Monte-Carlo Simulation of Neoclassical Transport in Magnetic Islands and Ergodic Region

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It is shown in Large Helical Device experiments that the transport modeling based only on the fluid description is not sufficient for expressing edge transport phenomena in a magnetic island. Existence of a bootstrap current around the island is strongly suggested. On the other hand, in recent tokamak experiments it is found that so-called stochastic diffusion theory based on the "field line diffusion" over estimates the radial energy transport in the collisionless edge plasma affected by resonant magnetic perturbations, though the perturbations induce a chaotic behavior in the field lines. These results imply that the conventional modeling of the edge transport should be reconsidered for a lower-collisionality case. In order to take a new look at the modeling of the edge transport, we investigate neoclassical effect on the transport in magnetic islands and ergodic region. By using a drift kinetic equation solver without an assumption of existence of nested flux surfaces (the KEATS code), it is possible for us to execute the investigation. The simulation results show that the radial energy flux of ions for a lower-collisionality case is quite small compared to the prediction of the stochastic diffusion theory, while the flux for a high-collisionality case is consistent with the prediction.

Keywords: neoclassical transport, edge plasma, magnetic island, ergodic region, Monte-Carlo simulation

1 Introduction

It is shown in Large Helical Device (LHD) experiments that the transport modeling based only on the fluid description is not sufficient for expressing edge transport phenomena in a magnetic island [1, 2, 3]. This result is given in the experiments of observing the healing of the $m/n = 1/1$ magnetic island in the edge, where m and n are the poloidal and toroidal mode numbers, respectively. A current depending on the pressure gradient (i.e. the bootstrap or Pfirsch-Schlüter current) is expected to explain the healing in the experiments. In results of a simulation study based on the fluid description [4], the healing phenomenon is not explained by the Pfirsch-Schlüter current only. The important role of a bootstrap current in the edge region is strongly suggested, and thus the kinetic modeling of the edge plasma is needed for understanding of the edge transport phenomena, where in the LHD experiments the temperature is $\gtrsim$500 eV and the plasma density $\sim 10^{19}$ m^{-3} in the island.

On the other hand, in recent tokamak experiments it is found that so-called stochastic diffusion theory based on the "field line diffusion" [5, 6] over estimates the radial energy transport in the edge region added resonant magnetic perturbations (RMPs) [7, 8]. This fact is discovered in the experiments of ELMs (edge localized modes) suppression by means of RMPs in collisionless tokamak plasmas [7, 8]. (Historically, the idea of suppressing the ELMs and controlling the edge transport by using RMPs has been proposed about 20 years ago [9].) When the RMPs induce a chaotic behavior in the field lines, the stochastic diffusion theory predicts that a thermal diffusivity is given as $\chi^a_{\rm ql} = \chi^a_{\parallel}|\delta B_{\rm r}/B_{\rm t}|^2$ or $v^a_{\rm th}\pi R_{\rm ax} q|\delta B_{\rm r}/B_{\rm t}|^2$ for the collisional or collisionless limit, where a is a particle species, $\chi^a_{\parallel} = 3.91 T_a\tau_a/m_a$ the parallel diffusivity, T_a the temperature, τ_a the collision time, m_a the particle mass, $v^a_{\rm th}$ the thermal velocity, $R_{\rm ax}$ the major radius of the magnetic axis, q the safety factor, $\delta B_{\rm r}$ the strength of RMPs, and $B_{\rm t}$ the toroidal component of the magnetic field. This prediction has been demonstrated in experiments on high-collisional tokamak plasmas [10]. However, in collisionless plasmas, the experimental thermal diffusivity $\chi_{\rm ex}$ is inconsistent with the prediction of the stochastic diffusion theory, e.g. $\chi^{\rm e}_{\rm ql}/\chi^{\rm e}_{\rm ex} \gg 10$ for the electron thermal diffusivity [8]. Small RMPs cause the complete suppression of the ELM events, and have a negligible effect on the energy confinement.

The above experimental results in torus plasmas imply that the conventional modeling of the edge transport should be reconsidered for a lower-collisionality case. There is no established theory describing radial transport in magnetic islands and ergodic region. In order to take a new look at the modeling of the edge transport, we investigate

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neoclassical effect on the transport in magnetic islands and ergodic region. Here, even in the three dimensional field line structure disturbed by RMPs, the Coulomb collision causes the transition between a passing particle orbit and a trapped particle orbit in toroidal and helical ripples (localized and/or blocked particle orbits) [11]; in the present paper we call it the neoclassical effect on the edge transport phenomena. Recently, we develop a new transport simulation code without an assumption of existence of nested flux surfaces; the code is named "KEATS" [12, 13]. The code is programmed by expanding a well-known Monte-Carlo particle simulation scheme based on the δf method [14, 15]. By using the KEATS code, it is possible for us to execute the investigation. In this paper we show the simulation results, applying the code to a torus plasma having the ergodic region in the edge.

2 Simulation Model

We consider that a guiding center distribution function of plasma f is separated into an equilibrium-like background f_0 and a kinetic part δf of the distribution, $f = f_0 + \delta f$, where the kinetic part δf is considered as a small perturbation from f_0. The zeroth-order distribution function f_0 is given as a local Maxwellian distribution $f_0 = f_{\rm M}(\boldsymbol{x}, \xi, v) = n\{m/(2\pi T)\}^{3/2} \exp\{-mv^2/(2T)\}$, where $\xi = v_{\parallel}/v$ is the cosine of the pitch angle, $v_{\parallel} = \boldsymbol{v} \cdot \boldsymbol{b}$, $\boldsymbol{b} = \boldsymbol{B}/B$ the unit vector along a field line, $\boldsymbol{B}$ a magnetic field, $B = |\boldsymbol{B}|$, $v = |\boldsymbol{v}|$, $n = n(\boldsymbol{x})$ the density, m the particle mass, and $T = T(\boldsymbol{x})$ the temperature. Applying the decomposition $f = f_{\rm M} + \delta f$ to the drift kinetic equation, we have the following equation of the kinetic part δf:

$$\frac{\rm D}{{\rm D}t}\delta f = -\left\{\boldsymbol{v}_{\rm d} \cdot \nabla f_{\rm M} - C_{\rm F} f_{\rm M}\right\}, \tag{1}$$

where the operator ${\rm D}/{\rm D}t$ is defined as ${\rm D}/{\rm D}t := \partial/\partial t + (\boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\rm d}) \cdot \nabla - C_{\rm T}$, $\boldsymbol{v}_{\parallel} = v_{\parallel}\boldsymbol{b}$ the parallel velocity, and $\boldsymbol{v}_{\rm d}$ the drift velocity of guiding center motion. The test particle collision operator $C_{\rm T}$ is given, for simplicity, as

$$C_{\rm T} = \frac{\nu_{\rm def}}{2}\frac{\partial}{\partial \xi}\left(1 - \xi^2\right)\frac{\partial}{\partial \xi}, \tag{2}$$

and it can be implemented numerically by random kicks in velocity space [16], which represents the Coulomb scattering process, where $\nu_{\rm def}$ is the deflection frequency. Here, we should note statistical accuracy of the operator $C_{\rm T}$ expressed by the Monte-Carlo method [16], in particular, around $|\xi| \approx 1$. The operator $C_{\rm F}$ is the field particle collision term, which represents local momentum conservation ($C_{\rm F}$ is needed to treat accurately the parallel transport):

$$C_{\rm F} = \nu_{\rm def}\frac{m}{T}\boldsymbol{v} \cdot \boldsymbol{u}_0, \tag{3}$$

and $\boldsymbol{u}_0$ is given as

$$\boldsymbol{u}_0 = \int {\rm d}^3v\, \nu_{\rm def}\, \boldsymbol{v}\, \delta f \bigg/ \int {\rm d}^3v\, \nu_{\rm def} \frac{mv^2}{3T} f_{\rm M}. \tag{4}$$

In general, effects of neutrals and an electric field are important in the edge transport, but in the present paper these effects are neglected for simplicity. (The modeling of a fluctuating field in the KEATS code is described in Refs. [13, 17].)

To solve Eq. (1) by Monte-Carlo techniques, we adopt the two-weight scheme of the δf formulation [14, 15]. In evolution of the δf part, the background $f_{\rm M}$ is assumed to be fixed because the background is in a quasi steady-state from the viewpoint of the δf part. The Monte-Carlo simulation code, KEATS, is programmed in an Eulerian coordinate system, i.e. so-called helical coordinates [4], thus the code does not need magnetic flux coordinates. Simulation results (e.g. estimation of particle and energy fluxes) of the KEATS code for a case of a simple tokamak field are agreed with ones of the "FORTEC-3D" code [15] which uses magnetic flux coordinates.

3 Simulation Results

For the investigation of neoclassical effect on the transport in the ergodic region, we use a magnetic configuration which is formed by adding RMPs into a simple tokamak field having concentric circular flux surfaces, where the major radius of the magnetic axis $R_{\rm ax} = 3.6$ m, the minor radius of the plasma $a = 1$ m, and the magnetic field strength on the axis $B_{\rm ax} = 4$ T. The Poincaré plots of the magnetic field lines on a poloidal cross section are shown in Fig.1. One can see the ergodic region in $r/a = 0.7 \sim 1$, where $r = \sqrt{(R - R_{\rm ax})^2 + Z^2}$. In the KEATS code, the number of test particles is $N_{\rm TP} = 16{,}000{,}000$.

To investigate effect of the existence of the ergodic region on the transport phenomena, we evaluate the ion energy flux $\boldsymbol{Q}_{\rm i}$ in two cases, i.e., in the configurations (a) having lower edge temperature $T_{\rm edge} \sim 200$ eV at a center of the ergodic region and (b) having higher edge temperature $T_{\rm edge} \sim 1$ keV. The temperature profile is given as $T_{\rm i} = T_{\rm ax}\{0.02 + 0.98 \exp[-4(r/a)^{\alpha}]\}$ with $T_{\rm ax} = 2$ keV and $\alpha = 2.5$ (case (a)) or 7.86 (case (b)), which neglects the existence of the ergodic region. The density profile is set homogeneous, $n_{\rm i} = {\rm const.} = 1 \times 10^{19}\ {\rm m}^{-3}$. The background $f_{\rm M}$ is fixed in the calculations. The radial profiles of the energy flux estimated from the KEATS computations for two cases (a) and (b) are shown in Figs.2a and b, respectively. For simplicity, the radial energy fluxes are given neglecting the existence of the ergodic region, because we have no magnetic coordinate system including several magnetic field structures as the core and ergodic regions. The energy flux $\boldsymbol{Q}_{\rm i}$ is averaged over concentric circular shell region in the whole toroidal angles as if there were nested flux surfaces. Here, in the KEATS computations the energy flux $\boldsymbol{Q}$ is given as

$$\boldsymbol{Q}(t, \boldsymbol{x}) = \overline{\int {\rm d}^3v\, \frac{mv^2}{2}(\boldsymbol{v}_{\parallel} + \boldsymbol{v}_{\rm d})\delta f}, \tag{5}$$

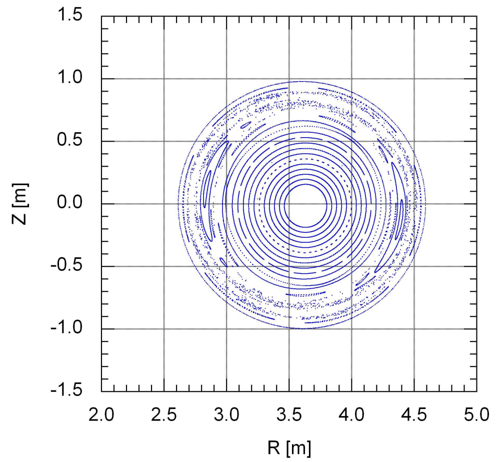

Fig. 1 Poincaré plots of magnetic field lines on a poloidal cross section.

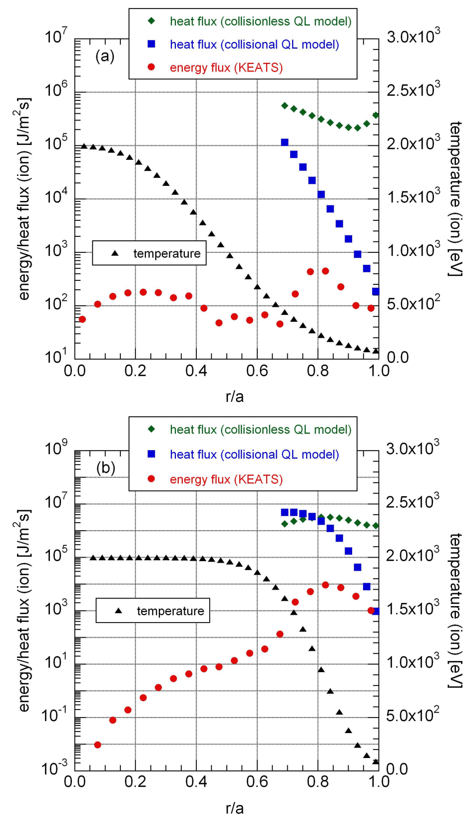

Fig. 2 Radial profiles of ion energy/heat flux for (a) lower and (b) higher edge temperatures, where $r = \sqrt{(R - R_{\rm ax})^2 + Z^2}$. The collisionless (or collisional) quasi-linear (QL) model means the prediction of the stochastic diffusion theory for the collisionless (or collisional) case.

where $\overline{\cdots}$ means the time-average, and the averaging time is longer than the typical time-scale of δf. The heat flux predicted by the stochastic diffusion theory is given as $\boldsymbol{q}_{\rm ql} = n\chi_{\rm ql}\nabla T$. From the results of the KEATS code, we find that the ion energy flux is affected by the RMPs, but the flux in the lower-collisionality region around $r/a \approx 0.8$ is quite small compared with the prediction of the stochastic diffusion theory, shown in both cases (a) and (b). On the other hand, in the high-collisionality region around $r/a \approx 1$, the flux is consistent with the prediction. These results can be explained theoretically, see Appendix.

4 Summary

We have been developing the neoclassical transport code, KEATS, to study the transport phenomena in the islands and ergodic region. We apply the code to the edge disturbed by resonant magnetic perturbations, and find that the ion energy flux estimated by the KEATS code for a lower-collisionality case is quite small compared to the prediction of the stochastic diffusion theory based on the "field line diffusion," while the flux for a high-collisionality case is consistent with the prediction.

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Appendix. Stochastic Analysis of Radial Transport in a Perturbed Field

A fluid equation in a steady-state corresponds to a stochastic differential equation described as $\mathrm{d}X_t^i = \gamma U^i(\boldsymbol{X}_t)\mathrm{d}t + c_j^i(\boldsymbol{X}_t)\mathrm{d}W_t^j$ and $i, j = 1, 2, 3$ [18, 19], where γ is a constant (e.g. $\gamma = 5n/2$ for the heat balance equation if n = const.), $\boldsymbol{U} = (U^1, U^2, U^3)$ a flow in a steady-state, $D^{ij} = c_k^i g^{k\ell} c_\ell^j$ a diffusion coefficient, $g^{k\ell}$ a metric coefficient, $\boldsymbol{X}_t = (X_t^1, X_t^2, X_t^3)$ a diffusion process, and $\boldsymbol{W}_t = (W_t^1, W_t^2, W_t^3)$ a Brownian process. It is assumed that a fluid is exposed to noise caused by resonant magnetic perturbations (RMPs), and that a fluid particle motion is described by an Itô process $\mathrm{d}Y_t^i = \gamma\tilde{U}^i(t,\omega)\mathrm{d}t + c_j^i(\boldsymbol{Y}_t)\mathrm{d}W_t^j$ instead of the process $\boldsymbol{X}_t$, where the flow is represented as $\tilde{\boldsymbol{U}}(t,\omega) = \boldsymbol{U}(\boldsymbol{Y}_t)$ + "noise", ω is a label of a fluid particle, "noise" is a random function having zero mean and finite strength, and the definition of an Itô process is given in Ref. [20]. It is known that an Itô process $\boldsymbol{Y}_t$ coincides in law with a diffusion process $\boldsymbol{X}_t$ if and only if $\mathrm{E}^{\boldsymbol{x}_0}[\tilde{\boldsymbol{U}}(t,\omega)|\mathcal{P}_t^Y] = \boldsymbol{U}(\boldsymbol{Y}_t)$ [20], where $\boldsymbol{X}_0 = \boldsymbol{Y}_0 = \boldsymbol{x}_0$ is a starting point of a fluid particle at $t = 0$, $\mathcal{P}_t^Y$ is the σ-algebra generated by the set $\{\boldsymbol{Y}_s; 0 \le s \le t\}$, and $\mathrm{E}^{\boldsymbol{x}_0}[\cdots|\mathcal{P}_t^Y]$ denotes the conditional expectation with respect to $\mathcal{P}_t^Y$. This theorem means that the "noise" cannot cause the diffusion of fluid particles. We should reconsider the reason why the noise created by RMPs can affect the transport.

Let us take the following collision operator:

$$C(f) = \nu_{\mathrm{col}} \frac{\partial}{\partial \boldsymbol{u}} \cdot \left\{ \boldsymbol{u} f + v_{\mathrm{th}}^2 \frac{\partial f}{\partial \boldsymbol{u}} \right\}, \tag{A.1}$$

where $\nu_{\mathrm{col}} = \nu_{\mathrm{col}}(\boldsymbol{x})$ is the collision frequency, v_{th} the thermal velocity, and $\boldsymbol{v} = \boldsymbol{U} + \boldsymbol{u}$ the velocity of a guiding center, and $\boldsymbol{U} = \boldsymbol{U}(\boldsymbol{x})$ the mean velocity [21]. The operator (A.1) is simpler, but is used only to get a rough idea of collisional effects [22]. We consider the motion of a guiding center along a field line for estimation of radial spreading the guiding centers by their parallel motions in a perturbed magnetic field. The guiding center motion exposed to the collisions (A.1) is given as an Ornstein-Uhlenbeck process:

$$\mathrm{d}\boldsymbol{x} = \boldsymbol{v}\mathrm{d}t = (\boldsymbol{U} + \boldsymbol{u})\mathrm{d}t, \tag{A.2}$$

$$\mathrm{d}\boldsymbol{u} = -\nu_{\mathrm{col}} \boldsymbol{u} \mathrm{d}t + \sigma \mathrm{d}\boldsymbol{W}_{\|t}, \tag{A.3}$$

where $\boldsymbol{U} = U_{\|}\boldsymbol{b}$, $\boldsymbol{u} = u_{\|}\boldsymbol{b}$, $\sigma = v_{\mathrm{th}}\sqrt{\nu_{\mathrm{col}}}$, $\boldsymbol{W}_{\|t}$ a Brownian process for the parallel direction, i.e. $\mathrm{d}\boldsymbol{W}_{\|t} = \boldsymbol{b}\mathrm{d}W_t$, and $\boldsymbol{b} = \boldsymbol{B}/B$ the unit vector along a field line. Here, the effects of ripples are neglected for simplicity. The solutions of Eqs. (A.2) and (A.3) are described respectively as

$$\boldsymbol{x} = \boldsymbol{x}_0 + \int_0^t (\boldsymbol{U} + \boldsymbol{u})\mathrm{d}s, \tag{A.4}$$

$$\boldsymbol{u} = \mathrm{e}^{-\nu_{\mathrm{col}} t} \boldsymbol{u}_0 + \int_0^t \mathrm{e}^{-\nu_{\mathrm{col}}(t-s)} \sigma \mathrm{d}\boldsymbol{W}_{\|s}, \tag{A.5}$$

where $\boldsymbol{x}_0$ and $\boldsymbol{u}_0$ are the initial values at $t = 0$.

One may consider that effect of a perturbation field on the motion is interpreted as noise on the motion. If the effect is expressed by a linear operator $\tilde{\boldsymbol{v}} = \tilde{N}\boldsymbol{v}$, then

$$\mathrm{d}\boldsymbol{x} = (\boldsymbol{v} + \tilde{\boldsymbol{v}})\mathrm{d}t = (\boldsymbol{v} + \tilde{N}\boldsymbol{v})\mathrm{d}t. \tag{A.6}$$

The solution of Eq. (A.6) is given as

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{x}_0 + \int_0^t (1 + \tilde{N}) \left\{ \boldsymbol{U} + \mathrm{e}^{-\nu_{\mathrm{col}} s} \boldsymbol{u}_0 \right\} \mathrm{d}s \\ &\quad + \int_0^t \mathrm{d}s \int_0^s \mathrm{e}^{-\nu_{\mathrm{col}}(s-h)} \sigma (1 + \tilde{N}) \mathrm{d}\boldsymbol{W}_{\|h}. \end{aligned} \tag{A.7}$$

For the collisional limit $t \gg 1/\nu_{\mathrm{col}}$, the diffusion (caused by the perturbation field) in configuration space is derived from Eq. (A.7):

$$\mathrm{d}\boldsymbol{x} \approx (1 + \tilde{N})\boldsymbol{U}\mathrm{d}t + \frac{v_{\mathrm{th}}}{\sqrt{\nu_{\mathrm{col}}}} (1 + \tilde{N}) \mathrm{d}\boldsymbol{W}_{\|t}, \tag{A.8}$$

i.e., for the collisional limit the diffusion in velocity space directly becomes the diffusion in configuration space. We should note that the diffusion in configuration space originates from the collisions in velocity space. When the RMPs are added to the original magnetic field having nested flux surfaces, the parallel motion of a guiding center may cause radial fluctuation in configuration space [5, 6]. If the noise $\tilde{\boldsymbol{v}} = (\tilde{v}^1, \tilde{v}^2, \tilde{v}^3)$ is given as

$$\tilde{v}^i = (\tilde{N}\boldsymbol{v})^i = \left| \frac{\delta B_{\mathrm{r}}}{B_{\mathrm{t}}} \right| \frac{\varepsilon^{ijk}}{\sqrt{g}} \hat{\theta}_j v_k \tilde{\phi}(t, i), \tag{A.9}$$

then the radial diffusivity $D_{\mathrm{r}} = D_{\|}|\delta B_{\mathrm{r}}/B_{\mathrm{t}}|^2$ is obtained in the fluid equations given from the drift kinetic equation having the collisions (A.1) for the collisional limit, where $\hat{\boldsymbol{\theta}}$ is the unit vector for the poloidal direction, δB_{r} the strength of the RMPs satisfying $|\delta B_{\mathrm{r}}/B_{\mathrm{t}}| \ll 1$, B_{t} the toroidal component of $\boldsymbol{B}$, $g = \det\left(g_{ij}\right)$ the square of Jacobian, ε^{ijk} the Levi-Civita symbol, $\tilde{\phi}(t, i)$ the ith component of a zero mean random vector having the mean square of $\mathrm{E}\left[\tilde{\phi}^2\right] = 1$ and being independent of $\mathrm{d}\boldsymbol{W}_{\|t}$, and $D_{\|} = v_{\mathrm{th}}^2/\nu_{\mathrm{col}}$ the parallel diffusivity. We should note that the noise term $\tilde{N}(\boldsymbol{U}+\boldsymbol{u}_0)$ cannot cause diffusion, as shown in the first paragraph in this section; see also Refs. [13, 17, 20].

The above discussion shows that for a high-collisional plasma (the characteristic time $t \gg 1/\nu_{\mathrm{col}}$), the guiding center motions become close to the prediction of the stochastic diffusion theory based on the "field line diffusion" [5, 6]. On the other hand, for a lower-collisionality plasma ($t \lesssim 1/\nu_{\mathrm{col}}$), the motions are not interpreted as the diffusion process predicted by the stochastic diffusion theory. These consequences are consistent with the results shown in Fig. 2 and also ones obtained in the test particle simulations [23].

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