Orthonormal Divergece-free Wavelet Analysis of Energy Transfer in Hall-MHD Turbulence

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Orthonormal divergence-free wavelet analysis is applied to the energy transfer process in Hall-MHD turbulence. The incompressible Hall-MHD equations are given by

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla P + \boldsymbol{j} \times \boldsymbol{b} + \nu \nabla^2 \boldsymbol{u}, \tag{1}$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times ((\boldsymbol{u} - \epsilon \boldsymbol{j}) \times \boldsymbol{b}) + \eta \nabla^2 \boldsymbol{b}, \tag{2}$$

where $\boldsymbol{u}$ is the bulk velocity field and satisfies $\nabla \cdot \boldsymbol{u} = 0$, $\boldsymbol{b}$ is the magnetic field, $\boldsymbol{j} := \nabla \times \boldsymbol{b}$ is the current density field, P is the total pressure, ν is the kinematic viscosity, η is the resistivity, and ϵ is the parameter for relative strength of the Hall term. The velocity field and the magnetic field are expanded into the orthonormal divergence-free wavelet modes[1]: $\boldsymbol{f}(\vec{x}, t) = \sum f_{j\epsilon\vec{l}\sigma}(t)\boldsymbol{\psi}_{j\epsilon\vec{l}\sigma}(\vec{x})$, where $\boldsymbol{f}$ stands for $\boldsymbol{u}$ or $\boldsymbol{n}$ and j, ϵ, $\vec{l}$, σ are the indices for spatial scale, anisotropy in the wavenumber space, position of wavelet, and helicity of wavelet, respectively. We analysed scale-to-scale wavelet energy transfers gievn by

$$\langle \boldsymbol{u}_i | \boldsymbol{u}_j | \boldsymbol{u}_k \rangle = -\int \boldsymbol{u}_i(\vec{x}, t) \cdot ((\boldsymbol{u}_j(\vec{x}, t) \cdot \nabla)\boldsymbol{u}_k(\vec{x}, t)) d^3\vec{x}, \tag{3}$$

$$\langle \boldsymbol{u}_i | \boldsymbol{b}_j | \boldsymbol{b}_k \rangle = -\int \boldsymbol{u}_i(\vec{x}, t) \cdot (\boldsymbol{j}_k(\vec{x}, t) \times \boldsymbol{b}_j(\vec{x}, t)) d^3\vec{x}, \tag{4}$$

$$\langle \boldsymbol{b}_k | \boldsymbol{b}_j | \boldsymbol{u}_i \rangle = -\int \boldsymbol{b}_k(\vec{x}, t) \cdot (\nabla \times (\boldsymbol{u}_k(\vec{x}, t) \times \boldsymbol{b}_j(\vec{x}, t))) d^3\vec{x}, \tag{5}$$

$$\langle \boldsymbol{b}_k | \boldsymbol{b}_j | \boldsymbol{b}_i \rangle = -\int \boldsymbol{b}_k(\vec{x}, t) \cdot (\nabla \times (\boldsymbol{j}_i(\vec{x}, t) \times \boldsymbol{b}_j(\vec{x}, t))) d^3\vec{x}, \tag{6}$$

where $\boldsymbol{f}_j(\vec{x}, t) = \sum_{\epsilon,\vec{l},\sigma} f_{j\epsilon\vec{l}\sigma}(t)\boldsymbol{\psi}_{j\epsilon\vec{l}\sigma}(\vec{x})$. Effect of the Hall terms to the redistribution of magnetic energy between scales is discussed.

References

[1] Kishida et al., PRL **83** (1999) 5487