Applications of HINT2 code to stellarator/heliotron

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To investigate the nature of three-dimensional (3D) MHD equilibrium and interpret the experiments, an equilibrium calculation code without the assumption of nested flux surfaces, HINT2 is developing. This code applied many stellarator/ heliotron devices and properties of MHD equilibrium were clarified. Some applications of HINT2 are shown and future plans are discussed.

Keywords: MHD, equilibrium, stochasticity, HINT2

1. Inroduction

The MHD equilibrium is the basis of both most theoretical considerations and physics interpretation of the experimental results. As a standard technique to calculate the 3D MHD equilibrium, inverse equilibrium solver VMEC [1], assuming the existence of perfect nested flux surfaces, is widely used. In such a technique, a flux coordinate system is directly constructed so as to satisfy the force balance or MHD equilibrium equation: $\vec{J} \times \vec{B} = \nabla p$. For the low-β equilibrium, if the vacuum magnetic field sustains clear flux surfaces, the standard technique is acceptable. However, by nature, the 3D MHD equilibrium may exhibit magnetic islands and stochastic regions in the plasma because of the absence of toroidal symmetry. For the high-β equilibrium, the degradation of flux surfaces by the finite β effect has to be determined, so that the standard technique based on the nested flux surfaces could not be directly applicable to them and other techniques are required such as HINT/HINT2 [2,3] and PIES [4] codes.

The HINT2 code is one of such solvers, where a relaxation method based on the dissipative MHD equations of the magnetic field. A special feature of HINT2 is the coordinates system, which uses a 'non-orthogonal rotating helical coordinate'. Since the helical coordinate system is Eulerian, HINT2 can treat magnetic islands and stochastic regions in plasmas.

The HINT2 code has been applied to the study of MHD equilibrium in many helical configurations, in order to clarify the properties inherent to 3D MHD equilibrium in various types of helical systems, which are LHD, W-7AS, W-7X, Heliotron-J. In this study, we show applications of HINT2 for high-β stellarator/heliotron researches.

2. Application of LHD

In order to understand how about HINT2 shows the finite-β MHD equilibrium, several quantities of a finite-β equilibrium with β~3.5% for an inward shifted (R_{ax}=3.6, γ=1.254, B_Q=100%) are shown in fig.1; (a) Puncture map of field lines, (b) rotational transform ι and distance along the magnetic field L_C, (c) a profile of the normalized plasma pressure p/p_0 (p_0 is the pressure at the magnetic axis), (d) contour lines of p/p_0, and (e) contour lines of square of the local residual force. Vertical lines in fig.1(a) and arrows in fig.1(b) and (c) indicate positions of the vacuum LCFS. In this finite β equilibrium, the initial pressure profile is set to $\mathrm{p} = \mathrm{p}_0(1-\mathrm{s})(1-\mathrm{s}^4)$, where s is the normalized toroidal flux defined as $s = \Phi/\Phi_{edge}$. In fig.1(a), a region colored with blue corresponds to the region with clear flux surfaces and a region with red (green) corresponds to Puncture map of stochastic magnetic field lines started from 4.419 < R < 4.55 (R > 4.55). For β~3.5%, a considerable part in the plasma periphery is ergodized. The position of the LCFS is much inside the vacuum LCFS. However, from the comparison among fig.1(a), (c) and (d), it is understood that the plasma pressure spreads over the stochastic region and the pressure gradient $\nabla \mathrm{p}$ exists in the stochastic region.

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This fact comes from the relaxation method of the pressure used in HINT2. The condition for a fixed $\boldsymbol{B}$ is written as

$$p^{i+1} = p = \frac{\int_{-L_{in}}^{L_{in}} \mathcal{F} p^{i} \frac{dl}{B}}{\int_{-L_{in}}^{L_{in}} \frac{dl}{B}} \tag{1}$$

$$F = \begin{cases} 1 \text{:for } L_C \geq L_{in} \\ 0 \text{:for } L_C < L_{in} \end{cases}$$

where i means a step number of iterations, L_C is the connection length of a magnetic field line starting each grid point (L_C is finite for open magnetic field lines), and L_{in} is the length along a magnetic field line followed from each grid point and prescribed as an input parameter. If the magnetic field line cannot be traced up to the prescribed value or the connection length L_C is shorter than L_{in} ($L_C < L_{in}$), the averaged plasma pressure is set to zero. That is, the distribution of relaxed plasma pressure depends on the relative magnitude between the connection length L_C and the parameter L_{in} prescribing the length of the field line trace. In other words, although HINT or HINT2 does not need the explicit boundary condition between plasma and the vacuum, the ratio of the connection length L_C to the length for pressure average L_{in} plays a role in controlling the distribution of the plasma pressure near the periphery. In fig.1, L_{in} is set to 30m, which corresponds to the length connecting the outside to the inside of the torus near the magnetic axis. It is seen from the comparison between fig.1(a) and (b) that the structure of magnetic field lines becomes stochastic outside the region with ι~1, because magnetic islands corresponding to some rational surfaces (i.e. n/m = 10/10, 10/9, 10/8, 10/7, 10/6, ...) overlap. Although these field lines are ergodized some magnetic field lines have long connection lengths L_C exceeding L_{in} as shown in fig.1(b), leading to the existence of the pressure and pressure gradient in the stochastic region. As mentioned above, in the case of HINT or HINT2 not assuming the existence of the nested flux surfaces, the relaxation method of the pressure profile mainly plays the role of determining the peripheral pressure profile instead of the plasma-vacuum boundary condition.

3. Application of Wendelstein 7-AS

The standard divertor configuration (SDC) for HDH-discharges has a boundary ι-value of the vacuum field of 5/9, where 9 natural islands form a clear separatrix. Additionally, correction coils located inside the vacuum vessel are used to increase the island size and keep a larger distance to the divertor plates. The corrugated separatrix boundary of such configurations can not be treated by free-boundary VMEC and thus

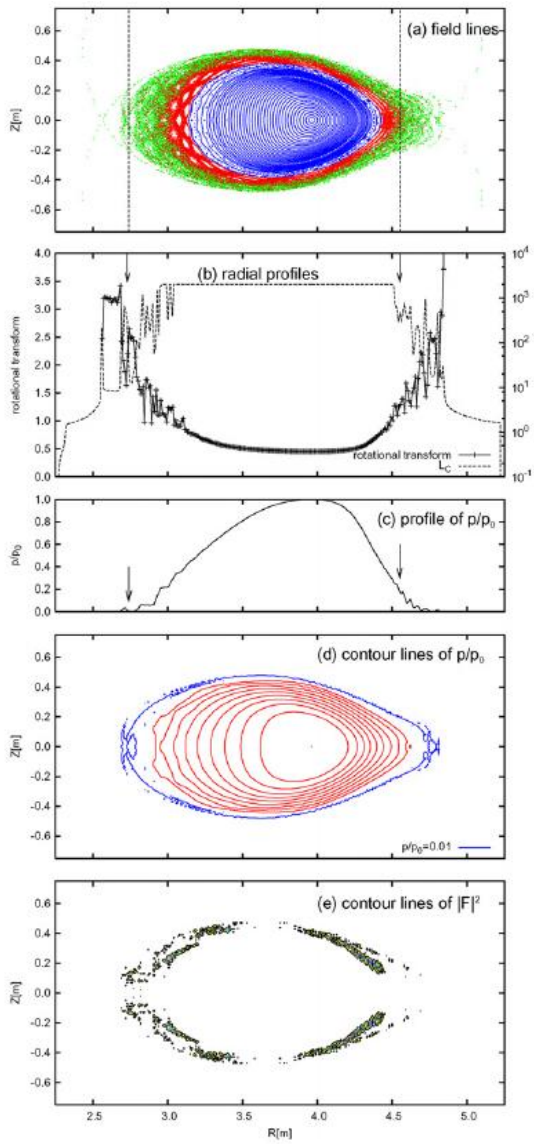

Fig.1 Various quantities of a finite-β equilibrium with β~3.5% on the horizontally elongated poloidal cross section in the inward-shifted LHD configuration: (a) Puncture map of magnetic field lines, (b) the rotational transform ι and the distance along the magnetic field L_C, (c) profile of the normalized plasma pressure, p/p_0, along the equatorial plane, (d) contour lines of p/p_0, and (e) contour lines of $|F|^2 = \left|\nabla p - \vec{J} \times \vec{B}\right|^2$.

equilibrium calculations were not available up to now. We compare the influence of different pressure profile forms on the configuration. The initial profiles used in this study are shown in Fig.2. They were choosen for variability and to have a zero pressure gradient at the boundary. Figure 3 shows how the initially flat ι-profile builds up shear by a central increase due to the Shafranov-shift. Thus, the 5/9-resonance appears in the ι-profile and islands develop whose position depends on the β and the underlying profile form. The broader profile (HDH-2) leads to larger islands closer to the boundary.

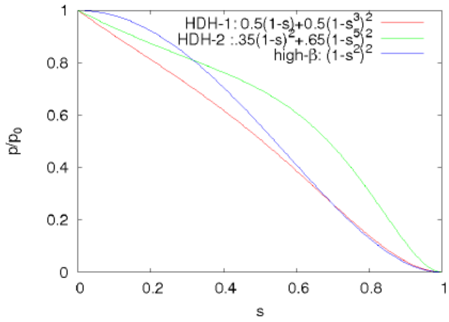

Fig2. Pressure profiles used in this study as the function of normalized toroidal flux *s*.

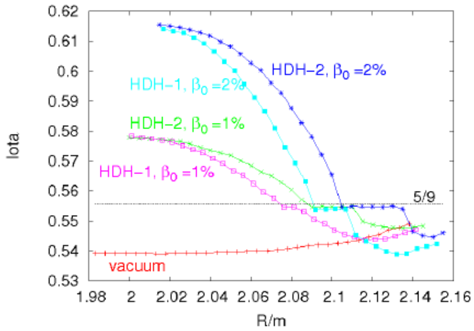

Fig.3 ι-profiles of the vacuum and finite-β calculations with different pressure profiles.

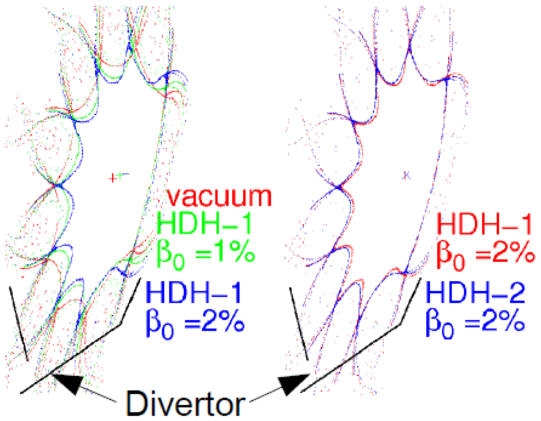

Fig.4 Axis position and separatrix depending on β (left) and pressure profiles (right).

The identations of the separatrix formed by the 5/9 boundary islands increase with β as seen in Fig.3. We also note that the x-points move poloidally which was already observed in the study performed with the old HINT-code [2]. Both effects are due to the poloidal expansion of the flux tubes due to the Shafranov-shift. Additionally, the island fans are moving radially on the target plates of the divertor which is also seen in the experiment. We note, that the separatrix structure seems not to depend strongly on the profile changes if the energy content is kept constant which is approximately fulfilled for the two profiles compared here.

4. Application of Wendelstein 7-X

Figure 4 shows the Large-Volume configuration for vacuum and for β~4.36%. Puncture plots of magnetic field lines are plotted at ϕ=0 plane. The profile of the rotational transform ι of this configuration has a low shear, clear flux surfaces in the core and clear 5/5 island structures in the edge region of the vacuum field. Thus, this configuration is sensitive to finite-β effects. The configuration had already been investigated with PIES [5]. Our study finds the following results; (i) the Shafranov shift and the change of ι is very small, (ii) the edge region is ergodized and the last closed flux surface (LCFS) retreats, that is, the plasma volume decreases.

In figs, green dots indicate the LCFS. The initial pressure distribution has been set to in this study, where *s* is the normalized toroidal flux Φ/Φ_{edge}.conforming to the profile used in Ref [5]. For b~4.36%, the edge region is ergodized and 5/5 islands evolve. Thus, the shape of the LCFS is changing according to the change of the edge region. However, clear flux surfaces are maintained inside 5/5 islands and other large resonances dot not appear.

We compare these results to those of PIES. In Ref [5], The LCFS of HINT2 is larger than the one of PIES. In both cases, 10/11 rational surfaces appear inside the LCFS. For PIES, the LCFS locates just outside 10/11 rational surface, whereas for HINT2, some more closed flux surfaces exist outside 10/11 rational surface. The LCFS of HINT2 locates outside 15/16 rational surface. In order to do a detailed comparison, the plasma volume inside the LCFS, V_{LCFS}, is studied. The volume V_{LCFS} for the vacuum is about 33.6m^3 (see Ref [5]). In the HINT2 calculation, V_{LCFS} is decreasing to ~ 31.6m^3, a volume reduction of ~ 6%. However, PIES sees a reduction of V_{LCFS} of ~ 21%, being much larger than the one of HINT2. This is a significant difference in the HINT2 and PIES results. There are some possibility explaining this difference. One is the numerical scheme of both codes. HINT2 is based on the relaxation method and calculations are done on a rectangular grid without the assumption of nested flux surfaces. On the other hand, PIES is an iterative solver on a quasi-flux coordinates system. This suggests differences in the effect of the finite pressure in the ergodic region. Another effect concerns the evolution of the pressure profile. Since HINT2 is based on the relaxation method, the pressure profile evolves during the relaxation process. The relaxed profile is almost the same as the initial profile but nevertheless slightly different. In order to confirm such effects more clearly, further studies and benchmarking are necessary.

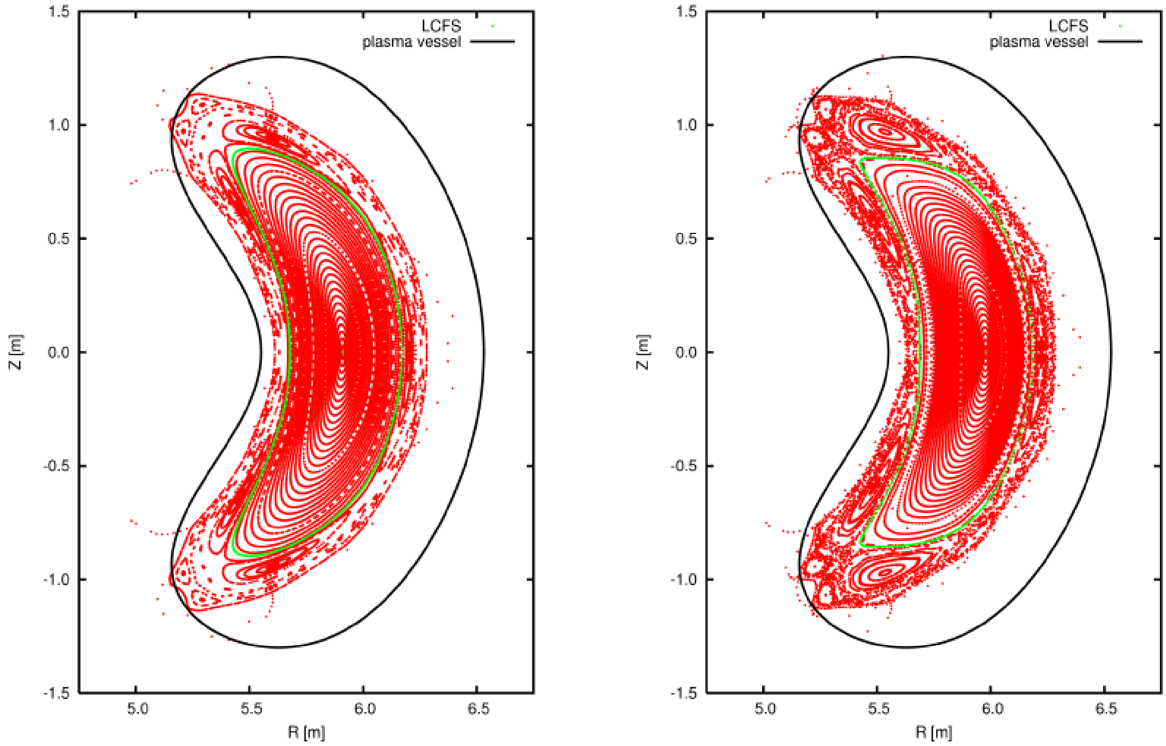

Fig.4 Puncture map of magnetic file lines for the vacuum field (left) and finite-β equilibrium (right).

5. Application of Heliotron J

Heliotron J device is an L=1/M=4 helical-axis heliotron with flat ι-profile and deep magnetic well for the vacuum field. Since the magnetic shear is low, the configuration is very sensitive to the pressure-induced perturbation and net toroidal currents. In experiments, the change of magnetic configuration due to the evolution of beam driven currents was observed and it is suggested the change of the magnetic configuration might be the trigger of a spontaneous transition [6]. In order to study effects of net toroidal currents to the configuration, the equilibrium including the net toroidal current is studied for the standard configuration. For the direction of toroidal currents is positive, the edge field structure is changed by appearing large magnetic islands. This affects the confinement through the plasma volume. However, for the negative direction, clear flux surfaces are kept by disappearing dangerous resonances.

6. Summary

HINT2 is useful and powerful tool to study 3D MHD equilibrium and it applies to many configurations.

Future plans of HINT2 are twofold. First direction is the further improvement of the code. One example is shown. A critical issue of stellarator/heliotron researches is the stochastic property of edge field lines due to the finite-β. Since HINT2 does not assume nested flux surfaces, the code can calculate the equilibrium with stochastic field. However, in the stochastic region, the plasma pressure is the flux surface quantity and the radial force is not balanced. In order to treat the plasma pressure in the stochastic region consistently, the anisotropic pressure is necessary. Second direction is the application to interpret the experiments.

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