Quantum Mechanical Plasma Scattering.

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We have shown the possibility of quantum mechanical plasma diffusion due to the uncertainty [1,2]. But it is necessary to solve the unsteady Schrödinger equation in order to analyze a real particle's motion. So we will numerically solve the Schrödinger equation.

The unsteady Schrödinger equation for wavefunction $\psi(\boldsymbol{r}, t)$ is given by

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\frac{1}{2m} \left(-i\hbar \nabla \right)^2 + U \right) \psi, \tag{1}$$

where $U = U(\boldsymbol{r})$ stands for the potential energy. We give $U = 0$, i.e. free particle, and the initial wave function as

$$\psi(\boldsymbol{r}, 0) = \frac{1}{\sqrt{\pi}\sigma_0} \exp\left(-\frac{(\boldsymbol{r} - \boldsymbol{r}_0)^2}{2\sigma_0^2} + i\boldsymbol{k}_0 \cdot \boldsymbol{r} \right), \tag{2}$$

where $\boldsymbol{r}_0$ is the initial center of ψ, σ_0 is the initial standard deviation and $\boldsymbol{k}_0$ is the initial wave vector, $\boldsymbol{k}_0 = m\boldsymbol{v}_0/\hbar$. Here, $\boldsymbol{v}_0$ is the initial velocity directed to $+y$. A great deal of the memory size and CPU time will be required in order to solve the Schrödinger equation for a plasma particle, because the de Broglie wavelength decreases inversely proportional to $\boldsymbol{v}_0$. Therefore, $\boldsymbol{v}_0$ is assumed to $v_0 = 10$ m/s. Figure 1 shows the time-dependent standard deviation $\sigma = \sigma(t)$, which increases by a factor of 3.4 during a time interval of $\Delta t = \Delta\ell/v_0 = n^{-1/3}/v_0$ with $n = 10^{20}$ m^{-3}. The calculated $\sigma(t)$ is in good agreement with the theoretical value given by

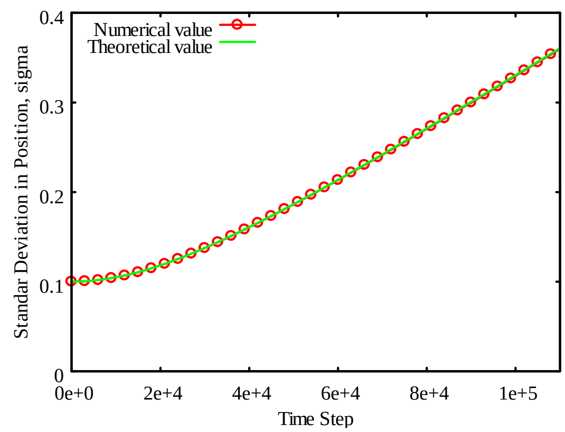

Figure 1: Standard deviation, σ, in position vs time t.

$$\sigma(t) = \sqrt{1 + \left(\frac{\hbar t}{\mu \sigma_0^2} \right)^2}\, \sigma_0. \tag{3}$$

Here, we have considered a hydrogen ion in a fusion plasma which has the density of $n = 10^{20}$ m^{-3} and a magnetic flux density $B = 3\,$T. The magnetic length $\ell_{\rm B} \equiv \sqrt{\hbar/eB} \sim \sigma_0$ is around one-tenth of the average interparticle separation $\Delta\ell = n^{-1/3}$, where $\ell_{\rm B}$ stands for the spread of a wave function in a direction perpendicular to magnetic field. In such a condition, the wave function expands to a diameter of $\Delta\ell$ in a 10^{-7}s. The analysis of faster particle and the Coulomb potential $U \propto r^{-1}$ will be shown at the conference.

[1] S. Oikawa, T. Oiwa, and T. Shimazaki, to be published in Plasma Fusion Res. **4** (2009).

[2] T. Shimazaki, S. Oikawa, and T. Oiwa, *this conference*.