A model of interaction between magnetic island and drift wave turbulence

Akihiro ISHIZAWA[1)], Patrick H. DIAMOND[2)] and Noriyoshi NAKAJIMA[1)]

[1)]*National Institute for Fusion Science, Toki 509-5292, Japan*

[2)]*University of California San Diego, La Jolla, California 92093-0424, USA*

October 15, 2007

A modified Rutherford equation of magnetic island is introduced. The island equation includes multi-scale interaction with drift wave turbulence and is coupled with turbulent wave energy equation. In this model, drift wave turbulence is prey and magnetic island corresponds to predator. The magnetic island suppresses the growth of turbulence by flattening temperature gradient because of the violation of magnetic surfaces. On the other hand, the turbulence affects perturbed neoclassical bootstrap current in the Rutherford equation through anomalous transport. In these interactions heat flux is fixed, and thus perpendicular thermal diffusion coefficient depends on the turbulence energy and the island width. A stabilizing effect of the turbulence on magnetic island growth is found and new critical island width of neo-classical tearing mode excitation is obtained.

Keywords: magnetic island, drift wave turbulence, multi-scale, NTM

DOI:

1 Introduction

Recently, multi-scale interaction between micro-turbulence and macro-scale magnetohydrodynamic (MHD) instability has been studied extensively [1, 2, 3]. The interaction would play crucial role in the analysis of neoclassical tearing mode (NTM), which limits the beta of tokamak plasmas[4, 5]. The NTM is driven by perturbed neoclassical bootstrap current density inside the separatrix of magnetic island, and it is unstable in high beta tokamak plasma, even if the current density profile is linearly stable against tearing mode. Thus, NTM is a nonlinear instability and it starts to grow when the width of island caused by an external perturbation exceeds a threshold.

The excitation mechanism of NTM is an open problem that is divided into two parts: the threshold and the trigger. The threshold which is called critical island width is evaluated as follows. When magnetic island appears the pressure gradient is reduced inside the island because of strong heat conductivity along magnetic field. This flattening of pressure gradient reduces neoclassical bootstrap current inside the magnetic island and destabilizes the magnetic island. When the island width is very small the flattening is not completed because it is not able to overcome perpendicular transport. Hence, competition between parallel and perpendicular heat transport determines critical island width [6]. The critical island width is also affected by the polarization current [7, 8]. On the other hand, the trigger problem is the process of producing the seed magnetic island caused by external phenomena. Once the width of seed island exceeds the critical island width, then the island grows as NTM. The external phenomena can be MHD modes of different helicities such as sawteeth and edge localized modes and induce the seed island through toroidal mode coupling [9, 10]. The external phenomena can also be micro-turbulence because the turbulence is able to produce seed island through nonlinear mode coupling [11].

The anomalous perpendicular transport due to drift wave turbulence should play crucial role in evaluating critical island width. In addition, magnetic island affects the turbulence simultaneously. We need a model which describes these mutual interaction between them. In this paper, we propose a simple model of interaction between drift wave turbulence and magnetic island. The point of our model is that we fix heat flux instead of thermal diffusivity coefficient when we evaluate temperature gradient. Then we introduce effects of drift wave turbulence on thermal diffusivity and introduce effect of magnetic island which reduces the growth of turbulence by flattening temperature gradient inside the island.

We present our model of interaction between magnetic island and the turbulence in Sec. 2. In Sec. 3 we evaluate critical island width by using the model. Finally we summarize results in Sec. 4.

2 Model of interaction between magnetic island and drift wave turbulence

We present magnetic island equation which couples with drift wave turbulence energy equation. The modified

author's e-mail: ishizawa@nifs.ac.jp

Rutherford equation of magnetic island evolution is [4]

$$\frac{\tau_R}{r_s}\frac{dw}{dt} = r_s\Delta' + \frac{L_s r_s}{B_0 w}\delta J_{BS}, \quad (1)$$

where w, r_s, B_0, τ_R, and L_s are island width, minor radius of resonant surface, uniform toroidal magnetic field, resistive diffusion time, and magnetic shear length, respectively. In this equation Δ' is the stability parameter of tearing mode and is negative so that it is stable against tearing mode. We remark that the island width is related to perturbed magnetic field as $\delta B = k_\theta B_0 w^2/L_s$. Here we assume that density profile is uniform n_0, then the perturbed bootstrap current is

$$\delta J_{BS} = \frac{\epsilon^{1/2} n_0}{B_\theta}\delta\left(\frac{dT_e}{dr}\right), \quad (2)$$

where ϵ, T_e, and B_θ are aspect ratio, electron temperature, and poloidal magnetic field, respectively. When we calculate temperature gradient

$$\frac{dT_e}{dr} = -\frac{Q_e}{\chi_\perp}, \quad (3)$$

we fix heat flux Q_e, and thus perpendicular heat diffusivity $\chi_\perp$ is variable. The heat diffusivity coefficient consists of diffusion due to perturbed magnetic field in the presence of magnetic island $\chi_{\perp island}$, anomalous transport by drift wave turbulence $\chi_{\perp turb}$, and neoclassical transport $\chi_{\perp neo}$ as,

$$\chi_\perp = \chi_{\perp turb} + \chi_{\perp island} + \chi_{neo}, \quad (4)$$

$$\chi_{\perp turb} = \chi_{\perp turb}(\varepsilon) = \chi_0\varepsilon(w), \quad (5)$$

$$\chi_{\perp island} = \chi_\parallel\left(\frac{\delta B}{B_0}\right)^2 = \chi_\parallel\left(\frac{k_\theta}{L_s}w^2\right)^2, \quad (6)$$

where $\chi_{\perp turb}$ is assumed to be proportional to turbulence energy ε, $\chi_\parallel$ is parallel thermal diffusivity, and k_θ is poloidal wave number. Then Eq. (3) is rewritten as,

$$\frac{dT_e}{dr} = \frac{-Q_e}{\chi_0\varepsilon(w) + \chi_\parallel(w^2 k_\theta/L_s)^2 + \chi_{neo}}. \quad (7)$$

By using Eqs. (7) and (2) the island equation (1) is written as,

$$\frac{d\hat{w}}{d\hat{t}} = \hat{\Delta}' + \frac{\hat{\beta}_e}{\hat{w}}\hat{Q}_e\left(\frac{1}{\varepsilon(0)+\hat{\chi}_{neo}} - \frac{1}{\varepsilon(\hat{w}) + \hat{w}^4/\hat{w}_d^4 + \hat{\chi}_{neo}}\right) \quad (8)$$

$$= \hat{\Delta}' + \frac{\hat{\beta}_e}{\hat{w}}\frac{\hat{Q}_e}{\varepsilon(0)+\hat{\chi}_{neo}}\left(1 - \frac{\hat{w}_d^4(\varepsilon(0)+\hat{\chi}_{neo})}{\hat{w}^4 + \hat{w}_d^4(\varepsilon(\hat{w})+\hat{\chi}_{neo})}\right), \quad (9)$$

where $\hat{\chi}_\perp(\varepsilon(\hat{w})) = \varepsilon(\hat{w})$. Normalizations are $\hat{w} = w/r_s$, $\hat{t} = t/\tau_R$, $\hat{\chi}_\parallel = \chi_\parallel/\chi_0$, $\hat{\chi}_{neo} = \chi_{neo}/\chi_0$, $\hat{k}_\perp = k_\theta r_s^2/L_s$, $\hat{L}_s = L_s/L_{T0}$, $\hat{B}_\theta = B_\theta/B_0$, $\hat{Q}_e = Q_e L_{T0}/T_e\chi_0$, $\hat{\beta}_e = \beta_e\epsilon^{1/2}\hat{L}_s/\hat{B}_\theta$, $\hat{w}_d = (\hat{\chi}_\parallel\hat{k}_\perp^2)^{-1/4}$.

In order to close interaction loop between magnetic island and the turbulence we need equation of turbulence wave energy[12]

$$\frac{d\varepsilon}{dt} = \gamma(\varepsilon, w)\varepsilon - \beta\varepsilon^2, \quad (10)$$

where

$$\gamma(\varepsilon, w) = \gamma_0 L_{T0}\left(\frac{1}{L_T(\varepsilon,w)} - \frac{1}{L_{Tcr}}\right)_{>0}, \quad (11)$$

where $(f)_{>0}$ is zero if f is negative. In this equation we include feedback from magnetic island to the turbulence. The strong parallel thermal diffusion flattens pressure gradient inside the separatrix of the island. This reduces growth rate γ by increasing length scale of temperature gradient

$$\frac{1}{L_T(\varepsilon,w)} = \frac{-1}{T_e}\frac{dT_e}{dr} \quad (12)$$

$$= \frac{1}{L_{T0}}\frac{\hat{Q}_e}{\hat{w}^4 + \hat{w}_d^4(\varepsilon(\hat{w})+\hat{\chi}_{neo})}. \quad (13)$$

When we calculate the temperature gradient we fix heat flux again. By substituting this equation to Eq. (11) we have equation of turbulence energy including the effect of magnetic island as,

$$\frac{1}{\tau_R\gamma_0}\frac{d\varepsilon}{d\hat{t}} = \left(\frac{\hat{Q}_e}{\hat{w}^4 + \hat{w}_d^4(\varepsilon(\hat{w})+\hat{\chi}_{neo})} - \frac{1}{\hat{L}_{Tcr}}\right)_{>0}\varepsilon - \hat{\beta}\varepsilon^2, \quad (14)$$

where $\hat{L}_{Tcr} = L_{Tcr}/L_{T0}$ and $\hat{\beta} = \beta/\gamma_0$. Hence, we have obtained a closed set of equations of magnetic island and the turbulence Eqs. (9) and (14).

3 Critical island width of NTM

We have established a model which is able to evaluate critical island width of NTM. We consider a situation that the turbulence saturates because drift frequency time, which is characteristic time scale of drift wave turbulence, is much faster than the resistive diffusion time $\tau_R\gamma_0 >> 1$, where $\gamma_0 \approx \omega_*$. Thus, we neglect left hand side of Eq. (14) and have the equation of turbulence energy as,

$$\varepsilon(w) = \frac{-1}{\hat{\beta}\hat{L}_{cr}} + \frac{1}{2}\left(-F(\hat{w}) + \sqrt{F(\hat{w})^2 + 4\hat{Q}_e/\hat{\beta}}\right), \quad (15)$$

where $F(\hat{w}) = \hat{w}^4/\hat{w}_d^4 + \hat{\chi}_{neo} - \frac{1}{\hat{\beta}\hat{L}_{cr}}$. Substituting this $\varepsilon(w)$ into Eq. (9) we have the island equation of our model.

We show curves of island growth $d\hat{w}/d\hat{t}$ calculated by our model and by the standard model

$$\frac{d\hat{w}}{d\hat{t}} = \hat{\Delta}' + \frac{\hat{\beta}_e}{\hat{w}}\frac{\hat{w}^2}{\hat{w}^2 + \hat{w}_d^2} \quad (16)$$

in Fig. 1. Here we set parameters $\hat{\Delta}' = -1$, $\hat{\beta}_e = 0.2$, $\hat{\chi}_{neo} = 0.1$, $\hat{Q}_e = 2$, $\hat{\chi}_\parallel\hat{k}_\perp^2 = 10^9$, $\hat{\beta} = 0.6$, and $\hat{L}_{Tcr} = 0.5$. The critical island width $w_{critical}/r_s$ is given by $d\hat{w}/d\hat{t} = 0$. The critical island width of our model is larger than the standard model. Thus, the turbulence has a stabilizing effect on the excitation of NTM. Notice that we cannot apply our model to evaluate saturated island width because a large magnetic island strongly reduces turbulence energy and makes it negative in our model. For the above parameter set the turbulence energy ε is positive when $w/r_s < 0.00548$. Figure 2 shows curves of parallel diffusion coefficient $\chi_{\parallel}$ as a function of critical island width. Our model implies $w_{critical}/r_s \propto \hat{\chi}_{\parallel}^{-1/3}$, while the standard model implies $w_{critical}/r_s \propto \hat{\chi}_{\parallel}^{-1/2}$. Figure 3 shows curves of plasma beta β_e as a function of critical island width. Our model implies $w_{critical}/r_s \propto \hat{\beta}_e^{-1/3}$, while the standard model implies $w_{critical}/r_s \propto \hat{\beta}_e^{-1}$, and thus our model suggests weak dependence of $w_{critical}$ on β_e compared to the standard model.

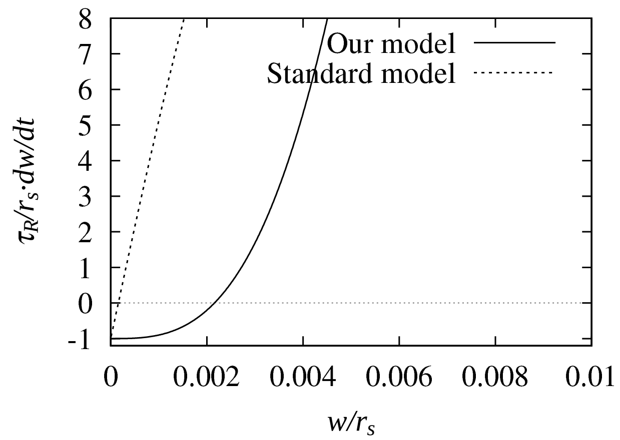

Fig. 1 Curves of island growth $d\hat{w}/d\hat{t}$ as a function of island width w/r_s. Solid curve indicates our model and dashed curve indicates standard model.

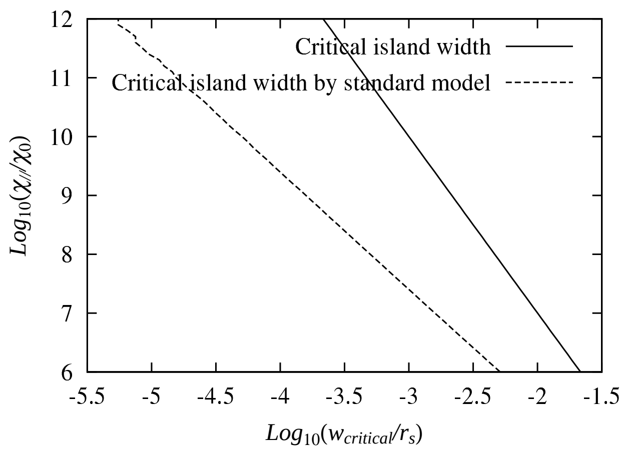

Fig. 2 Curves of parallel diffusion coefficient $\chi_{\parallel}$ as a function of critical island width. Solid curve indicates our model and dashed curve indicates standard model.

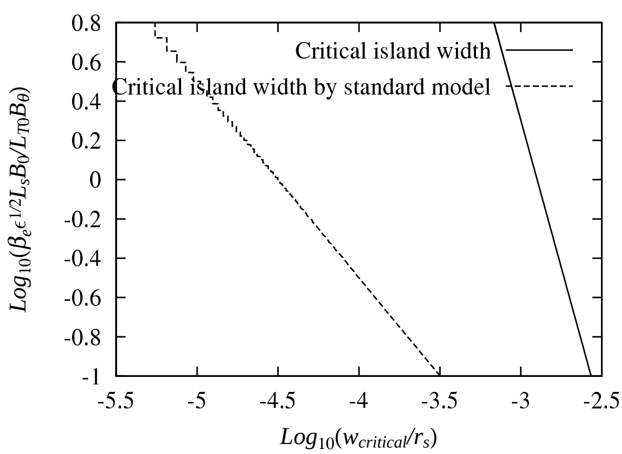

Fig. 3 Curves of plasma beta β_e as a function of critical island width. Solid curve indicates our model and dashed curve indicates standard model.

4 Summary and discussion

We have obtained a predator-prey model of interaction between magnetic island and drift wave turbulence. The turbulence affects perturbed bootstrap current in the island equation of NTM through anomalous perpendicular transport. When we evaluate bootstrap current we fix heat flux and make heat diffusivity depend on the turbulence and the island width. In order to close interaction loop between the magnetic island and the turbulence we introduce turbulence wave energy equation including effect of magnetic island. The magnetic island makes temperature gradient flatten inside it and reduces growth rate of the turbulence.

Our model predicts larger critical island width than the one by standard model. This implies that the drift wave turbulence has stabilizing effect on magnetic island excitation of NTM. In addition we found new β_e scaling of the critical island width of NTM $w_{critical}/r_s \propto \hat{\beta}_e^{-1/3}$. In order to compare our model with experimental observation we would include the polarization current effect in our future work.

[1] C. J. McDevitt and P. H. Diamond, Phys. Plasmas **13**, 032302 (2006).
[2] M. Yagi, S. Yoshida, S.-I. Itoh, H. Naitou, H. Nagahara, J.-N. Leboeuf, K. Itoh, T. Matsumoto, S. Tokuda and M. Azumi, Nucl. Fusion **45**, 900 (2005).
[3] A. Ishizawa and N. Nakajima, Phys. Plasmas **14**, 040702 (2007).
[4] R. J. La Haye, Phys. Plasmas **13**, 055501 (2006).
[5] C. C. Hegna, Phys. Plasmas **5**, 1767 (1998).
[6] R. Fitzpatrick, Phys. Plasmas **2**, 825 (1995).
[7] J. W. Connor, F. L. Waelbroeck, and H. R. Wilson, Phys. Plasmas **8**, 2835 (2001).
[8] R. Fitzpatrick and F. L. Waelbroeck, Phys. Plasmas **12**, 022307 (2005).
[9] R. J. La Haye and O. Sauter, Nucl. Fusion **38**, 987 (1998).
[10] C. C. Hegna, et al, Phys. Plasmas **6**, 130 (1999).
[11] S.-I. Itoh, K. Itoh, and M. Yagi, Phys. Rev. Lett. **91**, 045003 (2003).
[12] P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Controlled Fusion **47**, R35 (2005).