Constrained Interpolation Profile Method Applied to solving Magnetohydrodynamics Equations in terms of Vector Potential

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It is one of the key issues for realization of the nuclear fusion reactor to investigate Magneto-Hydro-Dynamics (MHD) instability or equilibrium. The fully three dimensional (3D) MHD studies have been done by solving the MHD equations[1][2]. Among these studies, however, there exist many studies in which the non-monopole condition ($\nabla \cdot \boldsymbol{B} = 0$) is not guaranteed numerically. It is pointed out that even small errors in satisfying the non-monopole condition cause large errors in the solution of the MHD equations[3].

In this paper, we express the induction equation in terms of the vector potential $\boldsymbol{A}$ to satisfy the non-monopole condition. The time evolution of the vector potential instead of the magnetic field is calculated by solving the following equation:

$$\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{V} \cdot \nabla \boldsymbol{A} = (\nabla \boldsymbol{A}) \cdot \boldsymbol{V} - \nabla \phi - \eta \boldsymbol{J}, \tag{1}$$

where ϕ is the scalar potential.

The MHD equations are composed of the hydrodynamic and the electromagnetic equations. All these equations include the advection term when the induction equation is reduced to Eq.(1). The Constrained Interpolation Profile (CIP) method[4] is suitable for numerically solving the equations with advection term. This method gives high accuracy and is applicable to curvilinear coordinates[5]. So, we adopt the CIP method to solve the MHD equations.

Based on these numerical models, we have developed a new 3D MHD simulation code and applied some test MHD problems. The details of the developed code will be shown. The results of the test problems will be discussed.

[1] H. Miura, T. Hayashi and T. Sato, Phys. Plasmas, **8**, 4870 (2001).
[2] N. Mizuguchi, T. Hayashi and T. Sato, Phys. Plasmas, **7**, 940 (2000).
[3] J. U. Brackbill and D. C. Barnes, J. Comput. Phys., **35**, 426 (1980).
[4] H. Takewaki, A. Nishiguchi and T. Yabe, J. Comput. Phys., **61**, 261 (1985).
[5] P. Y. Wang, T. Yabe, and T. Aoki, J. Phys. Soc. Japan, **62**, 1865 (1993).