Study of poloidal electric field generation by ECH in a helical plasma

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Poloidal electric field generated by ECH is investigated in a helical plasma. A linearized Fokker-Planck equation is solved by the adjoint method assuming a helically symmetic configuration for simplicity. It is found that the generated electric potential in the helical plasma is about 20% larger than that in the tokamak plasma. This indicates that about two times greater poloidal electic field is generated in a l=2 helical plasma.

Keywords: poloidal electric field, ECH, helical plasma

1 Introduction

Electron cyclotron heating (ECH) accelerates electrons perpendicularly and generates trapped particles, which tend to localize at the resonance region. Those resonant trapped particles would enhance a inhomogeneous electrostatic potential on a flux surface and generate a poloidal electric field resulting in the large radial transport due to the radial drift by $\boldsymbol{E} \times \boldsymbol{B}$drift.

M.Taguchi(1992)[1] evaluated the poloidal electric field generated by ECH in a tokamak plasma by solving an adjoint equation to the linearized Fokker-Planck equation with a quasi-linear diffusion term. Because of the deeper magnetic ripple by the helical coils the larger poloidal electric field would be generated in helical plasma than that in tokamaks.

In this paper, the poloidal electric field generated by ECH is investigated in a helical plasma in the collisionless regime. Applying the same method by Taguchi, the poloidal electric field is calculated assuming helically symmetric configuration and compared with one in a tokamak plasma.

2 Basic equation

We consider a toroidal plasma, where the magnetic field is expressed in the Booer coordinates (ψ, θ, ζ), where ψ, θ, ζ are the toroidal flux, the poloidal angle and the torodial angle, respectively. When the RF power is assumed to be weak, the gyrophase-averaged distribution function for the electrons is slightly distorted from the Maxwell distribution function $f_{e0} = n_{e0}(m_e/2\pi T_e)^{3/2} \exp(-m_e v^2/2T_e)$, where m_e and T_e are the electron rest mass and electron temperature. The distorted part f_{e1} is determined by the lineraized drift kinetic equation:

$$v_{\parallel}\boldsymbol{b} \cdot \nabla f_{e1} - C_e(f_{e1}) = ev_{\parallel}\boldsymbol{E} \cdot \boldsymbol{b}\frac{\partial f_{e0}}{\partial W} + Q_{\rm rf}(f_{e0}) - \frac{\partial f_{e0}}{\partial t} \tag{1}$$

where the energy $W = \frac{1}{2}mv^2$, the magnetic moment $\mu = v_{\perp}^2/2B$ and $\sigma = v_{\parallel}/|v_{\parallel}|$ as independent variables in velocity space. $\boldsymbol{b} = \boldsymbol{B}/B$,$v_{\parallel} = \boldsymbol{v} \cdot \boldsymbol{b}$, $v_{\perp} = (v^2 - v_{\parallel}^2)^{1/2}$, C_e is the linearized Fokker-Plack collision operator, and $Q_{\rm rf}$ and $\boldsymbol{E} = -\nabla\phi$ are the velocity-space diffusion and the poloidal electric field due to ECH, respectively. To calculate the electrostatic potential ϕ we introduce the adjoint equation

$$v_{\parallel}\boldsymbol{b} \cdot \nabla \tilde{f}_{m,n} + C_e(\tilde{f}_{m,n}) = -\frac{\nu_{ee} f_{e0} \mathrm{e}^{\mathrm{i}(m\theta+n\zeta)}}{\sqrt{g}} \tag{2}$$

where $\nu_{ee} = (4\pi n_{e0} e^4 \ln\Lambda)/m_e^2 v_e^3, v_e = (2T_e/m_e)^{1/2}$, $\ln\Lambda$ is the Coulomb logarithm, θ is the poloidal angle and $1/\sqrt{g}$ is Jacobian. We multiply (2) by f_{e1}/f_{e0}, integrating over velocity space and averaging over the flux surface. Then, the electrostatic potential ϕ can be expressed as

$$\frac{n_{e0}e}{T_e}\left\langle \frac{1}{\sqrt{g}}\mathrm{e}^{\mathrm{i}(m\theta+n\zeta)}\phi \right\rangle = -\frac{1}{\nu_{ee}}\left\langle \int \frac{\tilde{f}_{m,n}}{f_{e0}}\left[Q_{\rm rf}(f_{e0}) - \frac{\partial f_{e0}}{\partial t}\right] d\boldsymbol{v} \right\rangle + \left\langle \frac{n_{e1}}{\sqrt{g}}\mathrm{e}^{\mathrm{i}(m\theta+n\zeta)} \right\rangle \tag{3}$$

where

$$n_{e1} \equiv \int f_{e1} d\boldsymbol{v} \tag{4}$$

Here, $\langle A \rangle$ is the flux surface average of a quantity A:

$$\langle A \rangle = \oint \frac{d\theta d\zeta}{\sqrt{g}} A / \oint \frac{d\theta d\zeta}{\sqrt{g}}$$

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And we have used the relation

$$\left\langle \int f v_{\parallel} \boldsymbol{b} \cdot \nabla g d\boldsymbol{v} \right\rangle = -\left\langle \int g v_{\parallel} \boldsymbol{b} \cdot \nabla f d\boldsymbol{v} \right\rangle$$

and the adjoint property of the collision operator,

$$\int \frac{f_{e1}}{f_{e0}} C_e(\tilde{f}_{m,n}) d\boldsymbol{v} = \int \frac{\tilde{f}_{m,n}}{f_{e0}} C_e(f_{e1}) d\boldsymbol{v}$$

A similar relation can also be derived for ions. It becomes

$$\frac{n_{i0} e Z_i}{T_i} \left\langle \frac{1}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \right\rangle = -\left\langle \frac{n_{i1}}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \right\rangle \quad (5)$$

Z_i is the charge number of ion and T_i is the ion temperature.

We expand the electrostatic potential in a Fourier series:

$$\phi = \sum_{m,n=-\infty}^{\infty} \phi_{m,n} \mathrm{e}^{-\mathrm{i}(m\theta + n\zeta)} \quad (6)$$

The left hand side of (3) becomes

$$\begin{aligned} LHS &= \frac{n_{e0} e}{T_e} \left\langle \frac{1}{\sqrt{g}} \phi_{m,n} \right\rangle \\ &= \frac{n_{e0} e}{T_e} \frac{2\pi}{\oint \sqrt{g} d\theta d\zeta} \phi_{m,n} \end{aligned} \quad (7)$$

Using charge neutrality, the second term in the right hand side of (3) becomes

$$\begin{aligned} \left\langle \frac{n_{e1}}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \right\rangle &= \left\langle \frac{Z_i n_{i1}}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \right\rangle \\ &= -Z_i \frac{n_{i0} e Z_i}{T_i} \left\langle \frac{1}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \phi \right\rangle \\ &= -\frac{n_{e0} e Z_i}{T_i} \frac{2\pi}{\oint \sqrt{g} d\theta d\zeta} \phi_{m,n} \end{aligned} \quad (8)$$

Transposing the second term in the right hand side and arranging the equation give us

$$\frac{e\phi_{m,n}}{T_e} = -\frac{\nu_{\mathrm{rf}}}{\nu_{ee}} \frac{F_{m,n}}{1 + Z_i T_e / T_i} \quad (9)$$

$$F_{m,n} = \frac{\left\langle \int \frac{\tilde{f}_{m,n}}{f_{e0}} \left[Q_{\mathrm{rf}}(f_{e0}) - \frac{\partial f_{e0}}{\partial t} \right] d\boldsymbol{v} \right\rangle}{\left\langle \int \frac{W}{T_e} Q_{\mathrm{rf}} d\boldsymbol{v} \right\rangle} \frac{\oint \sqrt{g} d\theta d\zeta}{2\pi} \quad (10)$$

where

$$\nu_{\mathrm{rf}} \equiv \frac{1}{n_{e0} T_e} \left\langle \int W Q_{\mathrm{rf}} d\boldsymbol{v} \right\rangle \quad (11)$$

3 Solution of the adjoint equation

In the collisionless regime we expand $\tilde{f}_{m,n}$ as $\tilde{f}_{m,n} = \tilde{f}^{(0)}_{m,n} + \tilde{f}^{(1)}_{m,n} + \cdots$. Then the distribution functions $\tilde{f}^{(0)}_{m,n}$ and $\tilde{f}^{(1)}_{m,n}$ satisfy the equations

$$v_{\parallel} \boldsymbol{b} \cdot \nabla \tilde{f}^{(0)}_{m,n} = 0 \quad (12)$$

$$v_{\parallel} \boldsymbol{b} \cdot \tilde{f}^{(1)}_{m,n} + C_e(\tilde{f}^{(0)}_{m,n}) = -\frac{\nu_{ee} f_{e0}}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \quad (13)$$

This function $\tilde{f}^{(0)}_{m,n}$ is determined by the solubility condition for (13). The linearized Fokker-Planck collision operation C_e is approximated well for $v > (2-3)v_3$ by

$$\begin{aligned} C_e(f) \approx (1 + Z_i) \frac{\nu_{ee}}{x^3} \frac{v_{\parallel}}{B} \frac{\partial}{\partial \mu} \left(\mu v_{\parallel} \frac{\partial f}{\partial \mu} \right) \\ - f_{e0} \frac{\nu_{ee}}{x^2} \frac{\partial}{\partial x} \left(\frac{f}{f_{e0}} \right) \end{aligned} \quad (14)$$

where $x = v/v_e$. For simplicity we use this approximate collision term here. Then the solubility condition can be written as

$$\begin{aligned} 2(1 + Z_i) \frac{1}{x^3} \frac{\partial}{\partial \lambda} \left[\lambda \left\langle (1 - \lambda B)^{1/2} \right\rangle_b \frac{\partial \tilde{f}^{(0)}_{m,n}}{\partial \lambda} \right] \\ - \left\langle \frac{B}{(1 - \lambda B)^{1/2}} \right\rangle_b \frac{f_{e0}}{x^2} \frac{\partial}{\partial x} \left(\frac{\tilde{f}^{(0)}_{m,n}}{f_{e0}} \right) \\ = -\left\langle \frac{B}{(1 - \lambda B)^{1/2}} \frac{1}{\sqrt{g}} \mathrm{e}^{\mathrm{i}(m\theta + n\zeta)} \right\rangle_b f_{e0} \end{aligned} \quad (15)$$

$\lambda = 2\mu/v^2$. Here the bounce average $\langle A \rangle_b$ is defined by

$$\langle A \rangle_b = \begin{cases} \langle A \rangle & (0 \le \lambda < \lambda_c) \\ \iint \sqrt{g} d\theta d\zeta A / \oint \sqrt{g} d\theta d\zeta & (\lambda_c < \lambda \le \lambda_{\max}) \end{cases}$$

where $\lambda_c = 1/B_{\max} \lambda_{\max} = 1/B_{\min}$. In order to solve (15), we consider the following eigenvalue equation:

$$\begin{aligned} \frac{d}{d\lambda} \left[\lambda \left\langle (1 - \lambda B)^{1/2} \right\rangle_b \frac{dG}{d\lambda} \right] \\ + \frac{1}{2} \left\langle \frac{B}{(1 - \lambda B)^{1/2}} \right\rangle_b \kappa G = 0 \\ (0 \le \lambda < \lambda_c,\ \lambda_c < \lambda \le \lambda_{\max}) \end{aligned} \quad (16)$$

subject to the boundary conditions

$$G(\lambda_c - 0) = G(\lambda_c + 0)$$

$$\frac{dG(\lambda_c - 0)}{d\lambda} = \frac{dG(\lambda_c + 0)}{d\lambda}$$

$$G(0) = 1, \quad \frac{dG(\lambda_{\max})}{d\lambda}\ is\ finite$$

The eigenfunctions G_n with eigenvalues κ_n satisfy the orthogonality condition

$$\int_0^{\lambda_{\max}} \left\langle \frac{B}{(1-\lambda B)^{\frac{1}{2}}} \right\rangle_b G_n G_m d\lambda = 0 \quad (\kappa_n \neq \kappa_m)$$

Using this, we can express the solution of (16) in the form

$$\tilde{f}^{(0)}_{m,n} = \sum_{n=1}^{\infty} \frac{x^3}{(1+Z_i)\kappa_n + 3} G_n(\lambda) S_n f_{e0} \tag{17}$$

where $\kappa_0(=0) < \kappa_1 < \kappa_2 < \cdots$ and

$$S_n = \frac{\int_0^{\lambda_{\max}} G_n \left\langle \frac{B}{(1-\lambda B)^{\frac{1}{2}}} \frac{1}{\sqrt{g}} e^{i(m\theta + n\zeta)} \right\rangle_b d\lambda}{\int_0^{\lambda_{\max}} G_n^2 \left\langle \frac{B}{(1-\lambda B)^{\frac{1}{2}}} \right\rangle_b d\lambda}$$

Note that the eigenfunction $G_0 = 1$, so that $S_0 = 0$. Substituting the solution (17) into (9), we obtain

$$F_{m,n} = \frac{\oint \sqrt{g} d\theta d\zeta}{2\pi} \frac{1}{\left\langle \int \frac{W}{T_e} Q_{\rm rf} d\boldsymbol{v} \right\rangle} \times \sum_{n=1}^{\infty} \frac{S_n}{(1+Z_i)\kappa_n + 3} \left\langle \int x^3 G_n Q_{\rm rf} d\boldsymbol{v} \right\rangle \tag{18}$$

since

$$\left\langle \int x^3 G_n \frac{\partial f_{e0}}{\partial t} d\boldsymbol{v} \right\rangle \propto \int_0^{\lambda_{\max}} \left\langle \frac{B}{(1-\lambda B)^{\frac{1}{2}}} \right\rangle_b G_n G_0 d\lambda = 0 \quad (n \geq 1) \tag{19}$$

4 Evaluation of $\phi_{m,n}$

The quasi-linear diffusion term $Q_{\rm rf}$ for electron-cyclotron damping is written as

$$Q_{\rm RF} = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left[D v_\perp \left(\frac{v_\perp}{v_e} \right)^{2(l-1)} \times \delta \left(\omega - \frac{l\omega_c}{\gamma} - k_\parallel v_\parallel \right) \frac{\partial f_{e0}}{\partial v_\perp} \right] \tag{20}$$

where $\gamma = [a - (v/c)^2]^{-\frac{1}{2}}, c$ is the speed of light in vacuum and l is the harmonic number; ω and ω_c are the frequency of the injected wave and the non-relativistic electron-cyclotron frequency respectively. Throughout this paper we consider only the X-mode wave. Then D is taken to be independent of the velocity. Moreover, for simplicity, we approximate the relativistic resonance condition as

$$\omega - \frac{l\omega_c}{\gamma} - k_\parallel v_\parallel \approx \omega - l\omega_c \left(1 - \frac{v^2}{2c^2} \right) - k_\parallel v_\parallel = 0 \tag{21}$$

This resonance condition becomes a semicircle in velocity space

$$\left(\frac{v_\parallel}{v_e} - \frac{1}{2S} \right)^2 + \left(\frac{v_\perp}{v_e} \right)^2 = \frac{1 - 4u_0 S}{(2S)^2} \tag{22}$$

where $u_0 = (\omega - l\omega_c)/k_\parallel v_e$ is the normalized parallel velocity of the resonant electrons and $S = l\omega_c v_e / k_\parallel c^2$ relates the strength of the relativistic correction.

4.1 Helical symmetric configuration

We assume a helical symmetric plasma with magnetic configuration $B = B_h(1 - \epsilon_h \cos\theta')$ ($\theta' = M(\theta + \frac{N}{M}\zeta)$). Using $x = \lambda B_h(1 - \epsilon_h)$ we rewrite (16) and boundary condition .

$$x\alpha \frac{d^2}{dx^2} G + \left(x\frac{d\alpha}{dx} + \alpha \right) \frac{d}{dx} G + \frac{1}{2}\beta\kappa G = 0 \tag{23}$$

$$\alpha = \int_{\theta_1'}^{\theta_2'} \frac{1}{(1 - \epsilon_h \cos\theta')} \left(1 - \frac{1 - \epsilon_h \cos\theta'}{1 - \epsilon_h} x \right)^{1/2} d\theta' \tag{24}$$

$$\beta = \int_{\theta_1'}^{\theta_2'} \frac{1}{(1 - \epsilon_h)} \left(1 - \frac{1 - \epsilon_h \cos\theta'}{1 - \epsilon_h} x \right)^{-1/2} d\theta' \tag{25}$$

$$\frac{d\alpha}{dx} = -\frac{1}{2} \int_{\theta_1'}^{\theta_2'} \frac{1}{(1 - \epsilon_h)} \left(1 - \frac{1 - \epsilon_h \cos\theta'}{1 - \epsilon_h} x \right)^{-1/2} d\theta' \tag{26}$$

$$G \to \text{const} \times [1 - \frac{1}{2}\kappa(1 - x)] \qquad x \to 1 \tag{27}$$

$$G \to 1 - \frac{1}{2}\kappa x \qquad x \to 0 \tag{28}$$

We evaluate the eigenfunctions G_n and eigenvalues κ_n by a relaxation method. The eigenvalues κ_n are given in table 1.

$\epsilon \backslash n$	1	2	3	4	5
0.1	2.886	8.400	15417	26.643	40.196
0.3	2.377	4.983	12.837	20.314	31.568

Table 1 eigenvaluesκ_n

4.2 Results

We evaluated the poloidal electric field in a helical symmetric plasma and the results are compared with tokamak ones by Taguchi[1]. The poloidal electric field is represented as Eq.(9) and is analayzed in a helical plasma.

Figure 1 shows F_m values in a tokamak and a helical plasma changing the magnetic field ripple ϵ; 0.1, 0.3 and the normalized parallel velocity; 0, 0.5, -1.0. We can see that the F_m increases as the ϵ becomes larger in both helical and tokamak plasma cases. And the differences of F_m is larger in the larger ϵ cases. The value of $\phi_1^h/\phi_1^t \sim 1.25 - 2$ in the case of F_m with $\epsilon = 0.3$. The values of F_m with various ϵ of a helical plasma are shown in figure 2.

5 Conclusion

We have studied the poloidal electric field generated by ECH in a helical plasma. The linearized Fokker-Planck equation has been solved by the adjoint method assuming a helically symmetic configuration for simplicity. We have found that the generated electric potential of the helical plasma is about 20% larger than that of the tokamak configuration. This indicates that the two times greater poloidal electic field is generated in a l=2 helical plasma.

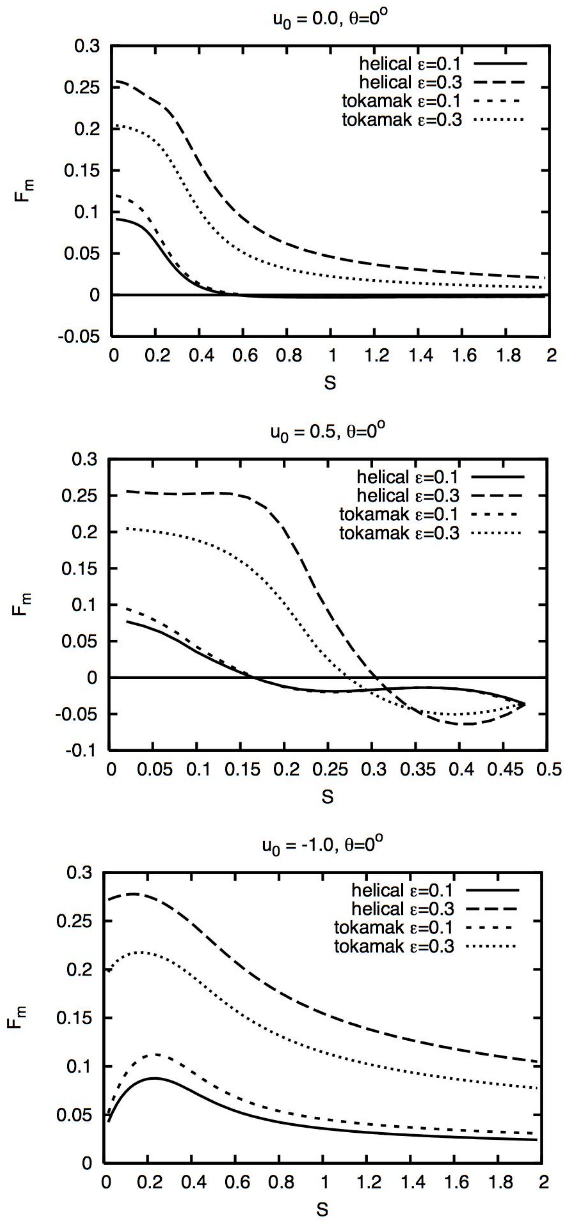

Fig. 1 Comparisons of F_m in the tokamak and helical plasma

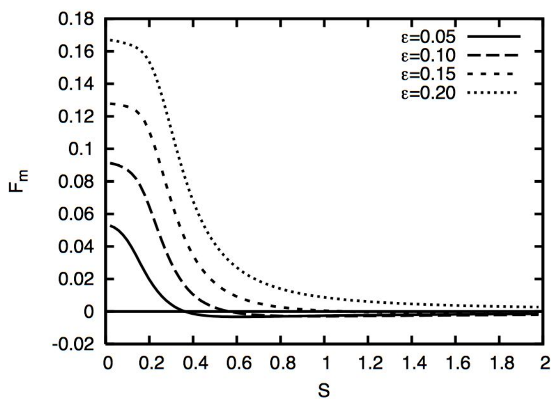

Fig. 2 Plots of F_m for helical plasma with $\epsilon = 0.05$, 0.1, 0.15, 0.2

[1] M.Taguchi, J.Plasma Physics. **47**(1992), part 2, 261
[2] S.Murakami, et al, Nucl. Fusion **40**(2000) 693