The response of toroidal plasmas to error fields

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The response of toroidal plasmas to three-dimensional magnetic error fields is studied by using Ideal Perturbed Equilibrium Code (IPEC). Since the toroidal plasmas are highly sensitive to small error fields, the perturbation theory is efficient and useful to describe the equilibrium $\nabla P = \boldsymbol{J} \times \boldsymbol{B}$ in the presence of error fields. The perturbed force balance equation in ideal MagnetoHydroDynamics (MHD) is solved by augmenting a stability code and by constructing the interface between plasma and external system. In an ideally perturbed equilibrium, a shielding current arises on a rational surface to prevent an island from opening. When error fields reach a critical magnitude, the shielding current will be dissipated and an island will open. This effect of error fields can be greatly mitigated by adjusting currents in auxiliary coils to reduce the shielding current, or equivalently the resonant field. From the coupling between the resonant field and error fields, the effects of various error fields on toroidal plasmas were studied. The most important external field is almost always localized on the outboard midplane, which gives an important implication to the study and control of the response of toroidal plasmas to error fields.

Keywords: toroidal plasma, MHD, perturbed equilibrium, error field, magnetic island

The magnetically confined toroidal plasmas, such as tokamaks and stellarators, are highly sensitive to externally driven magnetic perturbations. This sensitivity implies that the response of plasmas to small magnetic perturbations is a critical issue in design and control of equilibrium [1, 2, 3, 4, 5]. The small magnetic perturbations always exist in toroidal devices due to error fields, such as, imperfections of primary magnets and other conducting components. When an external perturbation occurs, plasma responds to it and relaxes to a new equilibrium state. Since the perturbations are in practice very small compared with the field of the original equilibrium, the perturbation theory is effective to describe the response of plasmas.

The perturbed force balance equation is given by

$$\boldsymbol{f}(\boldsymbol{\xi}) = -\nabla\delta P + \boldsymbol{J}\times\delta\boldsymbol{B} + \delta\boldsymbol{J}\times\boldsymbol{B} = \boldsymbol{0} \tag{1}$$

in ideal MagnetoHydroDynamics (MHD). Using Maxwell relations and adiabatic plasma response, $\delta\boldsymbol{J} = (\nabla\times\delta\boldsymbol{B})/\mu_0$, $\delta\boldsymbol{B} = \nabla\times(\boldsymbol{\xi}\times\boldsymbol{B})$ and $\delta P = -\boldsymbol{\xi}\cdot\nabla P - \gamma P(\nabla\cdot\boldsymbol{\xi})$, the force balance equation becomes a vector differential equation for the plasma displacement $\boldsymbol{\xi}$. Assuming that the plasma conserves its two independent profiles, rotational transform $\iota(\psi)$ and pressure $p(\psi)$, then the same equation can be derived from the theory of ideal, linear MHD stability. Through minimization of a perturbed potential energy $\delta W = -1/2\int dx^3\boldsymbol{\xi}\cdot\boldsymbol{f}(\boldsymbol{\xi})$, the exact same equation for $\boldsymbol{\xi}$ can be obtained. Therefore, an existing code in the stability analysis can be used to solve the problem of perturbed equilibria. Only the interface between plasma and external system is required to obtain an perturbed equilibrium given an external error field.

The Ideal Perturbed Equilibrium Code (IPEC) [6] modifies and augments the DCON ideal MHD stability code [7]. For a given axisymmetric equilibrium, the use of DCON gives a set of M plasma displacements $\boldsymbol{\xi}_i(\psi,\theta,\varphi)$, which is the set of M ideal MHD eigenmodes for a given toroidal harmonic number n, where $1 \leq i \leq M$ and M is the number of poloidal harmonics retained. Here (ψ,θ,φ) are magnetic coordinates which are straight on the field line. Each of these displacements $\boldsymbol{\xi}_i$ is associated with a certain deformation of the plasma boundary, $\boldsymbol{\xi}_i\cdot\boldsymbol{n}_b \equiv (\boldsymbol{\xi}_i\cdot\boldsymbol{n})(\psi_b,\theta,\varphi)$, where $\boldsymbol{\xi}_i$ is evaluated on the unperturbed plasma boundary at $\psi = \psi_b$ and $\boldsymbol{n}_b$ is the normal to the unperturbed plasma boundary. Each of these M displacements of the plasma boundary $\boldsymbol{\xi}_i\cdot\boldsymbol{n}_b$ defines a perturbed equilibrium if an external magnetic field produces a required force to support it. That is, the set of M ideal MHD eigenmodes found by DCON defines a set of M neighboring perturbed equilibria. Each of the neighboring equilibria is supported by an external magnetic field and has the same profiles of $\iota(\psi)$ and $p(\psi)$ as the unperturbed equilibrium; only the shape of the plasma has been changed.

A plasma displacement determines a magnetic perturbation $\delta\boldsymbol{B} = \nabla\times(\boldsymbol{\xi}\times\boldsymbol{B})$, so IPEC uses the displacement of the plasma boundary $\boldsymbol{\xi}\cdot\boldsymbol{n}_b$ to determine a part of the perturbed magnetic field that is normal to the unperturbed plasma boundary, $\delta\boldsymbol{B}\cdot\boldsymbol{n}_b$, and a part that is tangential to the plasma boundary, $\boldsymbol{n}_b\times\delta\boldsymbol{B}^{(p)}$. Since the normal field $\delta\boldsymbol{B}\cdot\boldsymbol{n}_b$ is continuous across the plasma boundary and the control surface, $\delta\boldsymbol{B}\cdot\boldsymbol{n}_b$ then gives a unique vacuum field

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outside the plasma, $\delta\boldsymbol{B}^{(vo)}$, that vanishes at infinity. The difference between the tangential field infinitesimally outside the control surface $\boldsymbol{n}_b\times\delta\boldsymbol{B}^{(vo)}$ and the tangential field on the plasma side of the control surface $\boldsymbol{n}_b\times\delta\boldsymbol{B}^{(p)}$ determines an external surface current on the control surface, $\mu_0\boldsymbol{K}^x=\boldsymbol{n}_b\times\delta\boldsymbol{B}^{(vo)}-\boldsymbol{n}_b\times\delta\boldsymbol{B}^{(p)}$. Once $\boldsymbol{K}^x$ is known, the externally produced normal magnetic field $\delta\boldsymbol{B}^x\cdot\boldsymbol{n}_b$ can be found by $\nabla\times\delta\boldsymbol{B}^x=\mu_0\boldsymbol{J}^x$ in vacuum. Note that the *external* field $\delta\boldsymbol{B}^x$ designated by superscript x is a vacuum field without plasma response, compared with the *total field* $\delta\boldsymbol{B}$.

Each of the M neighboring equilibria calculated by DCON has a unique distribution of the external normal magnetic field $\delta\boldsymbol{B}_i^x\cdot\boldsymbol{n}_b$, where $1\leq i\leq M$, that must be produced by currents outside the plasma to sustain that equilibrium. If an external magnetic perturbation, such as that due to a magnetic field error $\delta\boldsymbol{B}^x\cdot\boldsymbol{n}_b$, is specified on the unperturbed plasma boundary, this perturbation can be expanded as $\delta\boldsymbol{B}^x\cdot\boldsymbol{n}_b=\sum_{i=1}^{M}c_i\delta\boldsymbol{B}_i^x\cdot\boldsymbol{n}_b$, with expansion coefficients c_i. If this is done, the plasma displacement that gives the perturbed equilibrium produced by the field error is $\boldsymbol{\xi}(\psi,\theta,\varphi)=\sum_{i=1}^{M}c_i\boldsymbol{\xi}_i(\psi,\theta,\varphi)$. This is the method used by IPEC to find the perturbed equilibrium associated with a given magnetic field error [6].

The normal total magnetic fields of the M neighboring equilibria can be represented by

$$(\delta\boldsymbol{B}\cdot\boldsymbol{n}_b)(\theta,\varphi)=Re\left(\sum_m\Phi_m w(\theta)e^{i(m\theta-n\varphi)}\right),\qquad(2)$$

in Fourier space, where the weight function $w(\theta)=1/(\mathcal{J}(\theta)|\nabla\psi|(\theta))$ with the Jacobian $\mathcal{J}(\theta)$ is used for an orthogonal basis, by the definition of $\oint wf_mf_{m'}da=\delta_{mm'}$ on the boundary surface, with $f_m=e^{i(m\theta-n\varphi)}$.

The jump in the tangential field across the control surface just outside the plasma gives a surface current $\boldsymbol{J}=\boldsymbol{K}\delta(\psi-\psi_b)$. The surface current can also be expressed as $\boldsymbol{K}=\nabla\kappa(\theta,\varphi)\times\nabla\psi$ with a surface current potential $\kappa(\theta,\varphi)$. The potential $\kappa(\theta,\varphi)$ can be used for representing the surface current $\boldsymbol{K}$ by

$$\kappa(\theta,\varphi)=Re\left(\sum_m\mathcal{I}_m e^{i(m\theta-n\varphi)}\right)\qquad(3)$$

with the vector $\mathcal{I}$ having units of current. Combining the total fluxes $\boldsymbol{\Phi}_i$ and external currents $\mathcal{I}_i^x$ of the M neighboring equilibria, one can obtain a plasma inductance matrix $\boldsymbol{\Lambda}$ on the Fourier space, where the M poloidal harmonics are retained. $\boldsymbol{\Lambda}$ gives the relation between an total flux and an external current by $\boldsymbol{\Phi}=\boldsymbol{\Lambda}\cdot\mathcal{I}^x$. Similarly, the external normal magnetic perturbation $\delta\boldsymbol{B}^x\cdot\boldsymbol{n}_b$ producing the surface current $\boldsymbol{K}^x$ can be expanded and related to the current by $\boldsymbol{\Phi}^x=\boldsymbol{L}\cdot\mathcal{I}^x$, where $\boldsymbol{L}$ is a surface inductance matrix since it depends only on the shape of the boundary surface.

The linear relation between an total flux $\boldsymbol{\Phi}$ and an external flux $\boldsymbol{\Phi}^x$ can then be written as

$$\boldsymbol{\Phi}=\boldsymbol{P}\cdot\boldsymbol{\Phi}^x\qquad(4)$$

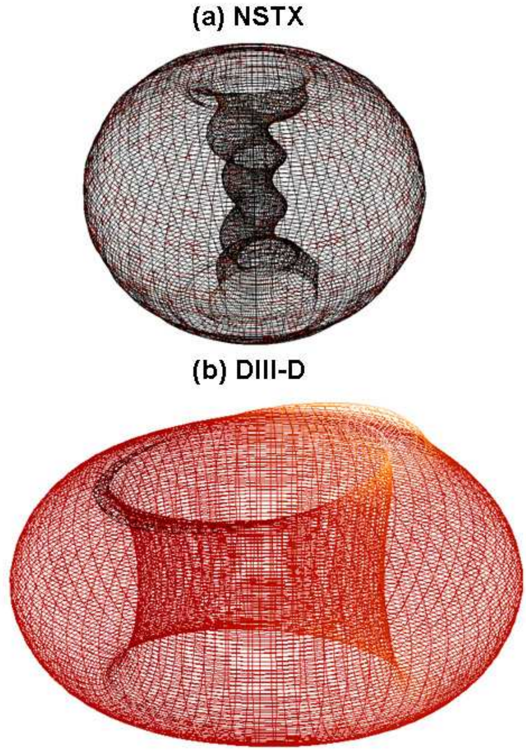

Fig. 1 The deformed plasma boundary of typical (a) NSTX (b) DIII-D tokamak plasmas due to each intrinsic error field. The scale is arbitrary.

with a permeability matrix $\boldsymbol{P}=\boldsymbol{\Lambda}\cdot\boldsymbol{L}^{-1}$. If a magnetic field error $\boldsymbol{\Phi}^x$ is specified on the boundary, one can expand it by $\boldsymbol{\Phi}^x=\sum_{i=1}^{M}c_i\boldsymbol{\Phi}_i^x=\sum_{i=1}^{M}c_i\boldsymbol{P}^{-1}\cdot\boldsymbol{\Phi}_i$, or equivalently by $\boldsymbol{\Phi}=\boldsymbol{P}\cdot\boldsymbol{\Phi}^x=\sum_{i=1}^{M}c_i\boldsymbol{\Phi}_i$ to obtain the perturbed equilibrium by $\boldsymbol{\xi}(\psi,\theta,\varphi)=\sum_{i=1}^{M}c_i\boldsymbol{\xi}_i(\psi,\theta,\varphi)$. Each actual flux $\boldsymbol{\Phi}_i$ is associated with a plasma displacement $\boldsymbol{\xi}_i$ through Eq. (2) and $\delta\boldsymbol{B}=\nabla\times(\boldsymbol{\xi}\times\boldsymbol{B})$. This is how IPEC constructs the interface and solve the perturbed equilibrium from a given error field [6]. Fig. 1 shows the computational examples, deformed plasma boundary of typical (a) NSTX [12] (b) DIII-D [13] plasmas due to each intrinsic error field.

An important consequence when plasma is ideally perturbed is a shielding current on a rational surface $\iota=n/m$ to prevent a magnetic island from opening. This indicates mathematically that the normal component of the resonant field has to be vanished on the rational surface, that is, $(\delta\boldsymbol{B}\cdot\boldsymbol{\nabla}\psi)_{mn}=0$. This constraint in ideal MHD enforces inner boundary condition and discontinuous tangential field across the rational surface. The jump of tangential field is [8]

$$\Delta_{mn}\equiv\left[\frac{\partial}{\partial\psi}\frac{\delta\boldsymbol{B}\cdot\boldsymbol{\nabla}\psi}{\boldsymbol{B}\cdot\boldsymbol{\nabla}\varphi}\right]_{mn}\qquad(5)$$

The shielding current $\boldsymbol{j}_s$ is related to the jump as

$$\boldsymbol{j}_s=\frac{\Delta_{mn}ie^{i(m\theta-n\varphi)}}{\mu_0 m(\oint dS\,B^2/|\boldsymbol{\nabla}\psi|^3)}\delta(\psi-\psi_{mn})\boldsymbol{B},\qquad(6)$$

where ψ is a toroidal flux. A total resonant field driving magnetic islands, $(\delta \boldsymbol{B} \cdot \boldsymbol{n})_{mn}$, can be defined by the field produced by the shielding current, $\nabla \times \delta \boldsymbol{B} = \mu_0 \boldsymbol{j}_s$. Note that $\boldsymbol{n}$ is normal to a magnetic surface at $\iota = n/m$, differently from $\boldsymbol{n}_b$ normal to the plasma boundary.

The sustainment of the shielding current is important to improve plasma performance since otherwise an island would open and destruct flux surfaces. This happens when error fields are larger than a critical magnitude. The importance of the shielding current, or equivalently the resonant field has been recently verified in tokamak experiments of locked modes [9]. The use of IPEC has shown that the external field driving the resonant field $(\delta \boldsymbol{B} \cdot \boldsymbol{n})_{mn}$ can be well described through perturbed equilibria.

A long standing supposition, which was supported by cylindrical theory [10, 11], is that the total resonant field driving islands, $(\delta \boldsymbol{B} \cdot \boldsymbol{n})_{mn}$, is proportional to the resonant component of the external field, namely, the external resonant field, $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n})_{mn}$. When this supposition was applied to mode locking experiments in DIII-D and NSTX, the results were paradoxical. When the control coil currents were optimized empirically, the external resonant field was often increased–not decreased as the standard supposition required. However, the IPEC calculation has shown that the total resonant fields were indeed decreased consistently when the control coil currents were optimized.

The failure in the previous method is due to the strong coupling between the resonant field $(\delta \boldsymbol{B} \cdot \boldsymbol{n})_{mn}$ and the external field $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b)$ specified here on the plasma boundary. The poloidal harmonic coupling is very broad and shifted to higher poloidal harmonics than expected, as shown in Fig. 2 [6, 9].

The most important external field driving the total resonant field has to be defined through the coupling, that is, by the first singular vector when decomposing the coupling matrix $\boldsymbol{C}$ between the total resonant field $(\delta \boldsymbol{B} \cdot \boldsymbol{n})_{mn}$ and the external field on the boundary $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b)_{mn}$. If one defines $\boldsymbol{\mathcal{B}}$ with the total resonant field on R rational surfaces, this is written as

$$\boldsymbol{\mathcal{B}} = \boldsymbol{C} \cdot \boldsymbol{\Phi}^x. \tag{7}$$

The i^{th} important mode can be also defined by the i^{th} singular mode in the SVD (Singular Value Decomposition) analysis of $\boldsymbol{C}$. Note again that each i^{th} important mode represents the external field on the plasma boundary, not the total field including plasma response.

A practical way to describe the important modes is to give the amplitude of the external field $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b)$ relative to the plasma boundary in real space. If the most important mode, or the first mode is highly dependent on equilibria when it is mapped in real space, the correction of the mode will be very difficult in practice. Fig. 3 shows the most important external field for $n = 1$ in the (a) DIII-D and (b) NSTX. The three-dimensional field can be constructed as $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b)(\theta, \phi) = C(\theta) cos(n\phi) + S(\theta) sin(n\phi)$, where ϕ is the polar toroidal angle. This has to be distinguished from a magnetic toroidal angle, φ.

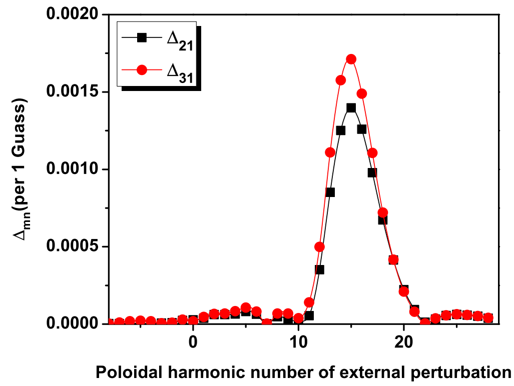

Fig. 2 (a) The typical poloidal harmonic coupling spectrum between the Fourier components of the external error field on the plasma boundary $(\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b)_{mn}$ and the total resonant field, which is here measured by Δ_{21} and Δ_{31}.

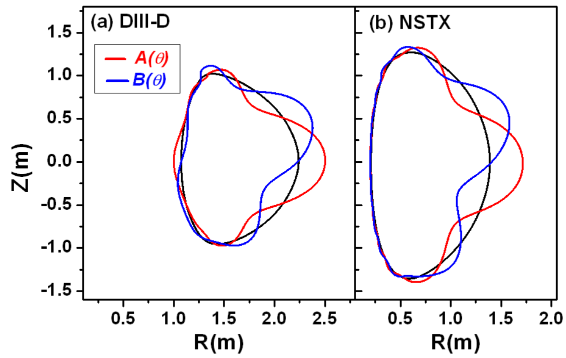

Fig. 3 The most important external field driving the total resonant field on rational surfaces, in the (a) DIII-D and (b) NSTX. The three dimensional distribution can be constructed by $\delta \boldsymbol{B}^x \cdot \boldsymbol{n}_b = A(\theta) cos(\phi) + B(\theta) sin(\phi)$ relative to the plasma boundary (black line).

An important implication is that the most important external field is localized on the outboard midplane. The localizations are very robust and almost regardless of various characteristics in equilibria [9]. For example, Fig. 4 (a) shows very little dependency of the localization on the plasma density, which is represented by the normalized beta, $\beta_n = \langle \beta_t \rangle I_p / a B_{t0}$, where β_t is the toroidal β, I_p is the plasma current, a is the minor radius and B_{t0} is the toroidal magnetic field at the magnetic axis. This explains well that the error-field control coils located on the outboard midplane could effectively mitigate the effect of error fields despite the limitation of poloidal harmonic controllability. The mitigation of error fields, therefore, can be optimized by developing the method to null the most important external field by designing proper control coils.

The detailed information of the coupling between the external field and the total resonant field can be used to many different purposes. For instance, one can find the external field only driving one magnetic island on a partic-

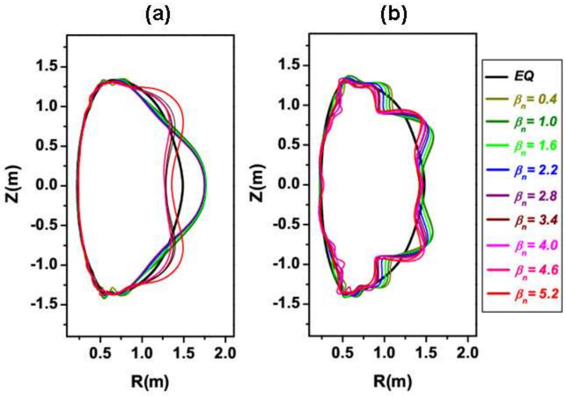

Fig. 4 (a) The most important $n = 1$ external field and (b) the $n = 1$ external field driving only $\iota = 1/2$ as a function of β_n in NSTX. The external fields are represented relatively to the plasma boundary as in Fig. 3.

ular rational surface, as shown in Fig. 4 (b). The external field beyond the most important part is typically not localized and may be too difficult to make relevant corrections in reality. Nonetheless, the specific pattern in the external field can be used to suppress islands in a particular region, for instance, the edge region. This is another important application of IPEC to the suppression of Edge Localized Mode (ELM), by applying intentional perturbations [14] in tokamaks.

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