}n_a\mathbf{u}_{\parallel a}/\langle n_a\rangle\\ \frac{2\mathbf{q}_{\parallel a}}{5\langle p_a\rangle}\\ n_a\mathbf{u}_{\parallel a2}/\langle n_a\rangle\end{bmatrix}$$
$$+\langle n_a\rangle\begin{bmatrix}1&0&0\\-1&\frac{3}{2}&0\\1&-\frac{3}{2}&\frac{15}{8}\end{bmatrix}\left[cE_s\frac{\nabla s\times\mathbf{B}}{\langle B^2\rangle}+u_{\parallel}^{(\mathrm{rigid})}\mathbf{b}\right]\bullet\nabla\begin{bmatrix}\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=0)}\\ T_{a1}^{(j=1)}\\ \frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=2)}\end{bmatrix}.$$
$$-\sum_b\begin{bmatrix}0&0&0\\0&e_{11}^{ab}&0\\0&-e_{11}^{ab}&e_{22}^{ab}\end{bmatrix}\begin{bmatrix}n_{b1}^{(j=0)}\langle T_b\rangle/\langle n_b\rangle\\ T_{b1}^{(j=1)}\\ n_{b1}^{(j=2)}\langle T_b\rangle/\langle n_b\rangle\end{bmatrix}$$
$$=\frac{c}{e_a}\nabla s\times\mathbf{B}\bullet\nabla\frac{1}{B^2}\begin{bmatrix}\langle T_a\rangle\left(\partial\langle p_a\rangle/\partial s+e_a\langle n_a\rangle\partial\langle\Phi\rangle/\partial s\right)\\ \frac{5}{2}\langle p_a\rangle\partial\langle T_a\rangle/\partial s\\ 0\end{bmatrix}\tag{1}$$

The parallel force balances, which are the Legendre-Laguerre moments of the LDKE with orders of l=0 and j=0,1,2, are given by

$$\langle n_a\rangle\begin{bmatrix}1&1&0\\0&\frac{5}{2}&\frac{5}{2}\\0&0&\frac{35}{8}\end{bmatrix}\mathbf{b}\bullet\nabla\begin{bmatrix}\frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=0)}\\ T_{a1}^{(j=1)}\\ \frac{\langle T_a\rangle}{\langle n_a\rangle}n_{a1}^{(j=2)}\end{bmatrix}\quad.\tag{2}$$
$$=\begin{bmatrix}F_{\parallel a1}\\F_{\parallel a2}\\F_{\parallel a3}\end{bmatrix}\equiv\sum_b\begin{bmatrix}l_{11}^{ab}&-l_{12}^{ab}&l_{13}^{ab}\\-l_{21}^{ab}&l_{22}^{ab}&-l_{23}^{ab}\\l_{31}^{ab}&-l_{32}^{ab}&l_{33}^{ab}\end{bmatrix}\begin{bmatrix}n_bu_{\parallel b}/\langle n_b\rangle\\ \frac{2q_{\parallel b}}{5\langle p_b\rangle}\\ n_bu_{\parallel b2}/\langle n_b\rangle\end{bmatrix}$$

In Eqs.(1)-(2), the energy scattering coefficients e_{ij}^{ab} and the friction coefficients l_{ij}^{ab} are calculated by the coefficients $M_{ab}^{ij}, N_{ab}^{ij}, P_{ab}^{ij}$ and Q_{ab}^{ij} which are listed in Eqs.(4.8)-(4.17),(5.21),(5.22) and (6.6)-(6.12) in Ref.[6]. In contrast to the flux surface averaged part of the momentum balance in Ref [5] using the 13M approximation, we included the Laguerre order of j=2 following the tokamak P-S transport theory [6]. An important purpose of this approximation is to include the energy scattering collision effects [6] and also the E×B drift effect in Eq.(1), which is peculiar to non-symmetric toroidal configurations. These effects were not included previous formulations for the multi-ion-species plasmas in Refs.[8,9] based on the 13M approximation. Eq.(1) includes $u_{//}^{(\mathrm{rigid})}\mathbf{b}\bullet\nabla$ corresponding to the parallel velocity of the moving frame in which the adiabatic invariant $\mu\equiv m_a v_\perp^2/2B$ and parallel particle velocity $v_\parallel=\pm v(1-\mu B/w)^{1/2}$, where $w\equiv m_a v^2/2$, are defined [10,11]. In general non-symmetric toroidal plasmas this velocity is given by

$$u_{\parallel}^{(\mathrm{rigid})}\equiv\frac{BcE_s}{2\chi'\psi'}\left(\frac{\psi'B_\zeta-\chi'B_\theta}{\langle B^2\rangle}+\frac{V'}{4\pi^2}H_2\right)\quad.\tag{3}$$

Here, H_2 is a constant on the flux surface used in Ref.[12]. This term vanish in symmetric configurations as follows and thus the present theory automatically includes the rigid rotation of the symmetric plasmas [4,6]. In symmetric configurations, there are only Fourier modes (m,n) of B and distribution functions satisfying

$$\frac{\chi'm+\psi'n}{\chi'm-\psi'n}=\mathrm{const}\equiv\frac{\chi'L+\psi'N}{\chi'L-\psi'N}\quad.\tag{4}$$

In these cases, H_2, $u_\parallel^{(\mathrm{rigid})}$, and $(u_\parallel^{(\mathrm{rigid})}\mathbf{b}+cE_s\nabla s\times\mathbf{B}/\langle B^2\rangle)\bullet\nabla$ become

$$H_2^{(\mathrm{symmetric})}=\frac{\chi'L+\psi'N}{\chi'L-\psi'N},\qquad u_\parallel^{(\mathrm{rigid})}=\frac{BcE_s}{\langle B^2\rangle}\frac{B_\zeta L+B_\theta N}{\chi'L-\psi'N}$$

$$\left(u_\parallel^{(\mathrm{rigid})}\mathbf{b}+cE_s\frac{\nabla s\times\mathbf{B}}{\langle B^2\rangle}\right)\bullet\nabla=\frac{cE_s}{\langle B^2\rangle\sqrt{g}}\left[\begin{array}{l}\frac{B_\zeta L+B_\theta N}{\chi'L-\psi'N}\left(\chi'\frac{\partial}{\partial\theta}+\psi'\frac{\partial}{\partial\zeta}\right)\\ \qquad-\left(B_\zeta\frac{\partial}{\partial\theta}-B_\theta\frac{\partial}{\partial\zeta}\right)\end{array}\right]\tag{5}$$

This operator vanishes for the "symmetric" Fourier components of $\propto\cos(m\theta-n\zeta)$ and $\propto\sin(m\theta-n\zeta)$ satisfying Eq.(4).

In Eq.(2), we do not include the poloidally and toroidally varying part of the parallel electric field determined by the charge neutrality [13]. A reason of it is that we cannot forbid an existence of l=0, $j\geq 3$ components in the electron distribution functions and in the electron force balance since we assume here cases with sufficiently high electron temperatures [1-3] giving long mean free paths of the electrons ($\tau_{ee}v_{Te}>>L_c$) even if the collisionality of the ions including the impurities may correspond to the P-S regime. When we allow the existence of l=0, $j\geq 3$ components in the electron distribution, the problem described by Eqs.(1)-(3) is not closed. In these cases with sufficiently electron temperatures, however, we can close this problem by the charge neutrality without the parallel electric field instead

author's e-mail: nishimura.shin@lhd.nifs.ac.jp

of including the higher order Laguerre terms. Although this procedure for the moment closure will be reported in a separated article, the conclusion of it is only replacing the friction coefficients $l_{3j}^{ea}=l_{j3}^{ae}$ in Eq.(2) by other newly defined friction coefficients $\lambda_{3j}^{ea}=\lambda_{j3}^{ae}$. In Eq.(1), the resulting P-S current under this charge neutrality, which should be a function only of total pressure gradient, is independent of the density and temperature perturbations δn_a, δT_a of individual species. By the momentum conservation of the friction forces, the total pressure perturbation given by Eq.(2) satisfies $\sum_a p_{a1}^{\mathrm{PS}}=\sum_a\left[\langle T_a\rangle n_{a1}^{(j=0)}+\langle n_a\rangle T_{a1}^{(j=1)}\right]=0$ (the total pressure is a flux surface quantity).

3. Numerical Examples

The problem described by Eqs.(1)-(3) including the replacement of $l_{3j}^{ea}=l_{j3}^{ae}$ by $\lambda_{3j}^{ea}=\lambda_{j3}^{ae}$ can be solved by a Fourier expansion method in the Boozer coordinates. In this section, we show a numerical example of the solution. By solving this problem, we can obtain the P-S diffusion coefficients defined by [5, 9],

$$\begin{bmatrix}\Gamma_a^{\mathrm{PS}}\\ q_a^{\mathrm{PS}}/T_a\end{bmatrix}=-\frac{c}{e_a}\begin{bmatrix}\langle\widetilde{U}F_{\|a1}\rangle\\ \langle\widetilde{U}F_{\|a2}\rangle\end{bmatrix}\equiv\sum_b\begin{bmatrix}(L^{\mathrm{PS}})_{11}^{ab} & (L^{\mathrm{PS}})_{12}^{ab}\\ (L^{\mathrm{PS}})_{21}^{ab} & (L^{\mathrm{PS}})_{22}^{ab}\end{bmatrix}\begin{bmatrix}X_{b1}\\ X_{b2}\end{bmatrix}. \quad (6)$$

The function $\widetilde{U}(\theta,\zeta)$ is defined in Appendix A in Ref.[5], and thermodynamic forces corresponding to radial gradients of the pressures and temperatures X_{a1}, X_{a2} also are defined in Eq.(10) in Ref.[5]. The Onsager symmetry $(L^{\mathrm{PS}})_{ji}^{ba}=(L^{\mathrm{PS}})_{ij}^{ab}$ is satisfied by the symmetric relations of the collision coefficients $e_{ji}^{ba}=e_{ij}^{ab}$ and $l_{ji}^{ba}=l_{ij}^{ab}$ [6], and the intrinsic ambipolar condition of Γ_a^{PS} also is satisfied by the momentum conservation of the friction. Because of the stellarator symmetry $B(-\theta,-\zeta)=B(\theta,\zeta)$, these coefficients are even functions of the radial electric field strength E_r. Figure.1 shows an example of the results. In this example, following Refs.[5,12], the magnetic field assumed there is that with $B=B_0[1-\varepsilon_{\mathrm{t}}\cos\theta_{\mathrm{B}}+\varepsilon_{\mathrm{h}}\cos(L\theta_{\mathrm{B}}-N\zeta_{\mathrm{B}})]$, $L=2$, $N=10$, $B_0=1\mathrm{T}$, $\chi'=0.15\mathrm{T\cdot m}$, $\psi'=0.4\mathrm{T\cdot m}$, $B_\theta=0$, and $B_\zeta=4\mathrm{T\cdot m}$. The contained ion assumed here is a mixture of protons (H^+) and fully ionized neon (Ne^{10+}), which is used for the charge exchange spectroscopic measurements and the impurity transport studies in the Large Helical Device (LHD) [3], with an ion density ratio corresponding to $Z_{\mathrm{eff}}=5.74$, and the assumed temperatures are $T_{\mathrm{e}}=T_{\mathrm{i}}=1\mathrm{keV}$. With these assumptions, a dependence of the diffusion coefficients on the density in a range of $n_{\mathrm{e}}\leq 5\times 10^{20}\mathrm{cm}^{-3}$ (up to the "SDC" [2] density regime) is investigated here. The mean free path of electron-electron collision is $v_{\mathrm{Te}}\tau_{\mathrm{ee}}=28.3\mathrm{m}$ corresponding to the plateau regime even at $n_{\mathrm{e}}=5\times 10^{20}\mathrm{cm}^{-3}$.

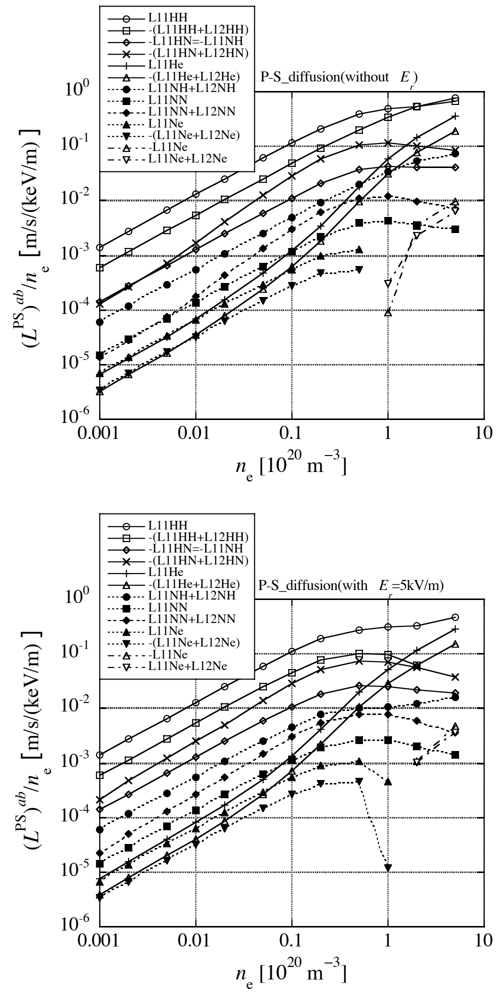

Fig.1 The impurity (Ne^{10+}) diffusion coefficients in (a) cases with energy scattering collision effects e_{ij}^{ab} and without the radial electric filed (E_r=0), (b) cases with both of the energy scattering collisions and a finite radial electric field of E_r =5kV/m.
The collisionless limit of $n_{\mathrm{e}}=10^{17}\mathrm{m}^{-3}$ in (a) coincides with the 13M approximation [8,9] without both effects. The particle species are denoted by e (a,b=e$^-$), H (a,b=H$^+$), and N (a,b=Ne^{10+}) in these figures.

Since Eq.(6) includes full non-diagonal coupling terms between particles species, there are 21 P-S diffusion coefficients even in this simple 2-ion-species model. We show here only coefficients relating to ion particle diffusions (i.e., a=H$^+$, Ne^{10+} and b=e$^-$, H$^+$, Ne^{10+}), since a main application area of the P-S transport is the impurity transport studies in high-density operations. It is well known that neoclassical theory without the temperature gradient terms predicts a "pessimistic" impurity accumulation but the temperature gradient terms prevent it. Although the final goal of the impurity transport studies is determining the steady-state impurity density profile including not only Γ_a^{PS} but also the banana-plateau and ripple diffusion fluxes Γ_a^{bn} and the

author's e-mail: nishimura.shin@lhd.nifs.ac.jp

classical diffusion fluxes $\Gamma_a^{\rm cl}$, such a calculation is complicated. We show here only comparison of the density gradient terms $(L^{\rm PS})_{11}^{ab}$ and the temperature gradient terms $(L^{\rm PS})_{11}^{ab}+(L^{\rm PS})_{12}^{ab}$ in $\Gamma_a^{\rm PS}$ as the component tests. In Fig.1(a) without the radial electric field (E_r=0), we can see the effects of the energy scattering collisions by deviations from pure $\propto n_{\rm e}$ scaling given by the 13M-moment approximation in Refs.[8,9]. It enlarges the negative value of (L11HN+L12HN) and positive value of (L11NH+L12NH). It also reduces the positive value of L11HH at $n_{\rm e}>10^{20}{\rm m}^{-3}$. These are favorable tendencies in viewpoint of the impurity control. We can see also an increase and an invert of ion diffusions driven by electron temperature gradient in this high-density range, which indicate an importance of the electron temperature controls in for the impurity controls. The finite radial electric field effects with E_r=5keV/m on these diffusion coefficients are only changes of order of unity as in Fig.1 (b) since Eqs.(1)-(3) make the P-S current independent of Er. Because of this characteristic the relative flow velocities between particle species $u_{//a}-u_{//b}$ determining the friction forces $F_{//a1}$ are insensitive even when the absolute values of the flow velocities of individual species are largely changed by the E×B term in Eq.(1). Nevertheless, there are non-negligible effects in view point of impurity density profiles determined by balances of temperature gradient terms and the density gradient terms. The aforementioned increases of (L11HN+L12HN) and (L11NH+L12NH) are enhanced by a radial electric field effect at ne~10^17m-3. The radial electric field effect at $n_{\rm e}>10^{20}{\rm m}^{-3}$ is complicated. These coefficients are reduced by the radial electric field at $n_{\rm e}>10^{20}{\rm m}^{-3}$ and thus the ion temperature gradient is not effective for the impurity control in this high density limit with finite radial electric fields. In this condition, controls of density and temperature profiles of the electron will be more important.

4. Concluding Remarks

By adding this poloidally and toroidally varying part of the momentum equations to a previously formulation handling the flux-surface averaged part [5], the development of the "non-symmetric version of NCLASS" [12] based on the concept of "stellarator moment equation approach" toward a future integrated simulation system [14] is almost completed in viewpoint of the "basic framework". (Although there are still many remaining technical problems on the viscosity coefficients in the flux-surface averaged part, this topic is discussed in another presentation in this conference P2-017). In non-symmetric configurations, a dependence of the P-S diffusions $\Gamma_a^{\rm PS}$, $q_a^{\rm PS}$ and the accompanied δn_a, δT_a on the E_r is predicted. Since a cause of this dependence on the E_r is the viscous damping of the "rigid rotation", it also should be noted that the basic idea of this theory is applicable to tokamaks with the rotation damping due to the symmetry-breaking by MHD activities, and so on [15]. It also should be noted that the plasma rotation assumed in Eq.(3), resulting density perturbations in Eqs.(1)-(2), and the assumed electro-static potential being a flux surface quantity are consistent with previous experimental results.[16-17]

author's e-mail: nishimura.shin@lhd.nifs.ac.jp

Acknowledgements

The authors thank Drs. Noriyoshi Nakajima, Kimitaka Itoh, Katsumi Ida, Mikirou Yoshinuma, Dave Mikkelsen, Mike Zarnstorff, Don Spong, Steve Hirshman, Masa Ono, Masayoshi Taguchi, and Rainer Burhenn for suggestions and discussions relating to this topic. The National Institute for Fusion Science (NIFS) coordinated research programs NIFS07KNXN103 and NIFS06KUHL007 also support this work.

References

[1] R.Burhenn, et al., Fusion Sci.Technol. **46**, 115 (2004)
[2] N.Ohyabu, T.Marisaki, S.Masuzaki, et al., Phys.Rev.Lett.**97**, 055002 (2006)
[3] Y.Nakamura, Y.Takeiri, R.Kumazawa, et al, Nucl.Fusion **43**, 219 (2003)
[4] W.A.Houlberg, K.C.Shaing, S.P.Hirshman, and M.C.Zarnstorff, Phys.Plasmas **4**, 3230 (1997)
[5] H.Sugama and S.Nishimura, Phys.Plasmas **9**, 4637 (2002)
[6] S.P.Hirshman and D.J.Sigmar, Nucl.Fusion **21**,1079 (1981)
[7] R.D.Hazeltine and J.D.Meiss, *Plasma Confinement* (Addison-Wesley, CA, 1992), p.196
[8] M.Taguchi, Phys.Fluids B **4**, 3638 (1992)
[9] H.Sugama and W.Horton, Phys.Plasmas **3**, 304 (1996)
[10] R.D.Hazeltine and A.A.Ware, Plasma Phys. **20**, 673(1978); R.D.Hazeltine and F.L.Hinton, Phys.Plasmas **12**, 102506 (2005)
[11] F.L.Hinton and S.K.Wong, Phys.Fluids **28**, 3082 (1985); H.Sugama and W.Horton, Phys.Plasmas **4**, 2215 (1997)
[12] S.Nishimura, H.Sugama, and Y.Nakamura, Fusion Sci.Technol.**51**, 61 (2007)
[13] R.D.Hazeltine and F.L.Hinton, Phys.Fluids **16**,1883 (1973)
[14] Y.Nakamura, M.Yokoyama, N.Nakajima, et al., Fusion Sci.Technol. **50**, 457 (2006)
[15] W.Zhu, S.A.Sabbagh, R.E.Bell, et al., Phys.Rev.Lett.**96**, 22002 (2006)
[16] K.Ida, T.Minami, et al., Phys.Rev.Lett. **86**, 3040 (2001); M.Yoshinuma, K.Ida, et al., in 16th International Stellarator Workshop (Toki, Japan, Oct.15-19, 2007)
[17] S.Nishimura, K.Ida, M.Osakabe, et al., Phys.Plasmas **7**, 437 (2000)
[18] K.Ida, A.Fujisawa, H.Iguchi, et al.,Phys.Plasmas **8**,1(2001)