Thermal Barrier Formation for Plasma Electrons and Ions in Kind of Connected Dip and Hump of Electrical Potential near ECR Points in Cylindrical Trap

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(Received day month year / Accepted day month year should be centered 10-point type)

The experimentally observed formation of localized solitary barriers for plasma electrons and ions near the electron cyclotron resonance points near the ends of the cylindrical trap is investigated analytically. A new method of confinement of plasma particles in cylindrical traps is proposed.

Keywords: barrier, ECR point, cylindrical trap, solitary perturbation, evolution equation.

1. Inroduction

In [1] the formation of thermal barrier of plasma particles was observed near the location of electron cyclotron resonance (ECR). In this paper, such thermal barriers and their behavior in plasma are investigated theoretically. Mechanisms of thermal barrier formation in the plasma near the two butt-ends of cylindrical magnetic traps are considered. These barriers can enhance confinement of the plasma electrons and ions in cylindrical magnetic traps. We consider general case the plasma with negative ions.

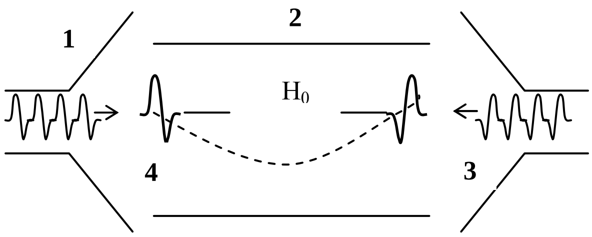

Fig. 1. Scheme of the thermal electrical barriers formation for plasma electrons and ions in ECR points on the edges of the magnetized cylindrical trap. Dotted line and symbol H(z) show the nonhomogeneous magnetic field. 1 is the qualitative scheme of the antenna for electromagnetic wave injection into the plasma trap; 2 is the cylindrical wall of the trap; 3 is the injected electromagnetic wave; 4 is the thermal barrier in kind of the dip and hump of the electric potential.

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2. Thermal barrier for plasma electrons

Consider two electromagnetic waves injected into a plasma trap through its butt-ends (see Fig. 1). Near the electron cyclotron resonance (ECR) points of the confining magnetic well, the energy of the waves are converted into that of the transverse motion of the electrons. We shall show that this energy conversion can be responsible for the observed [1] barriers of the plasma particles. A thermal barrier is a self-consistent structure consisting of a paired dip and hump of the electric potential (Fig. 1). The electron and ion phase spaces at the left barrier are shown in Figs. 2 and 3. The magnetic well H(z) has a minimum at the center of the trap, and it increases toward the edges of the trap. Near the ECR point the transverse electron velocity $V_\perp$ is increased up to $V_{\perp o}$ because of the resonant wave-particle interaction. The electrons are reflected by the magnetic mirror. Because of the inhomogeneous magnetic field H(z), the electron transverse velocity $V_\perp(z)=V_{\perp o}[H(z)/H_o]^{1/2}$, where H_o is the magnetic field at the ECR point, decreases in favor of its longitudinal velocity $V_{\|}(z)$ $V_{\|}(z)=V_{\perp o}(1-H(z)/H_o)^{1/2}$, which increases towards the center of the trap. This leads to an average electron flow towards the center, and thus an uncompensated increase of the electron density at the center of the trap.

The increase of the electron longitudinal velocity $V_{\|}$

with respect to that of the ions near the ECR point leads to a nonequilibrium state. The reflection of the electrons with nonequilibrium velocity distribution from the potential dip leads to a growth of the dip's amplitude. So these current-carrying electrons can excite the electric potential dip with amplitude φ_o on an ion mode with velocity V_c, close to zero, and are reflected from it.

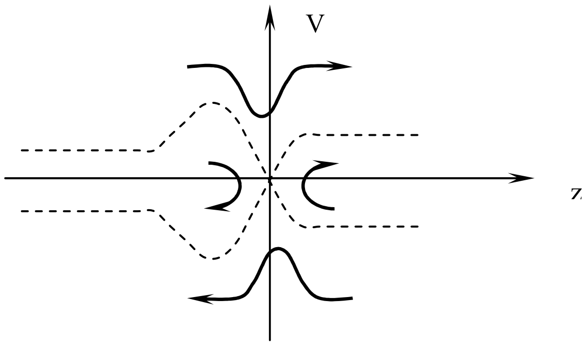

Fig. 2. Electron phase space. Dotted line is the separatrix, separating reflected and penetrated electrons. Arrows specify a direction of the electron movement.

From the electron Vlasov equation and ion hydrodynamic equations one can derive the evolution equation for the potential dip. In fact, we shall consider the slow evolution of the dip for its description. Taking into account that the resonant electrons are reflected from the dip, one can obtain from the Vlasov equation the expression for the steady state electron distribution function f_e

$$f_e=f_{oe}(-[V^2-2e(\varphi\pm\Delta\varphi)/m_e]^{1/2}\pm V_{||}), \qquad (1)$$

where f_{oe} is Maxwellian, $\Delta\varphi$ is the electric potential jump near the potential dip and has to be determined self-consistently. The plus and minus signs are for V greater and less than $A(\varphi)\mathrm{sign}(z)$, respectively. Here $A(\varphi)\equiv[2e(\varphi_o+\varphi)/m_e]^{1/2}$.

In the following we shall use the normalized quantities $\phi\equiv e\varphi/T_e$, $N_-\equiv n_{o-}/n_{o+}$, $N_e\equiv n_{oe}/n_{o+}$, $Q_\pm=q_\pm/e$, and $V_{s\pm}=(T_e/M_\pm)^{1/2}$. Furthermore, z is normalized by the electron Debye radius r_{de}, $V_{||}$ by the electron thermal velocity V_{the}, t by the inverse ion plasma frequency ω_{p+}^{-1}, and the speed V_c of the localized perturbation by the ion-acoustic velocity $(T_e/M_+)^{1/2}$. Here, T_e, is the temperature of electrons, n_{o-} and n_{o+} are the unperturbed densities of negative and positive ions, $q_\pm$ is the charge of positive and negative ions.

Integrating (1) on velocity, one can derive the electron density n_e in the first approximation on small $V_{||}$

$$n_e\approx n_{oe}\exp(\phi)[1-(2\Delta\phi/\sqrt{\pi})\beta_1-2V_{||}(2/\pi)^{1/2}\beta_2, \qquad (2)$$

$$\beta_1\equiv\int_o^\beta dx\,\exp(-x^2),\ \beta_2\equiv\int_o^\beta dx\,(x^2-\phi)^{1/2}\exp(-x^2)],$$

$$\beta\equiv(\phi_o+\phi)^{1/2}.$$

Far from the dip the plasma is quasineutral $n_e(z)\,|_{z\to\infty}=n_e(z)\,|_{z\to-\infty}=1-N_-$. Hence we derive, using (2), the expression for potential jump $\Delta\phi$ near the dip

$$\Delta\phi\approx V_{||}(2/\pi)^{1/2}(1-\exp(-\phi_o))/\beta_3, \qquad (3)$$

$$\beta_3\equiv[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\,\exp(-x^2)]$$

From hydrodynamic equations the expressions for densities of positive and negative ions can be obtained

$$n_{i\pm}=n_{\pm NL}+n_{\pm\tau},\ n_{\pm NL}=n_{o\pm}/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{1/2}, \qquad (4)$$

$$\partial n_{\pm\tau}/\partial z=\pm2(\partial\varphi/\partial t)(n_{o\pm}q_\pm/M_\pm V_c^3)\beta_4,$$

$$\beta_4\equiv[1-(\pm q_\pm)\varphi/M_\pm V_c^2]/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{3/2}$$

Substituting (2), (4) in Poisson's equation we can derive the nonlinear evolution equation

$$\partial^3{}_z\phi+\{\beta_5+\beta_6\}2\partial_t\phi/V_c^3+(\partial_z\phi/V_c^2)\{\beta_7+\beta_8\}-$$
$$-\{\exp(\phi)-\mathrm{sign}(z)V_{||}(2/\pi)^{1/2}\{\beta_9-$$
$$-\beta_{10}+(1-\exp(-\phi_o))\beta_{11}\beta_{12}/\sqrt{\pi}\}\}\partial_z\phi=0, \qquad (5)$$

$$\beta_5\equiv Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2}(1-\phi Q_+V^2_{s+}/V_c^2),$$

$$\beta_6\equiv Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}(1+\phi Q_-V^2_{s-}/V_c^2),$$

$$\beta_7\equiv Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2},$$

$$\beta_8\equiv Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2},$$

$$\beta_9\equiv(\phi_o/(\phi_o+\phi))^{1/2}\exp(-\phi_o),$$

$$\beta_{10}\equiv\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy(1-2y^2)\exp(-y^2)/(y^2+\phi)^{1/2},$$

$$\beta_{11}\equiv[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\exp(-x^2)]^{-1},$$

$$\beta_{12}\equiv[\exp(-\phi_o)/(\phi_o+\phi)^{1/2}+2(\phi_o+\phi)\exp(-\phi_o)+$$
$$+4\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy\ y(y^2+\phi)^{1/2}\exp(-y^2)].$$

Here $\partial_z\equiv\partial/\partial z$, $\partial_t\equiv\partial/\partial t$. (5) has been derived in approximation of slow dip evolution, i.e. in approximation of the small growth rate of the dip amplitude γ_{nl} .

From the nonlinear equation (5) the dip is shown to propagate with a slow velocity $V_c\approx0$. From (5) the growth rate of the dip's small amplitude can also be obtained

$$\gamma_{nl}\approx\omega_{p+}(V_{||}/V_{the})^{3/2}(q_+/e)(n_+/n_e)^{1/2}\beta_{13}, \qquad (6)$$

$$\beta_{13}\equiv\{1+[1/3-(n_e/n_+)(e/q_+)](e\varphi_o/T_e)(\pi/2)^{1/2}(V_{the}/2V_{||})\}.$$

One can see that the dip is formed at ratio of electron current-carrying to thermal velocity $V_{||}/V_{the}$ larger than the threshold. The threshold decreases at decreasing of ratio of electron and positive ion densities n_e/n_+ and is equal to zero at $n_e/n_+<q_+/3e$. The threshold is maximum at $n_e/n_+=1$.

3. Barrier formation for plasma ions in kind of electric potential hump near the ECR point

As the electrons are reflected from the dip and the ion flow passes with velocity V_{o+} freely, the uncompensated volume charge of ions is formed after the dip, in the field of which the ions slow down and are reflected. This volume charge forms perturbation of the electric potential hump. The ion flow enhances this hump of the electric potential. At first we describe the quasi-stationary properties of the hump, neglecting the nonequilibrium condition. Then taking into account the nonequilibrium condition we can obtain the hump's excitation, in other words the growth of the hump's amplitude. Further we will show that the electric potential hump is almost fixed in space.

We consider general case, when flow of the positive ions passes with velocity V_{o+} in the plasma with electrons and also with negative and motionless positive ions of small densities.

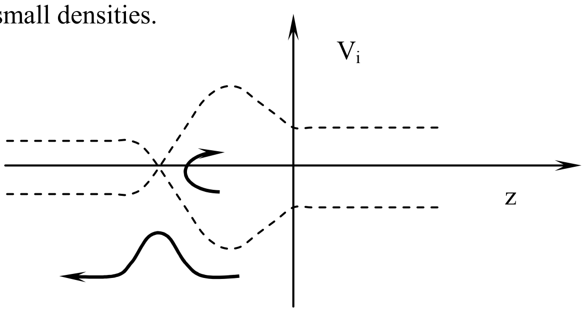

Fig. 3. Ion phase space. Dotted line is the separatrix, separating reflected and penetrated ions. Arrows specify a direction of the ion movement.

In linear approximation the perturbation excitation by the positive ion flow, propagating relative to negative and motionless positive ions of small densities, is described by the following dispersion ratio:

$$1+1/(kr_{de})^2-\omega^2_{p+}/(\omega-Kv_{o+})^2-\omega^2_{p-}/\omega^2-\omega^2_{pq}/\omega^2=0. \quad (7)$$

Here ω, k are the frequency and wave-vector of the perturbation; $\omega_{p\pm}$ are the plasma frequencies of the negative and the flow's positive ions; ω_{pq} is the plasma frequency of the positive motionless ions; r_{de} is the Debye radius of electrons; V_{o+} is the velocity of the positive ion flow.

From (7) we show that one can select the flow velocity such that following inequalities are correct

$$V_{ph}=\omega/k\approx(V_{o+}/2^{4/3})[(n_-m_+q_-^2/\ n_+m_-q_+^2)+(n_{+q}q_{+q}^2/\ n_+q_+^2)]^{1/3}<<V_{s+},$$

$$\lambda=2\pi/k=2\pi r_{de}/(V_{s+}^2n_+q_+^2/V_{o+}^2n_ee^2-1)^{1/2}>>r_{de}\ . \quad (8)$$

Hence the perturbation is almost motionless, that is $V_{ph}<<V_{s+}$. $V_{s+}=(T/m_+)^{1/2}$ is the ion-acoustic velocity of the flow positive ions. Here n_-, m_-, q_- (n_+, m_+, q_+) are the density, mass and charge of the negative (positive) ions.

From (7) we derive the growth rate of the perturbation excitation:

$$\gamma=(1.5)^{1/2}(V_{o+}/r_{de})[(n_-m_+q_-^2/n_+m_-q_+^2)+ +(n_{+q}q_{+q}^2/n_+q_+^2)]^{1/3}(V_{s+}^2q_+/V_{o+}^2e-1)^{1/2}. \quad (9)$$

On the non-linear stage of the instability development the electric potential perturbation φ represents the solitary hump of the finite amplitude φ_o.

The distribution function $f_e(v)$ of untrapped electrons, that are arranged outside the separatrix, looks like:

$$f_e(v)=[n_{oe}/V_{te}(2\pi)^{1/2}]\exp(e\varphi/T_e-m_ev^2/2T_e). \quad (10)$$

For trapped electrons, i.e. for electrons located inside the separatrix, the distribution function does not depend on energy because of an adiabaticity of the evolution.

Integrating the electron distribution function on velocity, we get following expression for electron density:

$$n_e=(n_o/(2\pi)^{1/2})(2/T)^{3/2}\int^\infty_o d\varepsilon(\varepsilon+e\varphi)^{1/2}\exp(-\varepsilon/T). \quad (11)$$

From the hydrodynamic equations for positive ions it is possible to get the following expression for their density:

$$n_+=n_{o+}/[1-2q_+\varphi/m_+(V_{o+}-V_h)^2]^{1/2}. \quad (12)$$

Here V_h is the velocity of the solitary perturbation.

As a result from (11), (12) and the Poisson's equation we have an equation for the spatial distribution of the electric potential ϕ of the perturbation of any amplitude:

$$\phi''=(2/\sqrt{\pi})\int^\infty_o da\,e^{-a}(a+\phi)^{1/2}-1/(1-2Q\phi/v_{oh}^2)^{1/2}. \quad (13)$$

$Q=q_+/e$, $\phi=e\varphi/T$, «'»$=\partial/\partial x$, $x=z/r_{de}$, $v_{oh}=(V_{o+}-V_h)/V_{s+}$.

From the apparent condition that the electric field of the electric potential hump is equal to zero for maximum potential $\phi'|_{\phi=\phi_o}=0$ and from (13) we obtain the hump velocity, v_{oh}:

$$v_{oh}^2/Q=(A-2)^2/2(A-2-\phi_o),$$

$$A\equiv(8/3\sqrt{\pi})\int^\infty_o da\,e^{-a}(a+\phi_o)^{3/2}. \quad (14)$$

In the approximation of small amplitude from (13), (14) we obtain:

$$v_{oh}^2\approx Q,\quad L\approx[15\sqrt{\pi}/4(1-1/\sqrt{2})]^{1/2}\phi_o^{-1/4}. \quad (15)$$

If V_{o+} is close to $(q_+/e)^{1/2}V_{s+}$, the perturbation is approximately motionless.

Taking into account the small densities of negative and motionless positive ions, we derive from the Poisson equation the evolution equation:

$$2\omega^2_{p+}\partial^3\varphi/\partial t^3/(V_{o+}-V_h)^3=-(\omega^2_{p-}+\omega^2_{pq})\partial^3\varphi/\partial z^3. \quad (16)$$

From (16) the growth rate of the non-linear perturbation amplitude is derived:

$$\gamma_{NL}\approx \quad (17)$$

$$\approx\omega_{p+}(e\varphi_o/T)^{1/2}[(n_{o-}m_+q^2_-/n_{o+}m_+q^2_+)+(n_{oq}q^2_{+q}/n_{o+}q^2_+)]^{1/3}.$$

4. The electron mechanism of the barrier formation for plasma ions near the ECR point

Let us consider the mechanism of the electric potential hump formation by current-carrying (i.e. with $V_{||}\neq 0$) plasma electrons near the dip of the electric potential.

The potential jump $\Delta\phi$, formed near the dip, accelerates electrons to the first front of the dip. Further we assume that the current-carrying velocity $V_{||}$ of the electrons on the back front of the dip is close but smaller than the electron thermal velocity V_{th}. Hence on the first front of the electric potential dip the electron current-carrying velocity becomes more than electron thermal velocity due to the flow continuity law. Hence on the first front of the dip the Bunemann instability is developed [2]. Due to Bunemann mechanism interaction of electron flow with this region an electric potential hump is excited.

Let us describe the solitary perturbation in kind of electric potential hump. We will show that it represents a nonlinear perturbation on a slow electron-sound mode. As it is slow, resonant electrons can be trapped by such perturbation.

From Vlasov's equation the expression for the perturbation of the electron distribution function is derived. Integrating this expression on velocity in case of a small amplitude of the solitary perturbation φ_o we get the expression for the perturbation of electron density

$$\delta n'=\partial_t\varphi\beta_{14}+\varphi'R(y)+\varphi\varphi'\beta_{15}, \qquad (18)$$

$$\beta_{14}\equiv[y+(1-2y^2)(1-R(y))/y],\ \beta_{15}\equiv[1-y^2+(3/2-y^2)(R(y)-1)],$$

$$R(y)\equiv 1+(y/\sqrt{\pi})\int_{-\infty}^{\infty}dt\exp(-t^2)/(t-y),\ y\equiv(V_{||}-V_h)/V_{th}\sqrt{2}.$$

Here a point means derivative in time, and a prime is a spatial derivative. V_h, φ are the velocity and the potential of soliton. $\phi\equiv e\varphi/T_e$. T_e is the electron temperature. Substituting (18) in Poisson's equation, we derive in stationary approximation an equation, describing spatial distribution of potential:

$$(\phi')^2=\phi^2R(y)-[1+(2y^2-3)R(y)]\phi^3/6. \qquad (19)$$

The soliton width is followed from (19) to be approximately equal $\Delta z=(48T_e/\varphi_o)^{1/2}$. The soliton width decreases with amplitude growth.

One can show that $V_h\approx 0$ if $V_{||}\approx 1.32V_{th}$.

In case of large amplitudes, $e\varphi_o/T_e>1$, from Vlasov equation we have the expression for electron distribution function $f=f_o[(u^2-2e\varphi/m)^{1/2}+V_h\mathrm{sign}(u)]$ for $|u|=|V-V_h|>(2e\varphi/m)^{1/2}$. Here f_o is Maxwell distribution function. Thus we obtain the equation for the soliton shape

$$(\phi')^2= \qquad (20)$$

$$=-\phi+(2/\sqrt{\pi})^{1/2}\int_{-\infty}^{\infty}dt(t-y)^2\exp(-t^2)\{[1+\phi/(y-t)^2]^{1/2}-1\}$$

From (20) we derive the soliton width

$$\Delta z=[2e\varphi_o/T_e(\sqrt{2}-1)]^{1/2} \qquad (21)$$

From (21) we conclude that the soliton width grows with φ_o. Therefore, it is necessary to take into account the electrons, trapped by the soliton field. Assuming the distribution of their density as $n_{tr}(z)=n_2\exp[e\varphi(z)/T_{tr}]$, we derive similarly to (21), that width and velocity of the soliton grow with amplitude growth (in difference from the case of small amplitudes of the solitary perturbation) Here T_{tr} , n_2 are the trapped electron temperature and density.

Thus, neglecting the ion mobility, this solitary perturbation is stationary and an electron one. However at taking into account of the ion mobility it is necessary to expect occurrence of slow growth of the perturbation's amplitude, as a result of Bunemann instability development. In the following order of the theory of disturbances from (18) we derive the correction of the next order to a spatial derivative of electron density

$$n_{e1}'=\partial_t\varphi[y+(1-2y^2)(1-R(y))/y] \qquad (22)$$

This expression as follows from a spatial derivative from Poisson's equation must be equal to a spatial derivative from the ion density perturbation n_i'. It is possible to find n_i' in a linear approximation from ion hydrodynamic equations

$$\partial^2_t n_i=(m_e/m_i)\phi'' \qquad (23)$$

Equating the second time derivative from (22) and the first spatial derivative from (23), we obtain

$$\partial^3_t\phi=(6m_e/m_i)\phi''' \qquad (24)$$

The solution of (24) we search as

$$\phi(z,t)=\phi_o(t)\eta[z-\int_{-\infty}^{t}dt_1\delta v_o(\phi_o(t_1))]\ . \qquad (25)$$

$\eta(z)$ is quasistationary shape of the perturbation. We assume that $\partial_t\phi_o(t)=\gamma\phi_o(t)$. In (25) the change of soliton velocity with change of its amplitude is taken into account.

Substituting $\partial_t\phi$ for $\gamma\phi-\delta v_h\phi'$, we obtain from (24)

$$\gamma\approx(m_e/m_i)^{1/3}\phi_o^{1/2} \qquad (26)$$

In the case of the electron mechanism of the barrier formation for plasma ions the inverse spatial sequence of the hump and dip, comparing with that one shown in Fig. 1, is realized.

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