# Langevin equation for guidng center motion and its application to neoclassical transport theory

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The Langevin equation for guiding center motion with Coulomb collisions is derived and applied to numerical calculation of the neoclasscial transport in toroidal plasmas. In the theory of Brownian motion, the Fokker-Planck equation is closely related to the Langevin equation [1]. This close relation allows us to reduce the drift kinetic equation to the Langevin equation for guiding center motion. The reduced set of equations corresponds to those used in the calculation of neoclassical transport coefficients by Monte Carlo methods. For these equations, numerical schemes for solving stochastic differential equations (SDEs) can be used [2] and the higher order or implicit schemes would improve the accuracy of numerical solutions.

Here, we begin with the derivation of the Langevin equation for guiding center motion and discuss the relation to the characteristics of the drift kinetic equations. As a result, the friction part of the Coulomb collision operator is incorporated into the guiding center equations and the diffusion part is described by the pre-point time discritization rule of the Ito stochastic process [1]. For numerical implementation, we developed SDE solvers based on both explicit and implicit schemes up to fourth-order (in the sense of ODEs). The numerical tests for stiff and nonstiff SDEs showed that, for the stiff SDEs, the implicit schemes give higher accuracy than explicit ones. Such stiffness would also be relevant to the equations used in the neoclassical transport theory. As applications for the plasma transport calculation, we examined the relaxation phenomena in velocity space and the linear neoclassical transport in axisymmetric systems. In summary, the guiding center motion with Coulomb collisions can be formulated as the Langevin equation and the numerical method for solving SDEs can be applicable to the Monte Carlo methods.

[1] J.Dunkel and P.Hänggi, Phys. Rev. E **72**, 037106 (2005)
[2] P.E.Kloeden and E.Platen, *"Numerical Solution of Stochastic Differential Equations"* (Springer-Verlag, Berlin, 1992)