Axisymmetric equilibria with flow in reduced single-fluid models

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Reduced magnetohydrodynamic equations for axisymmetric toroidal equilibria of flowing high-β plasmas are derived with asymptotic expansions in terms of the inverse aspect ratio in order to construct models suitable for the extension to include hot ion effects that are obtained with asymptotic expansions. Depending on the flow velocity, different orderings are applied. Singular points at the poloidal flow velocity equal to poloidal sound and poloidal Alfvén velocity are reproduced. The poloidal sound singularity appears in the higher order equations.

Keywords: magnetohydrodynamics, equilibrium, flow

Plasma flows are suggested to lead transport barriers and pedestals that show steep profiles in the steady states of magnetic confinement. Flowing equilibria have been studied to describe these phenomena in the framework of magnetohydrodynamics (MHD)[1, 2]. However, for such steep plasma profile features, small-scale effects not included in the ideal MHD model should be significant. The small scale effects arising due to the Hall current have been studied with two-fluid or Hall MHD models [6, 7, 8, 9, 10, 11]. However, these models are consistent with kinetic theory only for cold ions. In order to include the hot ion effects that are relevant to fusion plasmas, an extension of the model is necessary. However, a consistent treatment of hot ions in a two-fluid framework must include the ion gyroviscosity and other finite ion Larmor radius effects. These effects are obtained by asymptotic expansions in terms of the small parameter δ that is the ratio between the ion Larmor radius and the macroscopic scale length, and are much simplified in the slow dynamics ordering $v \sim \delta v_{th}$ where v and v_{th} are the flow and thermal velocities respectively [12, 13]. In this study, we obtain reduced sets of equations for MHD equilibria with flow with asymptotic expansions in order to construct models suitable for the extension to include hot ion effects. We shall study two cases of the flow velocity in the orders of the poloidal sound and poloidal Alfvén velocities. These are the characteristic velocities that bring singularities in the equilibrium equations [2, 3, 4, 5].

The equilibrium equations for single-fluid MHD are

$$\nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$\nabla \times \mathbf{E} = 0, \tag{2}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}, \tag{3}$$

$$\rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{j} \times \mathbf{B} - \nabla p, \tag{4}$$

$$\mu_0 \mathbf{j} = \nabla \times \mathbf{B} \tag{5}$$

$$\mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = 0, \tag{6}$$

where ρ is the mass density, $\mathbf{v}$ is the flow velocity, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{j}$ is the current density, and p is the pressure. Here we shall consider the corresponding toroidal axisymmetric equilibria, where the magnetic field $\mathbf{B}$ and the current density $\mathbf{j}$ can be written as

$$\mathbf{B} = \nabla \psi(R,Z) \times \nabla \varphi + I(R,Z) \nabla \varphi \tag{7}$$

$$\mu_0 \mathbf{j} = \nabla I \times \nabla \varphi - \Delta^* \psi \nabla \varphi, \tag{8}$$

where ψ is the poloidal magnetic flux and $\Delta^* \equiv R^2 \nabla \cdot [R^{-2} \nabla]$. The asymptotic expansion is defined in terms of the inverse aspect ratio $\varepsilon \equiv a/R_0 \ll 1$ where a and R_0 are the characteristic scale length of the minor and major radii respectively. The following high-β tokamak ordering is applied,

$$B_p \sim \varepsilon B_0, \quad p \sim \varepsilon \left(B_0^2/\mu_0\right). \tag{9}$$

The variables are expanded as

$$\psi = \psi_1 + \psi_2 + \psi_3 + \ldots,$$
$$I = I_0 + I_1 + I_2 + I_3 + \ldots,$$
$$p = p_1 + p_2 + p_3 + \ldots,$$
$$\rho = \rho_0 + \rho_1 + \ldots,$$
$$R = R_0 + x,$$

where $I_0 \equiv B_0 R_0$. The leading order of the force balance Eq. (4) yields

$$I_1 + \frac{\mu_0 R_0}{B_0} p_1 = \text{const.} \tag{10}$$

Here we consider the the flow velocity in the order of the poloidal sound speed,

$$v \sim (B_p/B_0) \sqrt{\gamma p/\rho} \sim \sqrt{\varepsilon} v_{Ap}, \tag{11}$$

where $v_{Ap} \equiv B_p/\sqrt{\mu_0 \rho}$ is the poloidal Alfvén velocity that is the order of the flow velocity for the usual reduced MHD (RMHD) [15]. Since

$$\rho v^2 \sim \varepsilon^2 p \sim \varepsilon^3 \left(B_0^2/\mu_0\right),$$

this requires the third-order accuracy of the total energy like the RMHD for finite aspect ratio tokamaks [15]. From

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the requirements $\nabla\cdot\mathbf{v}\sim\varepsilon v/a$ to eliminate the fast magnetosonic wave and then $\mathbf{v}\cdot\nabla p\sim\varepsilon^2 v/a$, the flow velocity $\mathbf{v}$ can be written as [15]

$$\begin{aligned}\mathbf{v} &\equiv \left(1+x/R_0\right)\nabla U\times\left(\mathbf{B}/B\right)+v_\parallel\left(\mathbf{B}/B\right)\\ &\equiv \mathbf{v}_p+v_\varphi R\nabla\varphi,\end{aligned} \tag{12}$$

$$\mathbf{v}_p \equiv \left[\frac{v_\parallel}{B}\nabla\psi+\left(1+\frac{x}{R_0}\right)\frac{I}{B}\nabla U\right]\times\nabla\varphi, \tag{13}$$

$$v_\varphi R \equiv \frac{Iv_\parallel}{B}-\left(1+\frac{x}{R_0}\right)\frac{\nabla\psi\cdot\nabla U}{B}. \tag{14}$$

The function U is expanded as

$$U=U_1+U_2+\dots$$

In the leading order, the poloidal flow is written in the standard stream function representation,

$$\mathbf{v}_p^{(0)}=R_0\nabla U_1\times\nabla\varphi, \tag{15}$$

and the toroidal flow velocity coicides with the parallel flow,

$$v_\varphi^{(0)}=v_\parallel. \tag{16}$$

The leading order of the φ-component of Ohm's law (3) yields

$$U_1=U_1\left(\psi_1\right), \tag{17}$$

and its next order is

$$R_0([U_2,\psi_1]+[U_1,\psi_2])=0, \tag{18}$$

which yields

$$U_2-U_1'\psi_2\equiv U_{2*}\left(\psi_1\right), \tag{19}$$

where $[a,b]\equiv(\nabla a\times\nabla b)\cdot\nabla\varphi$ is the Poisson bracket and the prime denotes the delivative of ψ_1. The first order of $\nabla\cdot\mathbf{v}$ is obtained from the projection of Faraday's law (2) along $\mathbf{B}$ as

$$(\nabla\cdot\mathbf{v})^{(1)}=\left[\frac{v_\parallel}{B_0}+2xU_1',\psi_1\right]. \tag{20}$$

The leading order of the pressure equation (6) yields

$$p_1=p_1[U_1(\psi_1)]=p_1(\psi_1), \tag{21}$$

and the next order is

$$R_0([p_2,U_1]+[p_1,U_2])=-\gamma p_1(\nabla\cdot\mathbf{v})^{(1)}. \tag{22}$$

Substituting Eq. (20) to Eq. (22), one obtains the equation for the second order pressure,

$$p_2-p_1'\psi_2+\gamma p_1\left(\frac{v_\parallel}{B_0R_0U_1'}+\frac{2x}{R_0}\right)\equiv p_{2*}\left(\psi_1\right). \tag{23}$$

Analogously, the continuity equation (1) gives the equations for the zeroth- and first-order density,

$$\rho_0=\rho_0[U_1(\psi_1)]=\rho_0(\psi_1), \tag{24}$$

$$\rho_1-\rho_0'\psi_2+\frac{\rho_0v_\parallel}{B_0R_0U_1'}+\frac{2x}{R_0}\rho_0\equiv\rho_*\left(\psi_1\right). \tag{25}$$

The projection of the force balance Eq. (4) along $\mathbf{B}$ is

$$\begin{aligned}&\mathbf{B}\cdot(\rho\mathbf{v}\cdot\nabla\mathbf{v}+\nabla p)\\ &=-\frac{\rho I}{R^2}\left\{\nabla\left[\frac{Iv_\parallel}{B}-\left(1+\frac{x}{R_0}\right)\frac{\nabla\psi\cdot\nabla U}{B}\right]\times\nabla\varphi\right\}\\ &\quad\cdot\left[\frac{v_\parallel}{B}\nabla\psi+\left(1+\frac{x}{R_0}\right)\frac{I}{B}\nabla U\right]\\ &-\left(1+\frac{x}{R_0}\right)\frac{\rho I}{BR^2}\left\{\frac{v_\parallel}{B}\Delta^*\psi+\left(1+\frac{x}{R_0}\right)\frac{I}{B}\Delta^*U\right.\\ &\quad\left.+\nabla\left(\frac{v_\parallel}{B}\right)\cdot\nabla\psi+\nabla\left[\left(1+\frac{x}{R_0}\right)\frac{I}{B}\right]\cdot\nabla U\right\}[U,\psi]\\ &-\frac{\rho}{2R^2}\left[\left[\frac{Iv_\parallel}{B}-\left(1+\frac{x}{R_0}\right)\frac{\nabla\psi\cdot\nabla U}{B}\right]^2,\psi\right]\\ &+\frac{\rho}{2}\left[v_\parallel^2+\left(1+\frac{x}{R_0}\right)^2\left[|\nabla U|^2-\left(\frac{\mathbf{B}}{B}\cdot\nabla U\right)^2\right],\psi\right]\\ &+[p,\psi]=0.\end{aligned} \tag{26}$$

The first order of Eq. (26) is

$$[\rho_0B_0R_0v_\parallel,U_1]+[p_2,\psi_1]+[p_1,\psi_2]=0 \tag{27}$$

which yields the equation for $v_\parallel$,

$$B_0R_0\rho_0U_1'v_\parallel+p_2-p_1'\psi_2\equiv p_{3*}\left(\psi_1\right), \tag{28}$$

which is the Bernoulli law in the present system. Equations (23), (25) and (28) show the coupling of $v_\parallel$, p_2 and ρ_1 due to the slow magnetosonic (sound) wave, since these are decoupled in the cold ($p_1\to0$) or incompressible ($\gamma\to\infty$) limits, and yield

$$v_\parallel=-\frac{(2x/R_0)\gamma p_1-(p_{2*}-p_{3*})}{(\beta_1-M_{Ap}^2)(B_0^2/\mu_0)}M_{Ap}v_A, \tag{29}$$

$$\begin{aligned}p_2=&p_1'\psi_2+\frac{(2x/R_0)M_{Ap}^2\gamma p_1}{\beta_1-M_{Ap}^2}\\ &-\frac{M_{Ap}^2p_{2*}-\beta_1p_{3*}}{\beta_1-M_{Ap}^2},\end{aligned} \tag{30}$$

$$\begin{aligned}\rho_1=&\rho_0'\psi_2+\rho_*+\frac{(2x/R_0)M_{Ap}^2}{\beta_1-M_{Ap}^2}\rho_0\\ &-\frac{p_{2*}-p_{3*}}{(\beta_1-M_{Ap}^2)(B_0^2/\mu_0)}\rho_0,\end{aligned} \tag{31}$$

where

$$\beta_1\left(\psi_1\right)\equiv\frac{\gamma p_1}{B_0^2/\mu_0}, \tag{32}$$

and $M_{Ap}^2(\psi_1) \equiv \mu_0 \rho_0 (R_0 U_1')^2$ is the poloidal Alfvén Mach number. The singularity is found for $\beta_1 = M_{Ap}^2$ where the poloidal flow velocity equals to the poloidal sound velocity. The projection of the force balance Eq. (4) along $\nabla\psi$ yields

$$|\nabla\psi|^2 \Delta^* \psi + I \nabla\psi \cdot \nabla I + \mu_0 R^2 \nabla\psi \cdot \nabla p + \mu_0 R^2 \nabla\psi \cdot (\rho \mathbf{v} \cdot \nabla \mathbf{v}) = 0. \quad (33)$$

The first and second orders of Eq. (33) are

$$|\nabla\psi_1|^2 \Delta_2 \psi_1 + 2\mu_0 R_0 x \nabla\psi_1 \cdot \nabla p_1 + I_1 \nabla\psi_1 \cdot \nabla I_1 + \mu_0 R_0^2 \nabla\psi_1 \cdot \nabla p_2 + B_0 R_0 \nabla\psi_1 \cdot \nabla I_2 = 0, \quad (34)$$

and

$$\begin{aligned} &|\nabla\psi_1|^2 \left(\Delta_2 \psi_2 - \frac{1}{R}\frac{\partial \psi_1}{\partial R} \right) + 2 (\nabla\psi_1 \cdot \nabla\psi_2) \Delta_2 \psi_1 \\ &+ \mu_0 x^2 \nabla\psi_1 \cdot \nabla p_1 + \nabla\psi_2 \cdot \nabla \left(I_1^2/2 \right) \\ &+ 2\mu_0 R_0 x (\nabla\psi_2 \cdot \nabla p_1 + \nabla\psi_1 \cdot \nabla p_2) \\ &+ \nabla\psi_1 \cdot \nabla \left(\mu_0 R_0^2 p_3 + R_0 B_0 I_3 + I_1 I_2 \right) \\ &- \mu_0 R_0^2 (\nabla\psi_1 \cdot \nabla U_1) \Delta_2 U_1 \\ &+ \mu_0 R_0^2 \nabla\psi_1 \cdot \nabla \left(|\nabla U_1|^2/2 \right) = 0, \end{aligned} \quad (35)$$

where

$$\Delta_2 \equiv \left(\frac{\partial^2}{\partial R^2} + \frac{\partial^2}{\partial Z^2} \right)$$

The projection of the force balance Eq. (4) along $\nabla\varphi$ is

$$\begin{aligned} &\nabla\varphi \cdot \left[\rho \mathbf{v} \times (\nabla \times \mathbf{v}) + \mu_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B} \right] \\ &= -\frac{\rho I}{R^2} \left\{ \nabla \left[\frac{I v_\parallel}{B} - \left(1 + \frac{x}{R_0} \right) \frac{\nabla\psi \cdot \nabla U}{B} \right] \times \nabla\varphi \right\} \\ &\cdot \left[\frac{v_\parallel}{B} \nabla\psi + \left(1 + \frac{x}{R_0} \right) \frac{I}{B} \nabla U \right] \\ &- (I/\mu_0 R^2)[I, \psi] = 0. \end{aligned} \quad (36)$$

The difference between Eqs. (26) and (36) is given by

$$\begin{aligned} &[p, \psi] + (I/\mu_0 R^2)[I, \psi] \\ &- \left(1 + \frac{x}{R_0} \right) \frac{\rho I}{B R^2} \left\{ \frac{v_\parallel}{B} \Delta^* \psi + \left(1 + \frac{x}{R_0} \right) \frac{I}{B} \Delta^* U \right. \\ &\left. + \nabla \left(\frac{v_\parallel}{B} \right) \cdot \nabla\psi + \nabla \left[\left(1 + \frac{x}{R_0} \right) \frac{I}{B} \right] \cdot \nabla U \right\} [U, \psi] \\ &- \frac{\rho}{2R^2} \left[\left[\frac{I v_\parallel}{B} - \left(1 + \frac{x}{R_0} \right) \frac{\nabla\psi \cdot \nabla U}{B} \right]^2, \psi \right] \\ &+ \frac{\rho}{2} \left[v_\parallel^2 + (1 + \frac{x}{R_0})^2 \left[|\nabla U|^2 - \left(\frac{\mathbf{B}}{B} \cdot \nabla U \right)^2 \right], \psi \right] \\ &= 0. \end{aligned} \quad (37)$$

The first order of Eq. (37) yields

$$p_2 + \frac{B_0}{\mu_0 R_0} I_2 \equiv g_* (\psi_1), \quad (38)$$

and the second order is

$$\begin{aligned} &\left[p_3 + \frac{B_0}{\mu_0 R_0} I_3, \psi_1 \right] + \frac{2x}{R_0} [p_2 - p_1' \psi_2, \psi_1] \\ &+ \frac{I_1}{\mu_0 R_0^2} [I_2 - I_1', \psi_1] - [g_*' \psi_2, \psi_1] \\ &+ \frac{\rho_0 U_1'^2}{R_0^2} \left[|\nabla\psi_2|^2/2, \psi_1 \right] = 0 \end{aligned} \quad (39)$$

which yields

$$\begin{aligned} &p_3 + \frac{B_0 I_3}{\mu_0 R_0} + \frac{I_1}{\mu_0 R_0^2} \left(I_2 - I_1' \psi_2 \right) + \frac{\rho_0 |\nabla U_1|^2}{2R_0^2} \\ &+ \left(\frac{x}{R_0} \right)^2 \frac{2M_{Ap}^2 \gamma p_1}{\beta_1 - M_{Ap}^2} - g_*' \psi_2 \equiv E_* (\psi_1). \end{aligned} \quad (40)$$

Substituting Eqs. (38) and (40) to Eqs. (34) and (35), we obtain the expanded Grad-Shafranov equation in the presence of poloidal-sonic flow,

$$\Delta_2 \psi_1 = -\mu_0 R_0^2 \left[\frac{2x}{R_0} p_1' + \left(\frac{\mu_0}{B_0^2} \frac{p_1^2}{2} + g_* \right)' \right], \quad (41)$$

$$\begin{aligned} &\Delta_2 \psi_2 + \mu_0 R_0^2 \left[\frac{2x}{R_0} p_1'' + \left(\frac{\mu_0}{B_0^2} \frac{p_1^2}{2} + g_* \right)'' \right] \psi_2 \\ &= \frac{1}{R} \frac{\partial \psi_1}{\partial R} + M_{Ap}^2 \Delta_2 \psi_1 + \frac{|\nabla\psi_1|^2}{2} \left(M_{Ap}^2 \right)' \\ &- \mu_0 R_0^2 \left[E_*' + \left(\frac{x}{R_0} \right)^2 p_1' \right. \\ &+ \left(\frac{x}{R_0} \right)^2 \left(\frac{2M_{Ap}^2 \gamma p_1}{\beta_1 - M_{Ap}^2} \right)' \\ &\left. - \frac{2x}{R_0} \left(\frac{M_{Ap}^2 p_{2*} - \beta_1 p_{3*}}{\beta_1 - M_{Ap}^2} \right)' \right]. \end{aligned} \quad (42)$$

The equation for ψ_1 (41) is same as for the static case while Eq. (42) for ψ_2 is modified by the flow and the singularity appears. In the cylindrical limit $x/R_0 \to 0$, the singularity can be removed when

$$p_{2*} = p_{3*} \equiv f_* (\psi_1) \gamma p_1 (\psi_1),$$

$$\rho_* (\psi_1) \equiv f_* (\psi_1) \rho_0 (\psi_1),$$

and

$$f_* (\psi_1) = f [U_1 (\psi_1)] \quad \text{and} \quad f [U_1 = 0] = 0.$$

Then, the equations for $v_\parallel$, p_2, ρ_1 and ψ_2 are rewritten as

$$v_\parallel = -\left(\frac{2x}{R_0} \right) \frac{\beta_1 M_{Ap} v_A}{\beta_1 - M_{Ap}^2}, \quad (43)$$

$$p_2 = p_1' \psi_2 - \frac{\left(f_* - 2x/R_0 \right) M_{Ap}^2 - f_* \beta_1}{\beta_1 - M_{Ap}^2} \gamma p_1, \quad (44)$$

$$\rho_1 = \rho_0' \psi_2 - \frac{\left(f_* - 2x/R_0\right) M_{Ap}^2 - f_* \beta_1}{\beta_1 - M_{Ap}^2} \rho_0, \tag{45}$$

$$\begin{aligned}
&\Delta_2 \psi_2 + \mu_0 R_0^2 \left[\frac{2x}{R_0} p_1'' + \left(\frac{\mu_0}{B_0^2} \frac{p_1^2}{2} + g_* \right)'' \right] \psi_2 \\
&= \frac{1}{R} \frac{\partial \psi_1}{\partial R} + M_{Ap}^2 \Delta_2 \psi_1 + \frac{|\nabla \psi_1|^2}{2} \left(M_{Ap}^2\right)' \\
&\quad - \mu_0 R_0^2 \left[E_*' + \left(\frac{x}{R_0} \right)^2 p_1' - \frac{2x}{R_0} \left(f_* \gamma p_1\right)' \right. \\
&\quad \left. + \left(\frac{x}{R_0} \right)^2 \left(\frac{2M_{Ap}^2 \gamma p_1}{\beta_1 - M_{Ap}^2} \right)' \right].
\end{aligned} \tag{46}$$

In comparson with the analysis of the transonic flow for low-β tokamaks [5, 2], the singularity at the poloidal flow velocity equal to poloidal sound velocity in the density and pressure and its dependence on toroidicity has been reproduced as higher-order effects and the singularity in the higher order magnetic structure has been found in the present study. However, in order to reproduce the radial discontinuity of the density and pressure found in Ref. [2], a local analysis assuming $\beta_1 - M_{Ap}^2 \sim \varepsilon M_{Ap}^2$ will be necessary. Finally we note that the hyperbolic region between the cusp velocity and the poloidal velocity of the slow magnetosonic wave as pointed out in Ref. [4] may be degenerated because the difference between them goes to higher order in the present ordering.

Next we consider the case for the poloidal-Alfvénic flow $v \sim v_{Ap}$ where the usual RMHD ordering applies. The first order of Eq. (33) is

$$\begin{aligned}
&|\nabla \psi_1|^2 \Delta_2 \psi_1 + 2\mu_0 R_0 x \nabla \psi_1 \cdot \nabla p_1 + I_1 \nabla \psi_1 \cdot \nabla I_1 \\
&+ \mu_0 R_0^2 \nabla \psi_1 \cdot \nabla p_2 + B_0 R_0 \nabla \psi_1 \cdot \nabla I_2 \\
&- M_{Ap}^2 [|\nabla \psi_1|^2 \Delta_2 \psi_1 - \nabla \psi_1 \cdot \nabla (|\nabla \psi_1|^2/2)] = 0.
\end{aligned} \tag{47}$$

The first order of Eq. (37) yields

$$p_2 + \frac{B_0}{\mu_0 R_0} I_2 + \rho_0 (R_0 U_1')^2 |\nabla \psi_1|^2/2 \equiv g_* (\psi_1). \tag{48}$$

Substituting Eq. (48) to Eq. (47), we obtain the equation for ψ_1 in the following form,

$$\begin{aligned}
&(1 - M_{Ap}^2) \Delta_2 \psi_1 - \frac{|\nabla \psi_1|^2}{2} \left(M_{Ap}^2\right)' \\
&= -\mu_0 R_0^2 \left[\frac{2x}{R_0} p_1' + \left(\frac{\mu_0}{B_0^2} \frac{p_1^2}{2} + g_* \right)' \right].
\end{aligned} \tag{49}$$

The singularity at the poloidal flow velocity equal to poloidal Alfvén velocity arises in the first order of the magnetic structure. This singularity is independent of the toroidicity [16].

We have shown reduced sets of equations for MHD equilibria with flow with asymptotic expansions and reproduced the singular points at the poloidal flow velocity equal to poloidal sound and poloidal Alfvén velocity. They will be extended to include hot ion effects by setting $\delta \sim \varepsilon$ for the poloidal-sonic flow, and $\delta^2 \sim \varepsilon$, as usual reduced two-fluid models [17, 18], for the poloidal-Alfvénic flow.

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