The Design Windows and Economical Potential of Heliotron Reactors
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Outline

- Background and Objectives
- HeliCos code - Basic equations and calculation flow
- The design windows boundary and $\beta$, $\gamma$ dependence
- Estimating magnet cost based on ITER data
- Economical potential of heliotron reactors
- Conclusion
Background

1) Many studies on system designs and economic evaluations of magnetic fusion reactor had been carried out such as the Generomac by Sheffield and the ARIES design studies. The most had concluded the smaller reactor with high beta was necessary for attractive fusion reactors from the point of view of mass-power density.

But with high beta we should consider decreasing magnetic field, on the other hand enlarging reactor size to ensure enough blanket space and to avoid too much heat flux and neutron loads.

2) As we have much experience of large size superconducting magnet from LHD and ITER construction, we should estimate the magnet cost based on some realistic database.

Objectives

1) To identify the equations dominating the design windows, and to develop the system design code linked with cost model.

2) To search the design windows and to estimate the economical potential of heliotron reactors.
System Design and Mass-Cost Estimating Code (HeliCos) - Major tasks and methodology-

- System Design Code
  - Standard Design Case
    - Weight-Cost Analysis

- Design Windows Analysis
  - Plasma ↔ Magnet ↔ Blanket
    - Identifying Critical Parameters
      - $\gamma, \beta$ dependence
      - $\Delta d$ limitation
      - J-Bmax condition
      - H factor in high $\beta$
      - Blanket & T sys.

- 4GW Standard Plant

- Cost Comparison to Previous Works
  - (The next task with detail cost)

- Mass-Cost Estimating Model (HeliCos)
  - Estimating COE (cost of electricity)

- Magnet Cost Analysis based on ITER
  - $\gamma, \beta$ dependence
  - $\Delta d$ limitation
  - J-Bmax condition
  - H factor in high $\beta$
  - Blanket & T sys.

- Conductor cost with $j, B_{\text{max}}$

- Mass-Cost Database

- FFHR-2m1 Design Study
- FFHR-2m2 based on LHD 3.6m case
HeliCos code - Major design parameters and relationships (1)

1) Basic geometry of plasma and helical coils given with Rp, γ, and Δd

→ The fat plasma increases plasma volume but decreases blanket space. → It depends on γ.

→ We can consider the similar shape of the LHD 3.6m inward shift cases for the high performance of plasma with variable γ (LHD → polarity l=2, field periods m=10, coil pitch parameter γ = (l/m)/(Rc/ac) is corresponding aspect ratio, γ=1.15~1.25 variable in LHD experiment)

→ The \( a_p \) and \( a_{pin} \) are given by the equations of regression of LHD data.

\( (a_p, a_c : \text{minor radius of plasma and coil}, \ a_{pin} : \text{inner minimum plasma radius}) \)

\[
\begin{align*}
    a_p &= a_c (-1.3577+1.603 \times \gamma) = 0.06292 \times \gamma^{4.5} R_p \rightarrow \tau_E \\
    a_{pin} &= (-1.2479 + 1.2524 \gamma) \times (R_c / 3.9)
\end{align*}
\]

→ Blanket space

\( \Delta d = a_c - (R_c-R_p) - a_{pin} - H/2 - \Delta t \quad --------(1) \)

Δt : thermal insulation space (Δt = 0.1 m)

H : The coil thickness depending on the \( I_{HC} \) and j

\( (I_{HC}, j : \text{current and current density of helical coils}) \)

→ \( I_{HC} = (R_p / B_0)/(2m) \times 10 \)

\( H = (I_{HC} / (j \times W/H))^{0.5} \ (W: \text{width of HC}) \)

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**FIG.1** The profile of plasma, helical coil and blanket. The required \( \Delta d \) gives the minimum \( R_p \)
Major design parameters and their relationships (2)

2) Fusion power given with \( B_0, \beta, \) and \( V_P \)

The fusion power is calculated by the volume integration of fusion power density \( p_f \) using the following \( <\sigma v>_D\) and the plasma profile assumptions in the HeliCos code.

\[
p_f = n_T n_D <\sigma v>_D V p \times 17.58(\text{MeV}) \times 1.6021 \times 10^{-19} \text{ (J/eV) } \times 10^{-3} \ \text{[GW]}
\]

\[
<\sigma v>_D = 0.97397 \times 10^{-22} \times \exp\{0.038245 (\ln(Ti))^3 - 1.0074 (\ln(Ti))^2 + 6.3997 \ln(Ti) - 9.75\}(\text{m}^3/\text{s})
\]

\( \rightarrow \) parabolic profile index \( a_n \) : plasma density, \( a_T \) : ion temperature

\( \rightarrow \) a good approximation \( <\sigma v>_D \propto T_i^2 \) for \( T_i \sim 10 \text{keV} \) using for sensitivity studies.

\[
P_f = \frac{0.06272}{(1+2a_n+2a_T)} \times n_e(0)^2 T_i(0)^2 V_P \times 10^{-6} \propto \beta^2 B_0^4 V_P \ \text{[GW]} \quad n_e:10^{19}/\text{m}^3, \ T_i: \text{keV} \quad (2)
\]

3) Power balance conditions given with scaling low ISS04 in H factor equations

The power balance is described using the required energy confinement time \( \tau_{Er} \).

\[
P_\alpha - R_{loss} = W_p / \tau_{Er} \quad (P_\alpha = 0.2 P_f, \ f_\alpha : \alpha \text{ heating efficiency, } R_{loss} \text{: Radiation loss})
\]

\[
W_p : \text{plasma stored energy, } W_p \propto n_e(0)T_i(0)V_P
\]

\[
\tau_{E,(ISS04)} = 0.134 (f_\alpha P_\alpha - R_{loss})^{-0.61} n_e^{0.54} B_0^{0.84} R_p^{0.64} a_p^{2.28} \tau_{2/3}^{0.41}
\]

\[
= 6.23 \times 10^{-5} R_p^{1.09} \gamma^{2.98} (p_f(1 - r_{loss}))^{-0.61} B_0^{0.84} n_e^{0.54} \ \text{[ms]}
\]

(Expressed only with the \( R_p \gamma, B_0, p_f = P_f / V_p, r_{loss} = R_{loss} / (0.2 f_\alpha P_f) \), \( r_{loss} \): radiation loss rate)

\[
H_f (ISS04) = \frac{\tau_{Er}}{\tau_{E,(ISS04)}} = 76.4 \times f_{np} \times R_p^{-1.09} \gamma^{-2.98} p_f^{-0.16} B_0^{-1.11} (1 - r_{loss})^{-0.66} \quad (3)
\]
Design points given with the cross points of the three basic equations

1) The function $B_0(R_p, \gamma, \Delta d, j)$ from the $\Delta d$-equation (1)

$$B_0 = (16j/R_p)((0.2633 - 0.1312 \gamma) R_p - 20.41(\Delta d + 0.1))^2 \ [T]$$  (4)

2) The function $B_0(R_p, \gamma, \beta, P_f)$ from the $P_f$-equation (2)

$$B_0 = 92.64 \ P_f^{1/4} \beta^{-1/2} \gamma^{-2.22} \ R_p^{-3/4} \ [T]$$  (5)

3) The function $B_0(R_p, \gamma, H_f, P_f)$ from the $H_f$-equation

$$H_f = 76.4 \times f_{np} \times R_p^{-1.09} \gamma^{-2.98} \ p_f^{-0.16} \ (1 - r_{loss})^{-0.66} \ B_0^{-1.11}$$  (3)

We can calculate the major design parameters, $B_0, \ R_p, \ \gamma, \ P_f$, based on the three equations (1), (2), (3).

The cross points of the three equations on the $B_0$-$R_p$ plane.

Figure 2 shows a cross point of those three equations, with the common assumptions of $\gamma=1.2, \ P_f=4GW, \ a_n=0.5, \ a_T=1, \ j=26 \ A/mm^2$, and with the constant key parameters in each equation, $H_f=1.09$ in (3), $\beta=5\%$ in (5) and $\Delta d=1.1m$ in (4).
Guidelines for analysis on the Design Windows

1) Logical results
   →from basic equations with the clear preconditions

2) Self consistent
   →with comprehensive view of plasma, magnets and reactor

3) Reasonable and affordable
   →Don’t design on the cliff
   →To open the design windows wide

→ Searching design windows logically with clear conditions
Major design parameters and calculation flows

- **[Plasma size, shape]**
  - \( a_p = f_0(R_p, \gamma) \)
  - \( V_p = f_0(R_p, \gamma) \)

- **[Pf, power balance, B_0]**
  - \( P_f = f_2(\beta, B_0, R_p, \gamma) \)
  - \( B_0 = f_2'(R_p, \gamma) \) Eq. 2

- **[Reacto system]**
  - \( \Delta d = f_1(R_p, \gamma, B_0, j) \) Eq. 1
  - \( \Delta d = f_1(R_p, \gamma, B_0, j) \)

**Equations**:

- Eq. 1: \( \Delta d = f_1(R_p, \gamma, B_0, j) \)
- Eq. 2: \( B_0 = f_2'(R_p, \gamma) \)
- Eq. 3: \( H_f = \frac{\tau_{Er}}{\tau_{EISS04}} = f_3(R_p, \gamma, B_0, P_f, n_e) \)

**Identifying Critical Parameters**

- \(<a_p>\)
- \(<a_{pin}>\)
- \(<\iota>\)

**Ergodic layers**

- \( \tau_{Er} = W_p / (0.2f_\alpha P_f - P_{loss}) \)

**Additional Information**

- Eq. of regression from LHD data
- Optimizing the coil configurations and the current.

**Diagram Notes**

- LCFS
- Ergodic layers
The minimum $R_p$ is given with $\Delta d$ constraints for each $\gamma$

- How to get design points of 4GW-\(\beta\) 5% plants with $\Delta d1.1m$-

![Diagram showing $B_0$-$R_p$ relationships depending on $\gamma$ and $\Delta d$.](image)

Fig.2-3 The $B_0$-$R_p$ relationships depending on $\gamma$ and $\Delta d$ [The minimum $R_p$ line is given by the cross points of the Pf=4GW lines and $\Delta d=1.1m$ lines for each $\gamma$.]
Design windows limited by the constraints of $\Delta d$ and $H$ factor

1) $\Delta d$ constraints give the lower boundary of $R_p$ (increasing $\gamma$, $a_p \rightarrow$ enlarging $R_p$)
2) $H$ factor constraints give the lower boundary of $B_0$ (larger $R_p$, $\gamma \rightarrow$ decreasing $B_0$)
3) The constraints of magnetic stored energy $W$ give the upper bounds of $B_0$

$\Delta d \geq 1.1 m, \ H_f \leq 1.16, \ W < 160 G$

Fig. 4: The design windows limited with $\Delta d \geq 1.1 m, \ H_f \leq 1.16, \ W < 160 G$, depending on $\gamma$ ($\beta$ 5%, Pf 4GW). $H_f = 1.16$ means the enhancement factor of 1.2 to LHD experiment [1]. $j = 26 A/mm^2$ precondition.
The heliotron reactor design windows depending on $\gamma$ and $\beta$

The design windows on $R_p$-$B_0$ plane limited with $H_f < 1.15$ and $W < 160$ GJ ($\Delta d = 1.1m$).

The low $\beta$ ($\sim 3\%$) conditions severely limit the fusion power less than $P_f = 2$ GW. In the high $\beta$ ($\sim 5\%$) conditions the large fusion power ($\sim 4$ GW) plants are not limited by $W$, but $H$ factor constraints restrict the small fusion power plants.
The heliotron reactor design windows strongly depending on $\beta$

The design windows clearly shown on $R_p$-$W$ plane limited with $H_f<1.15$ and $W<160$ GJ.

Fig. 5-2 The design windows of 2~4GW fusion power plants limited with the constraints of $\Delta d=1.1$m, $H_f\leq 1.15$, $W<160$ GJ. The $\gamma$ dependence are shown with the four points, $\gamma=1.15, 1.18, 1.20, 1.25$ on each line.

$\beta 3\% \rightarrow$ only $P_f=2$GW in $W\sim 160$GJ

$\beta 4\% \rightarrow B_0=4.5\sim 6$T $\rightarrow P_f=3\sim 4$GW, although $W=140\sim 150$GJ

$\beta 5\% \rightarrow$ We can consider the optimum design windows of $P_f=3.3\sim 4$GW plants with $R_p=14.6\sim 16.3$m, $B_0=4.4\sim 5.5$T, and $W=125\sim 140$GJ
<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Symbol (unit)</th>
<th>4GW standard plants $\beta=5%$, Hf=1.06-1.15</th>
<th>3GW Hf=1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil pitch parameter</td>
<td>$\gamma$</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>Coil major Radius</td>
<td>$R_c$ (m)</td>
<td>15.91</td>
<td>16.70</td>
</tr>
<tr>
<td>Plasma major radius</td>
<td>$R_p$ (m)</td>
<td>14.69</td>
<td>15.42</td>
</tr>
<tr>
<td>Plasma radius</td>
<td>$a_p$ (m)</td>
<td>1.78</td>
<td>2.27</td>
</tr>
<tr>
<td>Inner plasma radius</td>
<td>$a_{pin}$ (m)</td>
<td>0.78</td>
<td>1.09</td>
</tr>
<tr>
<td>Plasma volume</td>
<td>$V_p$ (m$^3$)</td>
<td>916</td>
<td>1565</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>$B_0$ (T)</td>
<td>5.74</td>
<td>5.02</td>
</tr>
<tr>
<td>Average beta</td>
<td>$\beta$</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Fusion power</td>
<td>$P_f$ (GW)</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>H factor to ISS04</td>
<td>$H_f$</td>
<td>1.064</td>
<td>1.094</td>
</tr>
<tr>
<td>Maximum field on coils</td>
<td>$B_{max}$ (T)</td>
<td>12.16</td>
<td>11.91</td>
</tr>
<tr>
<td>Coil current</td>
<td>$I_{HC}$ (MA)</td>
<td>42.18</td>
<td>38.67</td>
</tr>
<tr>
<td>Coil current density</td>
<td>$j$ (A/mm$^2$)</td>
<td>26.0</td>
<td>26.0</td>
</tr>
<tr>
<td>Helical Coil height</td>
<td>$H$ (m)</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>Blanket space</td>
<td>$\Lambda d$ (m)</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>Neutron wall loads</td>
<td>$f_n$ (MW/m$^3$)</td>
<td>2.9</td>
<td>2.2</td>
</tr>
<tr>
<td>Magnetic stored energy</td>
<td>$W$ (GJ)</td>
<td>144</td>
<td>131</td>
</tr>
</tbody>
</table>

*Effective ion charge $Z_{eff}=1.32$, **Alpha heating efficiency 0.9, and parabola profile index $a_n=0.5,a_r=1.0.$
Magnets Weight and Cost of Tokamak and Helical Reactor

<table>
<thead>
<tr>
<th></th>
<th>Weight (ton)</th>
<th>Cost (B yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Helical Coil</strong></td>
<td>15.7 k ton</td>
<td>210 B yen</td>
</tr>
<tr>
<td><strong>Poloidal Coil</strong></td>
<td>~ 1.6 times</td>
<td>~ 2 times</td>
</tr>
<tr>
<td><strong>Supps. &amp; Others</strong></td>
<td>11.2 k ton</td>
<td>110 B yen</td>
</tr>
<tr>
<td><strong>ITER TF</strong></td>
<td>50 GJ</td>
<td>10 k ton</td>
</tr>
<tr>
<td><strong>ITER CS</strong></td>
<td>140 GJ</td>
<td>11.2 k ton</td>
</tr>
<tr>
<td><strong>ITER PF</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Supps. &amp; Others</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FDR TF</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FDR CS</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FDR PF</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Supps. &amp; Others</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*estimated with the first version

**Tokamak Reactor**

<table>
<thead>
<tr>
<th></th>
<th>Weight (ton)</th>
<th>Cost (B yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ITER</strong></td>
<td>135 GJ</td>
<td>210 B yen</td>
</tr>
<tr>
<td><strong>SSTR</strong></td>
<td>50 GJ</td>
<td>110 B yen</td>
</tr>
</tbody>
</table>

2.5 times (ITER) ~ 2 times
## Table 2 The Cost Comparison of Helical Power Plants (Fusion Power 3~4 GW)

<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Symbol (unit)</th>
<th>4GW standard plants $\beta=5%$, Hf=1.06-1.15</th>
<th>3GW Hf=1.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil pitch parameter</td>
<td>$\gamma$</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>Fusion power</td>
<td>Pf (GW)</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>Weight of Blanket and shield</td>
<td>Mbs (ton)</td>
<td>8580</td>
<td>11360</td>
</tr>
<tr>
<td>Magnetic stored energy</td>
<td>W (GJ)</td>
<td>144</td>
<td>131</td>
</tr>
<tr>
<td>Weight of magnet s</td>
<td>Mmag (ton)</td>
<td>18000</td>
<td>16400</td>
</tr>
<tr>
<td>Magnet cost (%)***</td>
<td>Cmag (M$)</td>
<td>2079(34.6)</td>
<td>1893(31.0)</td>
</tr>
<tr>
<td>Blanket and shield cost (%)***</td>
<td>Cbs (M$)</td>
<td>889(14.8)</td>
<td>1177(19.3)</td>
</tr>
<tr>
<td>Total construction cost</td>
<td>C total (M$)</td>
<td>7270</td>
<td>7393</td>
</tr>
<tr>
<td>Net electric power</td>
<td>Pn (GW)</td>
<td>1604</td>
<td>1601</td>
</tr>
<tr>
<td>Total auxiliary power</td>
<td>Pa (GW)</td>
<td>109</td>
<td>112</td>
</tr>
<tr>
<td>Plant availability factor</td>
<td>$f_A$</td>
<td>0.680</td>
<td>0.706</td>
</tr>
<tr>
<td>Capital cost</td>
<td>mill/kWh</td>
<td>44.0</td>
<td>43.2</td>
</tr>
<tr>
<td>Operation cost</td>
<td>mill/kWh</td>
<td>26.8</td>
<td>27.1</td>
</tr>
<tr>
<td>Replacement cost</td>
<td>mill/kWh</td>
<td>8.18</td>
<td>8.19</td>
</tr>
<tr>
<td>Fuel cost</td>
<td>mill/kWh</td>
<td>0.023</td>
<td>0.022</td>
</tr>
<tr>
<td>COE(Cost of electricity)</td>
<td>mill/kWh</td>
<td>79.0</td>
<td>78.5</td>
</tr>
</tbody>
</table>

# The major assumption for calculating COE are FCR (Fixed charge rate); 5.78%, with 40 years plant life time and 3% discount rate, the ratio of operation and maintenance cost to construction cost; 4.5 % for the conventional components, but 1.5% for magnets considering the inherent characteristics of super conducting magnets.

### Availability factor is calculate as a function of neutron wall load.
The magnet cost, blanket-shield cost, and the COE

1) With increasing $R_p$ and $\gamma$, the blanket-shield cost increases but the magnet cost decreases, as $B_0$ decreases much with increasing $\gamma$ ($\sim V_p$).

2) The COE decreases strongly with increasing $\beta$ from 3% to 5%.

3) We should select the size ($R_p$, $\gamma$) and $B_0$ with considering the trade-off between magnet cost and blanket cost, and also plant availability.

Fig. 2. The $B_0$, magnet cost (Cmag), and blanket Cost (Cbs) depend on $R_p$, $\gamma$ and $\beta$. When $R_p$ and $\gamma$ increase, Cmag decreases but Cbs increases. Those plots on $R_p$ ($\gamma$) are given with $\Delta d=1.1$m. The COEs of helical reactors, which depend on $R_p$, $\gamma$ and $\beta$, show the bottom as the result of the trade-off between the Cmag and Cbs, i.e., $B_0$ versus plasma volume.
Conclusions

1) LHD-type helical reactors have the attractive design windows in rather large size of $R_p = 15\sim 16\text{m}$, with the sufficient blanket space and the reasonable magnetic stored energy of 120~140 GJ based on the physics basis of $H_f\sim 1.1$ and $\beta\sim 5\%$.

2) The $\beta$ dependence is very important for selecting the optimum fusion power with reasonable magnetic stored energy, so that the confirming good confinement in the near $\beta\sim 5\%$ plasma is the first priority of critical issues.

3) The $\gamma$ dependence is essential in Heliotron reactors that is critically sensitive not only for optimizing LCFS (plasma volume) but for selecting the optimum blanket design.

4) There are many remaining subject to be studied, in especially, the problem of the particle and heat loads on the diverter is a critical issue to be considered in the next analysis.
The roadmap for fusion energy

Fusion Power Plant

H plasma for physics

ITER

2025 Y

Diverter Blanket

2025 Y

Pre DEMO Helical

2015 Y

Pre DEMO Design

2040 Y

DEMO

2050 Y

SCM

2016 Y

JT60, JET, TFTR

LHD
Demonstrating the required $\beta \sim 5\%$ plasma in the next LHD experiment and the Pre-DEMO experiment is one of the most critical issues.

- The LCFS of LHD is very beautiful and it is also expected in the $\beta \sim 5\%$ heliotron reactor plasma.

- The LCFS volume could change largely depending on $\gamma$ and controlling poloidal coil currents, as shown by Tsuguhiro Watanabe. The LCFS plasma volume could be enlarged about 20% by optimizing the quadrupole component with removing the conditions of minimizing the leakage flux. It means the potential of decreasing magnetic stored energy from 130GJ to 110GJ.

- Even if in the high $\beta$ plasma largely affected with Shafranov shift, it is expected the high $\beta$ heliotron plasma could be achieved with selecting the $\gamma$ and controlling coil currents. That is critically sensitive not only for optimizing the LCFS but also for selecting the optimum blanket design.
Acknowledgements

I am much thankful to the circumstance of academic discussion of NIFS, and the collaborative work in the FFHR design group.