Preliminary Study on Uncertainty-Driven Plasma Diffusion II.

T. Shimazaki, S. Oikawa and T. Oiwa

Graduate school of Engineering, Hokkaido University, N-13, W-8, Sapporo 060-8628, Japan

sima143@fusion.eng.qe.hokudai.ac.jp

In quantum mechanics [1,2], the *size* of a charge q in the presence of a magnetic field $\boldsymbol{B}$, becomes the magnetic length $\ell_\mathrm{B} = \sqrt{\hbar/qB}$, where $\hbar = h/2\pi$, stands for Dirac constant. In typical fusion plasma with a temperature T and a density n, ℓ_B is as large as one tenth of the inter-particle separation $\Delta\ell \equiv n^{-1/3}$, which is considerably longer than the typical de Broglie wavelength, $\lambda \approx h/\sqrt{2mT}$. Here we will adopt an alternative method as described in what follows. Let us assume that the initial wave function of a *field* particle is Gaussian with the center at the origin:

$$f(\boldsymbol{r}) = \frac{1}{\pi^{3/2}\ell_\mathrm{B}^3}\exp\left(-\frac{r^2}{\ell_\mathrm{B}^2}\right). \tag{1}$$

If the *test* particle with the same charge q as the field particle has the similar distribution as Eq.(1), the probability $\mathrm{d}P(\boldsymbol{r})$ of finding the test particle within an infinitesimal volume $\mathrm{d}^3\boldsymbol{r}$ around a position $\boldsymbol{r}$ is given as

$$\mathrm{d}P(\boldsymbol{r}) = \frac{1}{\pi^{3/2}\ell_\mathrm{B}^3}\exp\left(-\frac{(\boldsymbol{r}-\boldsymbol{r}_0)^2}{\ell_\mathrm{B}^2}\right)\mathrm{d}^3\boldsymbol{r}. \tag{2}$$

The Coulomb potential energy in this case is given by

$$U(\boldsymbol{r}) = \frac{q^2}{4\pi\epsilon_0 r}\mathrm{erf}\left(\frac{r}{\ell_\mathrm{B}}\right), \tag{3}$$

We solve a set of classical equations of motion, in which the test particle q for several initial positions at $\boldsymbol{r} = \boldsymbol{r}_0$ with a velocity $\boldsymbol{v} = \boldsymbol{v}(0)$ in the presence of the potential field given by Eq. (3). For each initial position, Eq. (2) is used to mimic the quantum mechanical distribution of the test particle in order to calculate particle scattering in the plasma. For simplicity, initial speed is fixed to be the thermal velocity v_th and initial positions are restricted within the sphere of a radius $3\ell_\mathrm{B}$ centered at the initial position $\boldsymbol{r} = \boldsymbol{r}_0$. The test particle moves during $\Delta t = 2\Delta\ell/g_\mathrm{th}$ in classical mechanics. In the above calculation, we have ignored the effect of magnetic field $\boldsymbol{B}$, because $\Delta t \approx 10^{-13}$sec is much shorter than the cyclotron period of the order of 10^{-8} sec for protons in a plasma with $n = 10^{20}\mathrm{m}^{-3}$ and $T = 10$ keV. The variance of velocity change obtained in this study is

$$\left\langle(\Delta v)^2\right\rangle \approx 2.3\times10^{-9}v_\mathrm{th}^2. \tag{4}$$

The corresponding variance in classical mechanics is given by

$$\left\langle(\Delta v)^2\right\rangle = 4\pi b_0^2\ln\Lambda \approx 2.3\times10^{-11}v_\mathrm{th}^2, \tag{5}$$

where $b_0 = q^2/4\pi\epsilon_0\mu v_\mathrm{th}^2$ and $\ln\Lambda \approx 17$ are the impact parameter for $\pi/2$ scattering and the Coulomb logarithm. Thus, the calculated variance is one hundred times larger than this classical variation.

[1] L. D. Landau and E. M. Lifshitz, *Quantum mechanics: non-relativistic theory*, 3rd ed., translated from the Russian by J. B. Sykes and J. S. Bell (Pergamon Press, Oxford, 1977).
[2] S. Oikawa, T. Oiwa, and T. Shimazaki, to be published in Plasma Fusion Res. **4** (2009).