Application of Two Dimensional Extended Boundary Node Method to Potential Problem

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As is well known, the boundary-element method (BEM) is a powerful method for solving a potential problem. It has been widely used in the field of the nuclear fusion science and has yielded excellent results. However, the BNM has the inherent demerit. Before executing a BEM code, a boundary must be divided into a set of elements.

In order to resolve the above demerit, Mukherjee *et al.* have proposed the boundary-node method (BNM) [1]. Since the BNM is one of the meshless approaches, a preparation of input data can be extremely simplified. However, in the conventional BNM, integration cells must be employed for evaluating the influence coefficients. In other words, a concept of elements partly remains in the BNM.

In the field of computer graphics, the novel method has been recently proposed for representing the object surface [2]. In the method, the object surface has been represented in terms of an implicit function. If the above method were applied to the BNM, the demerit of the BNM could be completely resolved.

The purpose of the present study is to reformulate the BNM without using any integration cells and to numerically investigate the performance of the proposed method. Throughout the present study, the proposed method is called the eXtended Boundary-Node Method (X-BNM).

In the X-BNM, influence coefficients are directly calculated by using the vector equation of the boundary. Here, let us briefly explain about the method for determining the vector equation. First, the function $f(\boldsymbol{x})$ is determined so that the curve $f(\boldsymbol{x}) = 0$ may pass through all nodes on the boundary. Next, we numerically solve the following ordinary differential equation:

$$\frac{d\boldsymbol{x}}{ds} = \boldsymbol{R}\left(\frac{\pi}{2}\right) \cdot \frac{\nabla f}{|\nabla f|}, \tag{1}$$

where s and $\boldsymbol{R}(\theta)$ denotes a arclength along the boundary and a tensor representing a rotation through an angle θ, respectively. As a result, the vector equation $\boldsymbol{x} = \boldsymbol{x}(s)$ is determined numerically.

As a example of the potential problem, a 2D Poisson problem is adopted and its analytic solution is chosen as $u = 3e^{-(x^2+y^2)} - \cosh x \sin y + \cos x \sinh y$. In addition, the boundary $\partial\Omega$ is assumed as $\partial\Omega = \{\boldsymbol{x} \in \boldsymbol{R}^2 | f(\boldsymbol{x}) = x^2 + (y/2)^2 - 1 = 0\}$.

The performance of the X-BNM is numerically investigated by comparing with the dual-reciprocal boundary-element method (DRM). The results of computations show that the accuracy of the X-BNM is much higher than that of the DRM. In addition, it is found that the speed of the X-BNM is almost equal to that of the DRM if the number of boundary nodes exceeds a certain limit.

From the above results, we might conclude that the X-BNM is a powerful method for solving the potential problem.

[1] Y.X. Mukherjee, S. Mukherjee, *Int. J. Numer. Methods Eng.* **40** (1997), 797
[2] G. Turk, J.F. O'Brien, *Proc. ACM SIGGRAPH 99* (1999) 335