Resistive wall mode stability analysis including plasma rotation and error field

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A theory for the formation of static magnetic islands induced by error fields is developed especially taking into account the Alfvén resonance effects due to the plasma rotation in cylindrical geometry. The Alfvén resonance effect totally changes the mode structure in the ideal magnetohydrodynamic (MHD) regions, which results in the significant change of the behavior of magnetic islands. It is found that the magnetic island becomes wider and the toroidal torque becomes larger when the resistive wall is located closer to the plasma and when the mode-rational surface is closer to the plasma edge.

Keywords: resistive wall mode, plasma rotation, magnetic island, error field, tokamak

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1 Introduction

In recent tokamak experiments, stabilization of resistive-wall mode (RWM)[1, 2] is one of the central issues to sustain high-performance discharges. The stabilization of RWM by plasma rotation have been studied extensively, and it has been demonstrated that plasma rotation can stabilize RWM both experimentally[1, 3, 4] and theoretically[5, 6, 7, 8, 9, 10]. The critical rotation velocity for the stabilization of RWM had been considered as 1–2% of the Alfvén velocity at $q = 2$ surface[11] where q is the safety factor[2].

However, it was recently demonstrated experimentally that the critical rotation velocity can be greatly reduced if the error field resonant to the mode is carefully reduced[3, 4]. The resulting critical rotation velocity was about 0.3% of the Alfvén velocity, which is one-order smaller than the previous results. In DIII-D experiments, it was shown that the critical rotation velocity in the NBI torque reduction experiments is much smaller than that in the magnetic braking experiments in which the error field is increased to brake the plasma rotation[4]. Therefore, it is obvious that the error field plays a crucial role in this critical-rotation problem.

The formation of magnetic islands by an error field was studied in a rotating plasma theoretically[12]; the effect of the plasma rotation was included as the convection term in the nonlinear evolution equation of the magnetic island width, or so-called modified Rutherford equation[13]. However, the plasma rotation generates twin Alfvén resonances due to the Doppler shift at both sides of the mode-rational surface. When the plasma rotation is not so slow, the island region at the mode-resonance surface can be well separated from the Alfvén resonance positions. Therefore, we need to take into account the effect of Alfvén resonances in the ideal magnetohydrodynamic (MHD) region. Therefore, we developed a theory for this problem by including the Alfvén resonance effect[14]. It was found that the Alfvén resonance effect changes the tearing mode parameter Δ' significantly, which results in, most importantly, the complete rotation suppression of the magnetic islands. In the present paper, we study the effect of the position of the wall and the mode-rational surface on this problem.

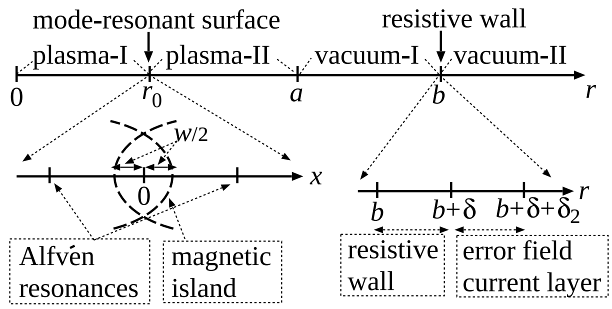

Fig. 1 Schematic picture of the geometry.

2 Matching of independent solutions

Here we consider a cylindrical plasma with a mode-resonant surface inside the plasma. Let us assume zero beta plasma, where beta is the ratio of the plasma pressure to the magnetic pressure. The plasma column is surrounded by a vacuum region, and a resistive wall is located in the vacuum region. Just outside the resistive wall, we assume a thin error field current layer. The schematic picture of the geometry is shown in Fig. 1

We apply the matching techniques: in almost whole

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region, non-dissipative and stationary equation is solved. Then, the solutions are connected across interfaces: the magnetic islands, the plasma-vacuum interface, the resistive wall and the error field current layer. The resistive wall and the error field current layer is assumed much thiner than the system size.

As for the time dependence, the characteristic growth rate of the mode seems to be much smaller than the plasma rotation frequency by the experimental observations[3, 4]. Therefore, in the following formulation, $\partial/\partial t + \mathbf{v}\cdot\nabla$ is approximated by $\mathbf{v}\cdot\nabla$, i.e., we consider a stationary state.

When the plasma is not rotating, we usually solve the Newcomb equation[15] in the plasma region except for the resistive layer. Then, we can calculate Δ'[16] which describes the tearing mode stability. The important difference of our study from the previous works is that we solve the following equation which includes the effect of equilibrium plasma rotation Ω_0 as[16]:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left[(\rho n^2\Omega_0^2 - F^2)r\frac{\mathrm{d}}{\mathrm{d}r}(r\xi)\right] - \left[m^2(\rho n^2\Omega_0^2 - F^2) - r\frac{\mathrm{d}F^2}{\mathrm{d}r}\right]\xi = 0, \tag{1}$$

where ξ is the radial displacement of the plasma, ρ is the equilibrium mass density, F is defined as $F := \mathbf{k}\cdot\mathbf{B}$ where $\mathbf{k} := m\nabla\theta - (n/R_0)\nabla z$ is the wave-number vector, m and n are the poloidal and toroidal mode numbers, respectively, and R_0 is the plasma major radius. The cylindrical coordinate system (r, θ, z) is used. This equation has the singularity, i.e., the Alfvén resonances at two radial positions where $F^2 - \rho(n\Omega_0)^2 = 0$. It is noted that $F = 0$ at the mode-resonant surface, which is in between the twin Alfvén resonances. Since the $\rho(n\Omega_0)^2$ term is negligible in almost whole region except for the region near the Alfvén resonances, and since the separation of the Alfvén resonances from the mode-resonant surface is small, it is sufficient to assume constant Ω_0 at the mode-resonant surface in Eq. (1). The mass density ρ is also assumed constant for simplicity.

By integrating Eq. (1) in the plasma-I region (see Fig. 1) ranging from the magnetic axis $r = 0$ to the edge of the magnetic islands $r = r_0 - w/2$, we obtain an independent solution in the plasma-I region as $\xi^{\mathrm{pI}}(r)$. The suffix "pI" denotes "plasma-I". Similar notations will be used in the following. The boundary condition at $r = 0$ eliminates one of the two independent solutions; Eq. (1) has two independent solutions in principle. Similarly, by integrating Eq. (1) from the plasma edge $r = a$ to the other edge of the magnetic islands $r = r_0 + w/2$, we obtain two independent solutions $\xi_1^{\mathrm{pII}}(r)$ and $\xi_2^{\mathrm{pII}}(r)$, which are normalized so that $\xi_1^{\mathrm{pII}}(w/2) = \xi_2^{\mathrm{pII}}(w/2) = -\xi^{\mathrm{pI}}(-w/2)$ where the suffix denotes $x := r - r_0 = w/2$ or $-w/2$. The boundary conditions at $r = a$ will be mentioned below. Then, we obtain

$$\xi(r) = c^{\mathrm{pI}}\xi^{\mathrm{pI}}(r), \tag{2}$$

$$\xi(r) = c_1^{\mathrm{pII}}\xi_1^{\mathrm{pII}}(r) + c_2^{\mathrm{pII}}\xi_2^{\mathrm{pII}}(r), \tag{3}$$

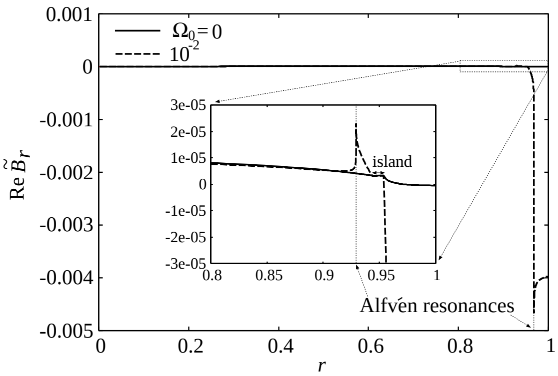

Fig. 2 Real part of the perturbed radial magnetic field.

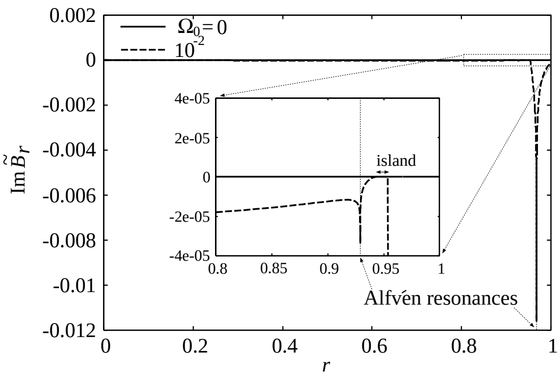

Fig. 3 Imaginary part of the perturbed radial magnetic field.

for the plasma-I and -II regions, respectively.

In integrating Eq. (1), we meet the singularity on the real r axis where $F^2 - \rho(n\Omega_0)^2 = 0$. This is resolved by adding a small artificial growth rate; this method was employed in Ref. [10]. We adopted the adaptive 4th-order Runge-Kutta method for the integration.

Here we show an example of the solution. Figures 2 and 3 shows the real and imaginary parts of the perturbed radial magnetic field, respectively. In the plasma region, it is expressed by $\tilde{B}_r = \mathbf{B}\cdot\nabla\xi = \mathrm{i}F\xi$. The wall position was chosen as $b = 1.1$ and the magnetic island width was $w = 0.01$. These length are normalized to the plasma minor radius $r = a$. For the finite rotation case, the plasma rotation frequency was chosen as $\Omega_0 = 10^{-2}$ normalized to the Alfvén time. We see the spiky behavior at the Alfvén resonances for the finite rotation case. Then it is easy to expect that Δ' is significantly changed from that without the Alfvén resonance effect. It is noted that $\tilde{B}_r$ is continuous across the resonance positions. The numerical solution of ξ was also benchmarked to the analytic solution of Eq. (1) near the Alfvén resonances[14].

In the vacuum region, the perturbed magnetic field is expressed as $\tilde{\mathbf{B}} = \nabla\psi$, where ψ also has a spatial dependence $\mathrm{e}^{\mathrm{i}(m\theta-(n/R_0)z)}$. Then, ψ satisfies the Laplace equation $\nabla^2\psi = 0$. The independent solutions can be written as linear combinations of r^m and r^{-m}. Then, we have

$$\psi(r) = c_1^{\rm vI}\psi_1^{\rm vI}(r) + c_2^{\rm vI}\psi_2^{\rm vI}(r), \tag{4}$$

$$\psi(r) = c^{\rm vII}\psi^{\rm vII}(r), \tag{5}$$

in the vacuum-I and -II regions, respectively. Here, the independent solutions are chosen such that $d\psi_1^{\rm vI}(b)/dr = 0$ and $\psi_2^{\rm vI}(r) = \psi^{\rm vII}(r) \propto r^{-m}$. Thus $\psi_1^{\rm vI}$ is related to ideal wall solution, and $\psi_2^{\rm vI} = \psi^{\rm vII}$ is related to the no wall solution.

The boundary conditions at $r = a$ are the continuity of perturbed radial magnetic field and the total pressure. By using these conditions, we can set convenient boundary conditions such that $c_1^{\rm pII} = c_1^{\rm vI}$ and $c_2^{\rm pII} = c_2^{\rm vI}$.

Now, let us define Δ'_∞ and Δ'_b related to the no wall and ideal wall solutions, respectively; $\Delta'_\infty := (\partial\tilde{B}_r/\partial r)|_{-w/2}^{w/2}/\tilde{B}_r$ where $\tilde{B}_r$ and its jump are evaluated by using the no wall solution. Similarly, Δ'_b is defined by using the ideal wall (at $r = b$) solution. Then, the total Δ' can be written as[17]

$$\Delta' = \Delta'_\infty - \frac{c_1^{\rm pII}}{c^{\rm pI}}(\Delta'_\infty - \Delta'_b), \tag{6}$$

where $c^{\rm pI}$ is related to the magnetic island width[16] and $c_1^{\rm pII}$ is related to the error field.

The island equation can be written as[12]

$$-{\rm i}\, n\Omega_0 w = \frac{\eta}{\mu_0}\Delta', \tag{7}$$

where we seek the static islands instead of rotating ones. By giving the error field strength, Eq. (7) can be solved to determine the island width w and the phase of the error field. It is noted that this formulation has a limit that the island width must be much smaller than the separation between the twin Alfvén resonances.

At the resistive wall, we solve the diffusion equation of the perturbed magnetic field. By introducing the error field term just outside the resistive wall, we have the following relation,

$$c_1^{\rm vI}\psi_1^{\rm vI}(b) + \psi_{\rm err} = 0. \tag{8}$$

It is also used that the perturbed radial magnetic field is continuous across the wall and the current layer, thus we obtain $c_2^{\rm vI} = c^{\rm vII}$.

If the island width and the error field are obtained, we can calculate the perturbed magnetic field and therefore the toroidal torque[18, 19]. We calculated it on the error field current layer, which has the same magnitude and the opposite direction to the total torque on the plasma:

$$\tau_\varphi = {\rm i}\,\pi^2 R_0^2 bk(\psi_{\rm err}\psi'^*(b) - \psi_{\rm err}^*\psi'(b)). \tag{9}$$

3 Effect of wall position on island width and torque

In the following, we will show numerical results based on the above formulation. The quantities shown below are normalized by using the minor radius a, toroidal magnetic field B_z, and the Alfvén time $\tau_{\rm A} := a/(B_z/\sqrt{\mu_0\rho})$. We adopted an equilibrium with toroidal current density profile as $j_{\rm t}(r) = j_{\rm t0}(1 - r^2)$. The edge safety factor was chosen as $q_a = 2.2$. For this current density profile, we have $q_0 = q_a/2 = 1.1$. The rational surface of $m/n = 2/1$ locates near the plasma edge, and the tearing mode is stable when an ideal wall is located at the plasma edge whereas unstable without the wall.

Figure 4 shows the magnetic island width w as a function of the plasma rotation frequency Ω_0. The magnitude of the error field was assumed to be $|\psi_{\rm err}| = 5 \times 10^{-3}$. The width w decreases as Ω_0 is increased. It is noted that we stopped the calculation when w exceeds one half of the separation of the twin Alfvén resonances because of the validity of Eq. (7). When Ω_0 is small, the separation of the Alfvén resonances is also small, and then the island width is also limited to small value. Thus no data is shown in the small Ω_0 and large w range. If we neglect the Alfvén resonance effect in Eq. (1), w is larger than the values in Fig. 4, and is proportional to $\Omega_0^{-1/3}$. In our case, w was found to be proportional to Ω_0^{-1} when w is relatively large[14]. Especially, it was found that the formation of magnetic islands can be completely suppressed by the plasma rotation when $\psi_{\rm err}$ is small [14]. It is seen that the curves for $b = 1.1$, 1.15 and 1.2 terminate around $\Omega_0 \gtrsim 0.015$. This is becomes of the complete suppression of the island formation. These are the consequences of the Alfvén resonance effect. It is found from Fig. 4 that w becomes larger if the wall position $r = b$ becomes closer to the plasma edge. This is naturally understood since the effect of the error field should be stronger when the wall is closer to the plasma.

Figure 5 shows the total torque on the plasma, which is denoted by $-\tau_\varphi$. The torque is in the direction to slow down the plasma rotation. It was found that the Alfvén resonance effect makes the torque larger by an order of magnitude, as well as changes the dependence on Ω_0; it increases as Ω_0 is increased[14]. If we neglect the Alfvén resonance effect in Eq. (1), the torque is proportional to $\Omega_0^{-2/3}$. From Fig. 5, we find that the torque becomes larger if the wall is located closer to the plasma. This is also naturally understood similarly to the island width case; bigger magnetic perturbation leads stronger torque.

4 Effect of edge safety factor on island width and torque

Finally, we show the effect of the edge safety factor on the magnetic island width and the torque. By increasing the edge safety factor q_a, the $m/n = 2/1$ mode-resonant surface moves inward in radius. Then, the effect of the error field is expected to become weaker. Figure 6 shows the magnetic island width w as a function of the plasma rotation frequency Ω_0 for the edge safety factor $q_a = 2.2$, 2.3, 2.4 and 2.5. The toroidal current density profile is

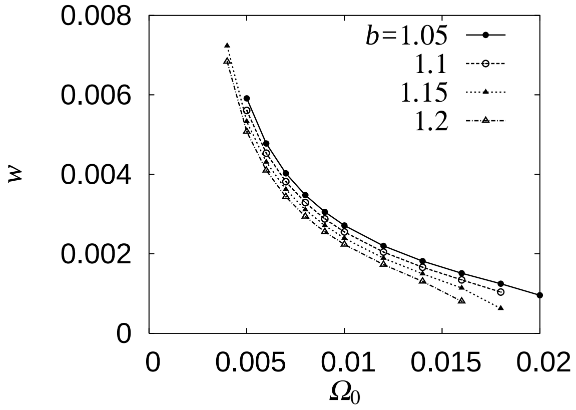

Fig. 4 Magnetic island width v.s. plasma rotation frequency for several wall positions.

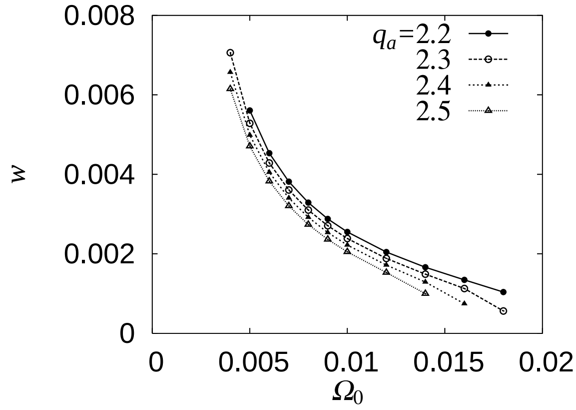

Fig. 6 Magnetic island width v.s. plasma rotation frequency for several edge safety factors.

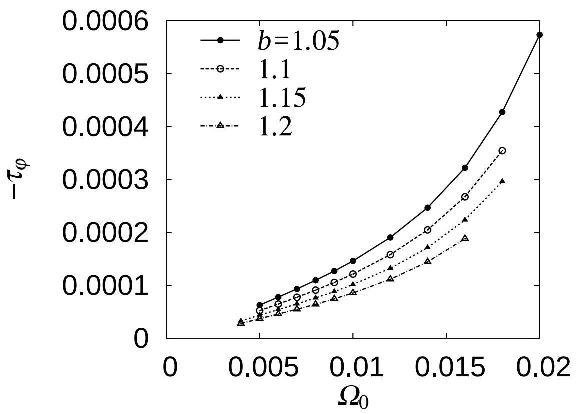

Fig. 5 Toroidal torque v.s. plasma rotation frequency for several wall positions.

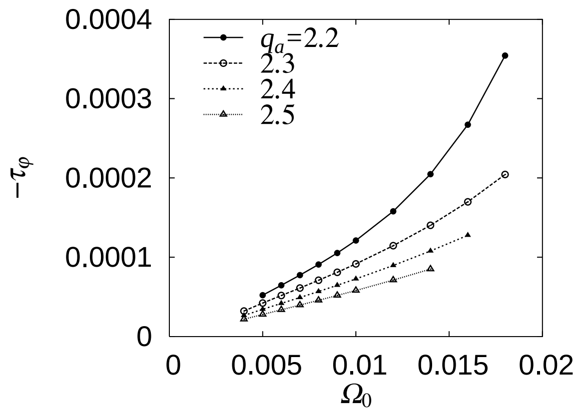

Fig. 7 Toroidal torque v.s. plasma rotation frequency for several wall positions.

fixed as $j(r) = j_{t0}(1 - r^2)$ and j_{t0} was changed. The error field and the wall position were chosen as $|\psi_{\rm err}| = 5 \times 10^{-3}$ and $b = 1.1$, respectively. It is found that w indeed becomes smaller for larger q_a. Figure 7 shows the toroidal torque. As naturally understood, we find the torque becomes smaller if q_a is increased.

5 Summary

We developed a theory for the formation of static magnetic islands induced by the error field for a rotating plasma in cylindrical geometry, especially taking into account the Alfvén resonance effect. The Alfvén resonance effect was shown to change the tearing mode parameter Δ' significantly, which results in reduction or even complete rotation suppression of the magnetic islands. In this paper, it was shown that the island width and the resulting toroidal torque increases if the resistive wall is closer to the plasma as well as if the mode-rational surface is closer to the plasma edge.

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